Algorithms: Solutions 6

```
                X
                X
                X
                X
                X
                X
                X
                X
                X
                X
                X
                X
                X
                X
                X
                X
                X
                X
                X
number of       X           X
homeworks       X           X
                X           X
                X           X
          X  X  X           X
          X  X  X  X        X
          X  X  X  X        X
    X  X  X  X  X  X  X     X
    --------------------------
    2  3  4  5  6  7  8  9  10
              grades
```

Problem 1
Suppose that we apply RB-INSERT to add a node to a red-black tree, and then immediately call RB-DELETE to remove this node. Can the resulting tree differ from the initial tree?

We use the example in Figure 14.4 (page 269) of the textbook to demonstrate that the new tree may differ from the original. The example shows the insertion of a node with value 4 into a red-black tree. If we then delete the value 4, we obtain a tree that differs from the initial tree. Note that, in this case, the deletion does not involve color changes or rotations.

Problem 3
Determine an asymptotic bound for the running time of the following algorithm:

```
NUMBER-TYPER(n)
if n ≤ 1
    return
for i = 1 to n
    do print i
for i = 1 to n − 1
    do NUMBER-TYPER(i)
```

The recurrence for the running time is

$$T(n) = \sum_{i=1}^{n-1} T(i) + \Theta(n),$$

and we unwind it as follows:

$$\begin{aligned}
T(n) &= \sum_{i=1}^{n-1} T(i) + c \cdot n \\
&= \sum_{i=1}^{n-2} T(i) + T(n-1) + c \cdot n \\
&= \sum_{i=1}^{n-2} T(i) + \sum_{i=1}^{n-2} T(i) + c \cdot (n-1) + c \cdot n \\
&= 2 \cdot \sum_{i=1}^{n-2} T(i) + c \cdot (n-1) + c \cdot n \\
&= 2^2 \cdot \sum_{i=1}^{n-3} T(i) + 2c \cdot (n-2) + c \cdot (n-1) + c \cdot n \\
&= 2^3 \cdot \sum_{i=1}^{n-4} T(i) + 2^2 c \cdot (n-3) + 2c \cdot (n-2) + c \cdot (n-1) + c \cdot n \\
&\quad \vdots \\
&= \sum_{i=1}^{n-1} 2^{n-i-1} c \cdot i + c \cdot n \\
&= \sum_{i=1}^{n-1} 2^{n-i-1} c + \sum_{i=2}^{n-1} 2^{n-i-1} c + \sum_{i=3}^{n-1} 2^{n-i-1} c + \ldots + \sum_{i=n-1}^{n-1} 2^{n-i-1} c + c \cdot n \\
&= \sum_{j=0}^{n-2} 2^j c + \sum_{j=0}^{n-3} 2^j c + \sum_{j=0}^{n-4} 2^j c + \ldots + \sum_{j=0}^{0} 2^j c + c \cdot n \\
&= (2^{n-1} - 1)c + (2^{n-2} - 1)c + (2^{n-3} - 1)c + \ldots + (2^1 - 1)c + c \cdot n \\
&= 2^{n-1}c + 2^{n-2}c + 2^{n-3}c + \ldots + 2^1 c - (n-1)c + c \cdot n \\
&= 2^{n-1}c + 2^{n-2}c + 2^{n-3}c + \ldots + 2^1 c + c \\
&= 2^n c - c \\
&= \Theta(2^n)
\end{aligned}$$