

Algorithms: Solutions 7

Problem 1

Write an algorithm for finding the k th smallest element of a given array.

The following algorithm uses the same PARTITION procedure as QUICK-SORT. The average-case time of this algorithm is $\Theta(n)$, but the worst case is $\Theta(n^2)$.

```
SELECT( $A, n, k$ )
 $p \leftarrow 1$ 
 $r \leftarrow n$ 
while  $p < r$ 
    do  $q = \text{PARTITION}(A, p, r)$ 
        if  $k \leq q$ 
            then  $r \leftarrow q$ 
            else  $p \leftarrow q + 1$ 
return  $A[k]$ 
```

You may find a more sophisticated algorithm in Section 9.3 of the textbook; the worst-case time of the textbook algorithm is $\Theta(n)$.

Problem 2

Suppose that $A[1..n]$ is an array of integer numbers, and some value k occurs at least $\lfloor n/2 \rfloor + 1$ times in this array. Write an efficient algorithm for finding this value.

The “frequent” element is the median of the array; that is, it is the $\lfloor n/2 \rfloor$ -th smallest element. We can find it using the SELECT algorithm from Problem 1, with $k = \lfloor n/2 \rfloor$.