

Algorithms: Assignment 2

Due date: September 14 (Thursday)

Problem 1 (3 points)

Give an example of functions $f(n)$ and $g(n)$ that satisfy all of the following conditions:

$$f(n) = O(g(n))$$

$$f(n) \neq \Theta(g(n))$$

$$f(n) \neq o(g(n))$$

Problem 2 (4 points)

Give a precise mathematical proof of the following asymptotic bounds:

(a) $\sqrt{n} = o(n)$

(b) for any constant $a > 0$,
 $(n + 1)^a = \Theta(n^a)$

Problem 3 (3 points)

Prove the following transitivity property of asymptotic bounds:

$$\text{if } f(n) = \Theta(g(n)) \text{ and } g(n) = \Theta(h(n)), \text{ then } f(n) = \Theta(h(n)).$$

Problem 4 (bonus)

*This problem is optional, and it does not affect your grade for the homework; however, if you solve it, then you will get 2 bonus points toward your **final grade** for the course. You cannot submit this bonus problem after the deadline.*

Suppose that we have four algorithms, called A_0 , A_1 , A_2 , and A_3 , whose respective running times are n , n^2 , $\lg n$, and 2^n . If we use a certain old computer, then the maximal sizes of problems solvable in an hour by these algorithms are s_0 , s_1 , s_2 , and s_3 .

Suppose that we have replaced the old computer with a new one, which is k times faster. Now the maximal size of problems solvable in an hour by A_0 is $k \cdot s_0$. What are the maximal problem sizes for the other three algorithms, if we run them on the new computer?