

Algorithms (COT 6405): Solutions 3

Problem 1

Write an algorithm that combines INSERTION-SORT and MERGE-SORT.

The following algorithm calls INSERTION-SORT for array segments whose length is at most k ; the running time of this algorithm is $\Theta(n \cdot k + n \cdot \lg(n/k))$.

```
INSERTION-SORT( $A, p, r$ )
for  $j \leftarrow p + 1$  to  $r$ 
    do  $key \leftarrow A[j]$ 
         $i \leftarrow j - 1$ 
        while  $i \geq p$  and  $A[i] > key$ 
            do  $A[i + 1] \leftarrow A[i]$ 
                 $i \leftarrow i - 1$ 
         $A[i + 1] \leftarrow key$ 

COMBINED-SORT( $A, p, r, k$ )
if  $r - p < k$ 
    then INSERTION-SORT( $A, p, r$ )
    else  $q \leftarrow \lfloor (p + r)/2 \rfloor$ 
        COMBINED-SORT( $A, p, q, k$ )
        COMBINED-SORT( $A, q + 1, r, k$ )
        MERGE( $A, p, q, r$ )
```

Problem 2

Write an algorithm that inputs an integer array $A[1..n]$ and an odd integer number k , and determines whether k can be represented as the sum of two elements of the array.

The following algorithm uses two subroutines, MERGE-SORT (described in class) and BINARY-SEARCH (described in Assignment 2); the running time of this algorithm is $O(n \cdot \lg n)$.

```
SUM-SEARCH( $A, n, k$ )
MERGE-SORT( $A, 1, n$ )  $\triangleright$  sort the array  $A[1..n]$ 
for  $i \leftarrow 2$  to  $n$ 
    do if BINARY-SEARCH( $A, n, k - A[i]$ )  $\neq 0$ 
        then return TRUE  $\triangleright k$  is the sum of two elements of  $A[1..n]$ 
return FALSE  $\triangleright k$  is not the sum of any two elements
```