Algorithms: Assignment 4

Due date: September 28 (Thursday)

Problem 1 (3 points)
Let $A[1..n]$ be a *sorted* array of n distinct numbers. Write an efficient algorithm INDEX-SEARCH(A, n) that finds an index i such that $A[i] = i$. If the array does not have such an element, the algorithm should return 0.

Problem 2 (3 points)
Suppose that $A[1..n]$ is a heap and you need to change the value of its ith element. Give an efficient algorithm CHANGE-ELEMENT(A, i, n, k) that sets $A[i] \leftarrow k$ and updates the heap structure appropriately. Specify the running time of your algorithm.

Problem 3 (4 points)
A *d-ary heap* is like a binary heap, but instead of 2 children, nodes have d children.

(a) How would you represent a d-ary heap with n elements in an array? What are the expressions for determining the parent of a given element, PARENT(i), and a j-th child of a given element, CHILD(i, j), where $1 \le j \le d$?

(b) Write an efficient implementation of HEAPIFY and HEAP-INSERT for a d-ary heap, and give the running time of your algorithms in terms of n and d.

Problem 4 (bonus)
This problem is optional, and it allows you to get 2 bonus points toward your final grade for the course. You cannot submit this bonus problem after the deadline.

What is the height of a d-ary heap of n elements in terms of n and d? You need to give an *exact* expression for the height, without using the Θ-notation.