

Competition Programming and Problem Solving (15-295)

Examples of interview questions

Fast math

The following three questions require a near-instant reply, with little thinking time.

- Can the sum of two prime numbers be a prime number?
- The arc in Figure 1 is centered in O, and the lengths of the two parts of the lower horizontal segment are 6 and 4; determine the length x of the diagonal.
- The cube in Figure 2 is cut into $5 \times 5 \times 5$ small cubes; determine how many of these small cubes are on the surface of the large cube.

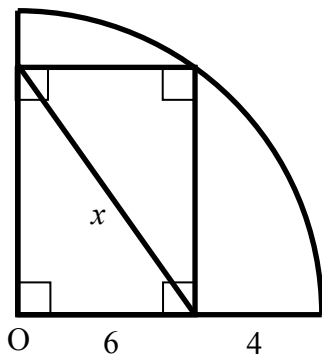


Figure 1

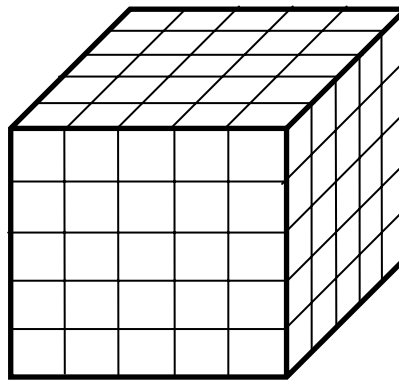


Figure 2

Probabilities

Three dice: Consider a game of chance in which you toss three six-sided dice. If one or more dice show “six,” you win; else, you lose. Determine your probability of winning.

Twenty balls: Consider a game that involves ten black balls and ten white balls. First, you distribute these twenty balls between two boxes; you have complete control over the placement of each ball. After you have placed the balls, your opponent *randomly* chooses one of the two boxes, without seeing their contents, and then *randomly* selects a ball from the chosen box. If the selected ball is white, you win; else, you lose. Describe the initial placement of the balls that maximizes your chances of winning.

Fifty-two cards: Consider a game that involves a randomly shuffled deck of fifty-two cards; twenty-six cards have a black suit, and the other twenty-six have a red suit. Your opponent shows you the top card of the deck, then the next card, and so on, one card at a time. At any point, you can say “I am betting on the next card,” and then check the suit of the next card in the deck. If it is red, you win; else, you lose. If you never say “I am betting...,” then your win or loss is determined by the suit of the last card in the deck. Describe your strategy in this game, and prove that there is no better strategy.

Planning

Dark bridge: Alice, Bob, Cathy, and David need to cross a narrow bridge at night. They are on the left side of the river, and they all need to reach its right side. They have one lantern, and they cannot walk without it. Furthermore, since the bridge is narrow, at most two people can walk at one time. Thus, after the first two people cross the bridge, one of them must return with the lantern; then, two more people can cross the bridge. In general, they will be crossing the bridge in groups of one or two, carrying their lantern back and forth. Different people walk with different speeds; Alice can cross the bridge in one minute, Bob takes two minutes, Cathy needs five minutes, and David spends ten minutes. When two people walk together, their speed is determined by the slower walker; for example, if Alice and Cathy cross the bridge together, they spend five minutes. Find a sequence of moves that enables these four people to cross the bridge in the minimal possible time.

Ropes: Suppose that you have five short ropes and a cigarette lighter. If you take any one of the five ropes and ignite it at (either) one end, the flame reaches the other end in exactly four minutes. The speed of the flame moving along the rope may not be constant; for example, it may reach the middle of the rope in less than two minutes, and then spend more than two minutes in the other half. Suppose further that you are cooking a complex dish, which requires exactly three minutes of cooking, but you do not have any timer except the ropes. How can you use these ropes to determine when to stop cooking?

Coins

Digital scales: Suppose that you have digital scales, which show the exact weight in grams, and you also have ten large piles of coins. You know that the coins in one of the piles are counterfeit, and each counterfeit coin weighs 9 grams, whereas the coins in the other nine piles weigh 10 grams each. Explain how to identify the counterfeit pile in one weighing, using the *minimal possible number* of coins for this weighing.

Easy balance scales: Suppose that you have balance scales and twenty-four coins, one of which is counterfeit, and you know that the counterfeit coin weighs less than the other coins. Explain how to identify the counterfeit coin in three weighings.

Hard balance scales: Suppose that you have balance scales and twelve coins, one of which is counterfeit. The weight of the counterfeit coin is different from the other coins, but you do not know whether it is lighter or heavier.

- Explain how to identify the counterfeit coin in four weighings.
- Determine whether it can be identified in three weighings, and justify your answer.

Estimation

Estimate the following values as accurately as you can:

- The weight of the planet Earth.
- The cost of a brand-new Boeing 737.
- The number of live cows in the United States.
- The total wealth of the humankind; that is, the total value of all money and property owned by all people.