

Algorithms: Solutions 5

Problem 1

Write an algorithm that randomly re-orders an array $A[1..n]$.

```
SHUFFLE( $A, n$ )  
for  $i \leftarrow n$  downto 2  
    do  $j \leftarrow \text{RANDOM}(1, i)$   
        exchange  $A[i] \leftrightarrow A[j]$ 
```

Problem 2

Consider a programming language that allows only integer numbers and supports addition, subtraction, and multiplication. Write an algorithm $\text{DIVIDE}(n, m)$ that computes $\lfloor n/m \rfloor$, where n and m are positive integers.

Simple algorithm

The following computation takes $\Theta(\lceil n/m \rceil)$ time and $\Theta(1)$ space.

```
SLOW-DIVIDE( $n, m$ )  
 $ratio \leftarrow 1$   
while  $ratio \cdot m \leq n$   
    do  $ratio \leftarrow ratio + 1$   
return  $ratio - 1$ 
```

Fast algorithm

The following recursive algorithm runs in $\Theta(\lg \lceil n/m \rceil)$ time. The space complexity is not constant; the algorithm requires $\Theta(\lg \lceil n/m \rceil)$ memory for the stack of recursive calls.

```
FAST-DIVIDE( $n, m$ )  
if  $n < m$   
    then return 0  
 $ratio \leftarrow 2 \cdot \text{FAST-DIVIDE}(n, 2 \cdot m)$   
if  $n - ratio \cdot m < m$   
    then return  $ratio$   
    else return  $ratio + 1$ 
```