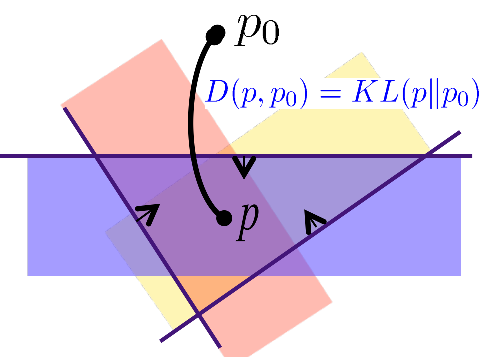
School of Computer Science
Carnegie Mellon

Probabilistic Graphical Models

Posterior Regularization: an integrative paradigm for learning GMs

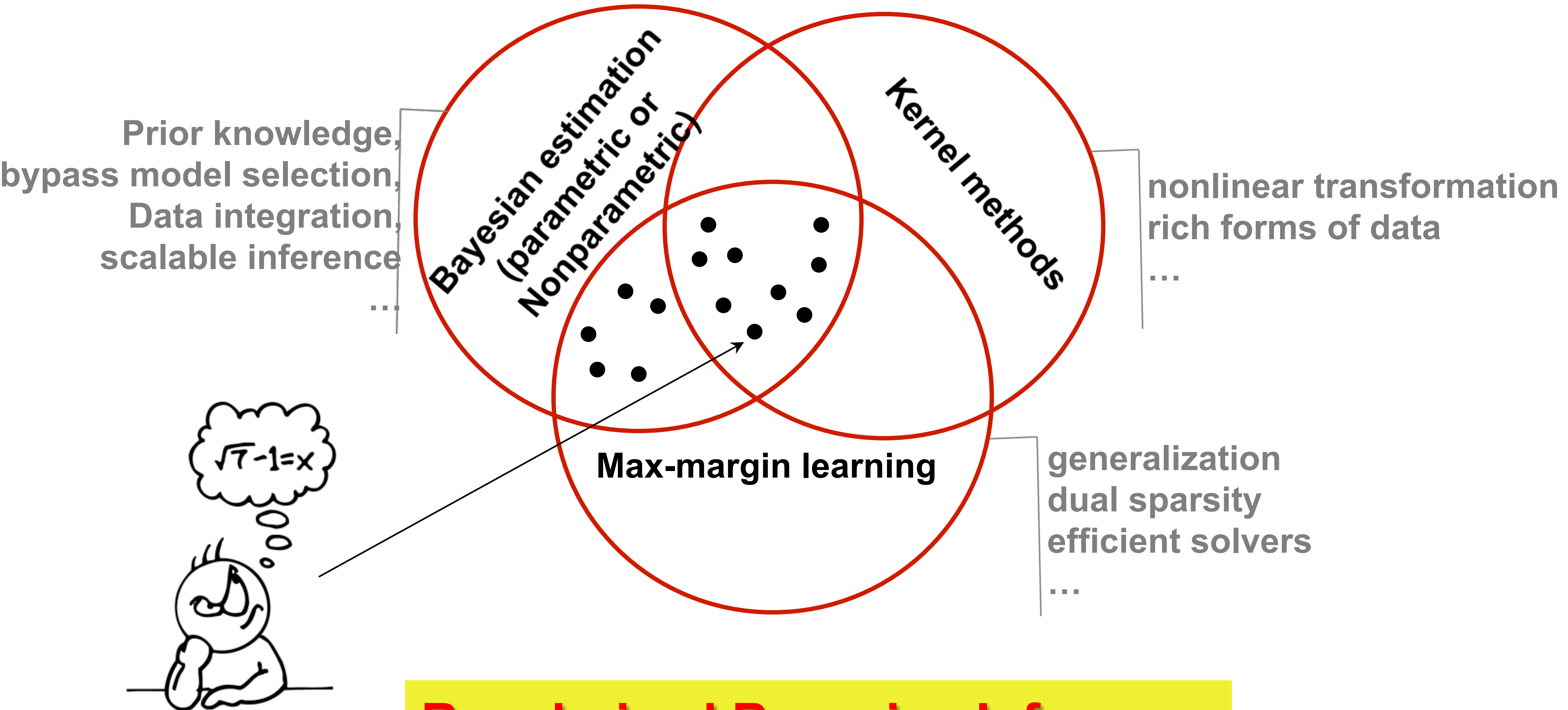


Matt Gormley
(Slides by Jun Zhu and Eric Xing)

April 13, 2016

Reading: Zhu, Chen, & Xing (2014)

Learning GMs



Regularized Bayesian Inference

Bayesian Inference

- A coherent framework of dealing with uncertainties

$$p(\mathcal{M}|\mathbf{x}) = \frac{p(\mathbf{x}|\mathcal{M})\pi(\mathcal{M})}{\int p(\mathbf{x}|\mathcal{M})\pi(\mathcal{M})d\mathcal{M}}$$

  - ***M*: a model from some hypothesis space**
  - **x: observed data**

Thomas Bayes (1702 – 1761)

- Bayes' rule offers a mathematically rigorous computational mechanism for combining prior knowledge with incoming evidence

Parametric Bayesian Inference

$\mathcal{M}$ **is represented as a finite set of parameters** θ

- A parametric likelihood: $\mathbf{x} \sim p(\cdot|\theta)$
- Prior on θ : $\pi(\theta)$
- Posterior distribution

$$p(\theta|\mathbf{x}) = \frac{p(\mathbf{x}|\theta)\pi(\theta)}{\int p(\mathbf{x}|\theta)\pi(\theta)d\theta} \propto p(\mathbf{x}|\theta)\pi(\theta)$$

Examples:

- **Gaussian distribution prior + 2D Gaussian likelihood → Gaussian posterior distribution**
- **Dirichilet distribution prior + 2D Multinomial likelihood → Dirichlet posterior distribution**
- **Sparsity-inducing priors + some likelihood models → Sparse Bayesian inference**

Nonparametric Bayesian Inference

$\mathcal{M}$ is a richer model, e.g., with an infinite set of parameters

- A nonparametric likelihood: $\mathbf{x} \sim p(\cdot|\mathcal{M})$
- Prior on $\mathcal{M}$: $\pi(\mathcal{M})$
- Posterior distribution

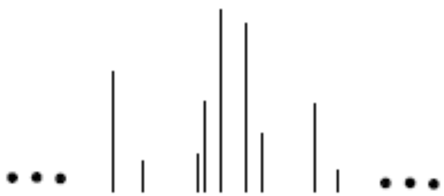
$$p(\mathcal{M}|\mathbf{x}) = \frac{p(\mathbf{x}|\mathcal{M})\pi(\mathcal{M})}{\int p(\mathbf{x}|\mathcal{M})\pi(\mathcal{M})d\mathcal{M}} \propto p(\mathbf{x}|\mathcal{M})\pi(\mathcal{M})$$

Examples:

→ see next slide

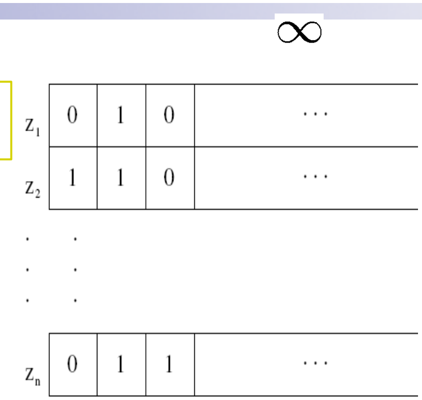
Nonparametric Bayesian Inference

probability measure



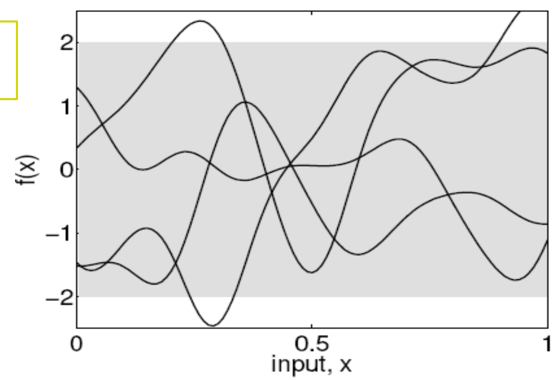
Dirichlet Process Prior [Antoniak, 1974]
+ Multinomial/Gaussian/Softmax likelihood

binary matrix



Indian Buffet Process Prior [Griffiths & Gharamani, 2005]
+ Gaussian/Sigmoid/Softmax likelihood

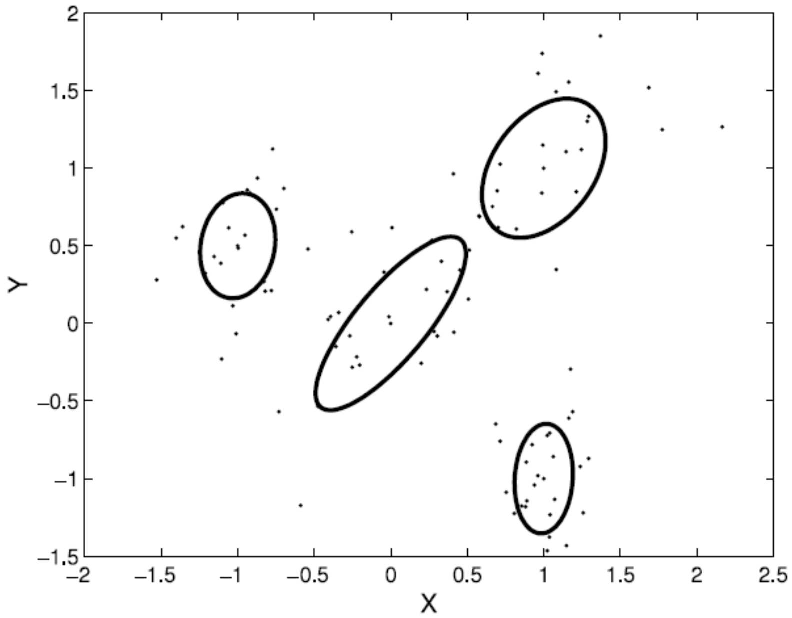
function



Gaussian Process Prior [Doob, 1944; Rasmussen & Williams, 2006]
+ Gaussian/Sigmoid/Softmax likelihood

Why Bayesian Nonparametrics?

- Let the data speak for themselves
- Bypass the model selection problem
  - let data determine model complexity (e.g., the number of components in mixture models)
  - allow model complexity to grow as more data observed



Can we further control the posterior distributions?

posterior **likelihood model** **prior**

$$p(\mathcal{M}|\mathbf{x}) = \frac{p(\mathbf{x}|\mathcal{M})\pi(\mathcal{M})}{\int p(\mathbf{x}|\mathcal{M})\pi(\mathcal{M})d\mathcal{M}}$$

It is desirable to further regularize the posterior distribution

- An extra freedom to perform Bayesian inference
- Arguably more direct to control the behavior of models
- Can be easier and more natural in some examples

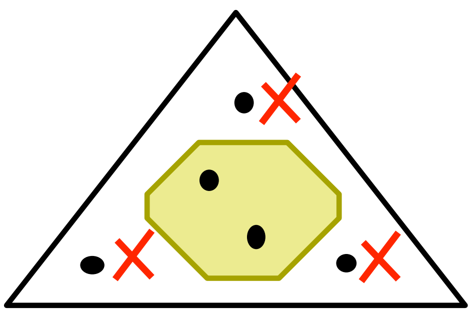
Can we further control the posterior distributions?

posterior — likelihood model — prior

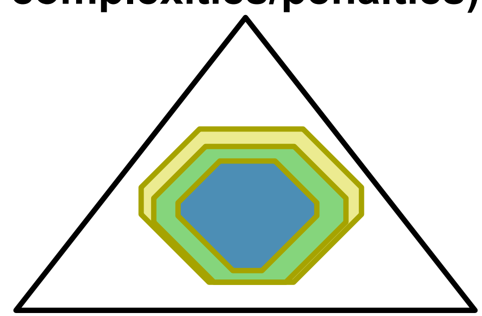
$$p(\mathcal{M}|\mathbf{x}) = \frac{p(\mathbf{x}|\mathcal{M})\pi(\mathcal{M})}{\int p(\mathbf{x}|\mathcal{M})\pi(\mathcal{M})d\mathcal{M}}$$

- Directly control the posterior distributions?
  - Not obvious how …

hard constraints
(A single feasible space)



soft constraints
(many feasible subspaces with different complexities/penalties)



A reformulation of Bayesian inference

posterior — likelihood model — prior

$$p(\mathcal{M}|\mathbf{x}) = \frac{p(\mathbf{x}|\mathcal{M})\pi(\mathcal{M})}{\int p(\mathbf{x}|\mathcal{M})\pi(\mathcal{M})d\mathcal{M}}$$

- **Bayes' rule is equivalent to:**

$$\min_{p(\mathcal{M})} \quad \mathrm{KL}(p(\mathcal{M})\|\pi(\mathcal{M})) - \mathbb{E}_{p(\mathcal{M})}[\log p(\mathbf{x}|\mathcal{M})]$$
$$\text{s.t.}: \quad p(\mathcal{M}) \in \mathcal{P}_{\text{prob}},$$

A direct but trivial constraint on the posterior distribution

E.T. Jaynes (1988): "this fresh interpretation of Bayes' theorem could make the use of Bayesian methods more attractive and widespread, and stimulate new developments in the general theory of inference"

[Zellner, Am. Stat. 1988]

Regularized Bayesian Inference
(Ganchev et al.'10)

$$\inf_{q(\mathrm{M}),\xi} \mathrm{KL}(q(\mathrm{M})\|\pi(\mathrm{M})) - \int_{\mathcal{M}} \log p(\mathcal{D}|\mathrm{M})q(\mathrm{M})d\mathrm{M} + U(\xi)$$
$$\text{s.t.}: q(\mathrm{M}) \in \mathcal{P}_{\text{post}}(\xi),$$

where, **e.x.**,

$$\mathcal{P}_{\text{post}}(\xi) \stackrel{\text{def}}{=} \Big\{ q(\mathrm{M}) |\ \forall t = 1, \cdots, T,\ h\big(Eq(\psi_t;\mathcal{D})\big) \leq \xi_t \Big\},$$

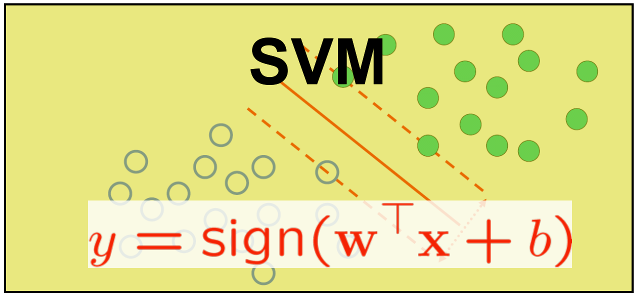
and

$$U(\xi) = \sum_{t=1}^{T} \mathbb{I}(\xi_t = \gamma_t) = \mathbb{I}(\xi = \gamma)$$

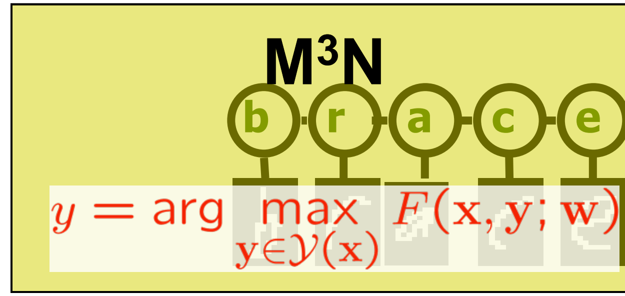
Solving such constrained optimization problem needs convex duality theory

So, where do the constraints come from?

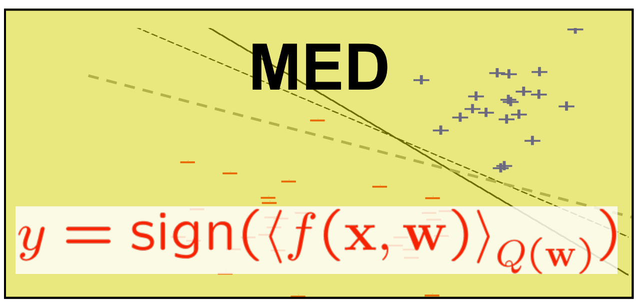
Recall our evolution of the Max-Margin Learning Paradigms



$$\min_{\mathbf{w},\xi} \quad \frac{1}{2}||\mathbf{w}||^2 + C\sum_{i=1}^{m} \xi_i$$
$$y^i(\mathbf{w}^\top \mathbf{x}^i + b) \geq 1 - \xi_i, \quad \forall i$$



$$\min_{\mathbf{w},\xi} \quad \frac{1}{2}||\mathbf{w}||^2 + C\sum_{i=1}^{m} \xi_i$$
$$\mathbf{w}^\top[\mathbf{f}(\mathbf{x}^i, \mathbf{y}^i) - \mathbf{f}(\mathbf{x}^i, \mathbf{y})] \geq \ell(\mathbf{y}^i, \mathbf{y}) - \xi_i, \quad \forall i, \forall \mathbf{y} \neq \mathbf{y}^i$$



$$\min_{Q} \quad \text{KL}(Q||Q_0)$$
$$y^i \langle f(\mathbf{x}^i) \rangle_Q \geq \xi_i, \quad \forall i$$

MED-MN ?

= SMED + "Bayesian" M³N

Maximum Entropy Discrimination Markov Networks

- Structured MaxEnt Discrimination (SMED):

$$\text{P1}: \quad \min_{p(\mathbf{w}),\xi} \; KL(p(\mathbf{w})||p_0(\mathbf{w})) + U(\xi)$$
$$\text{s.t.} \quad p(\mathbf{w}) \in \mathcal{F}_1, \; \xi_i \geq 0, \forall i.$$

generalized maximum entropy or *regularized* KL-divergence

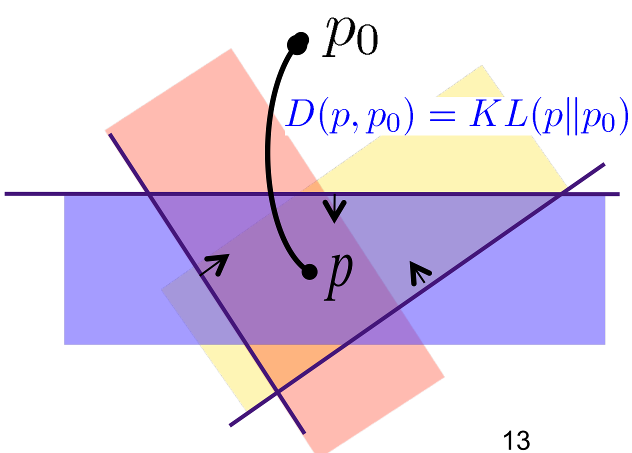
- Feasible subspace of weight distribution:

$$\mathcal{F}_1 = \left\{p(\mathbf{w}) : \int p(\mathbf{w})[\Delta F_i(\mathbf{y};\mathbf{w}) - \Delta \ell_i(\mathbf{y})] \, d\mathbf{w} \geq -\xi_i, \; \forall i, \forall \mathbf{y} \neq \mathbf{y}^i\right\},$$

expected margin constraints.

- Average from distribution of M[3]Ns

$$h_1(\mathbf{x}; p(\mathbf{w})) = \arg\max_{\mathbf{y}\in\mathcal{Y}(\mathbf{x})} \int p(\mathbf{w}) F(\mathbf{x},\mathbf{y};\mathbf{w}) d\mathbf{w}$$



Can we use this scheme to learn models other than MN?

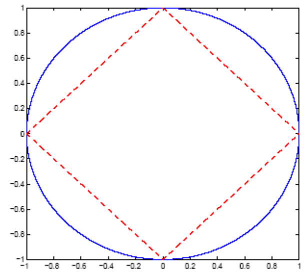
Recall the 3 advantages of MEDN

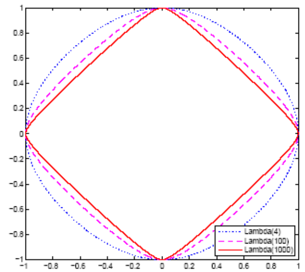
- An averaging Model: PAC-Bayesian prediction error guarantee (Theorem 3)

$$\Pr_Q(M(h,\mathbf{x},\mathbf{y}) \le 0) \le \Pr_{\mathcal{D}}(M(h,\mathbf{x},\mathbf{y}) \le \gamma) + O\Big(\sqrt{\frac{\gamma^{-2}KL(p||p_0)\ln(N|\mathcal{Y}|) + \ln N + \ln \delta^{-1}}{N}}\Big).$$

- Entropy regularization: Introducing useful biases
  - Standard Normal prior => reduction to standard M^{3}N (we've seen it)
  - Laplace prior => Posterior shrinkage effects (sparse M^{3}N)

$$\min_{\mu,\xi} \sqrt{\lambda}\sum_{k=1}^{K}\Big(\sqrt{\mu_k^2 + \frac{1}{\lambda}} - \frac{1}{\sqrt{\lambda}}\log\frac{\sqrt{\lambda\mu_k^2+1}+1}{2}\Big) + C\sum_{i=1}^{N}\xi_i$$
$$\text{s.t. } \mu^\top \Delta\mathbf{f}_i(\mathbf{y}) \ge \Delta\ell_i(\mathbf{y}) - \xi_i;\ \xi_i \ge 0,\ \forall i,\ \forall \mathbf{y} \ne \mathbf{y}^i.$$

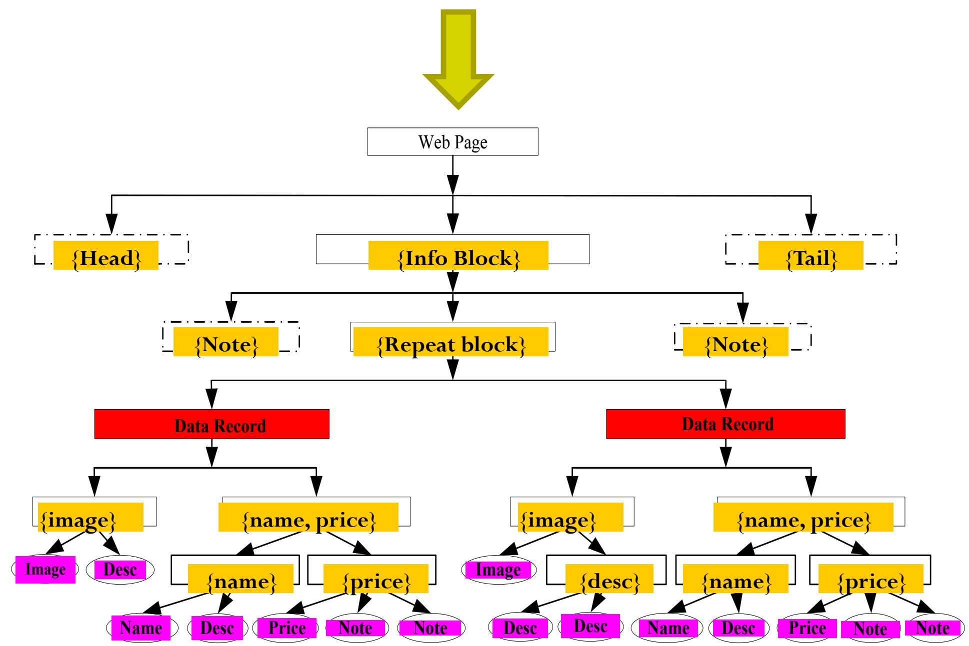




- Integrating Generative and Discriminative principles (next class)
  - Incorporate latent variables and structures (PoMEN)
  - Semisupervised learning (with partially labeled data)

Latent Hierarchical MaxEnDNet

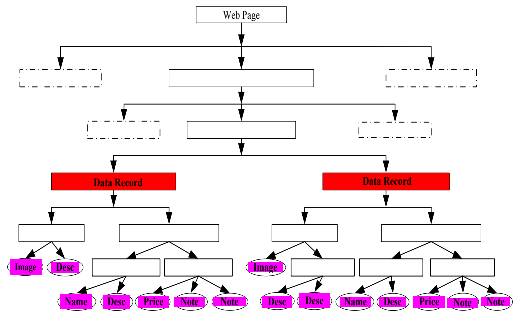
- Web data extraction
  - Goal: *Name*, *Image*, *Price*, *Description*, etc.



- **Hierarchical labeling**
- **Advantages:**
  - Computational efficiency
  - Long-range dependency
  - Joint extraction

Partially Observed MaxEnDNet (PoMEN) (Zhu et al, NIPS 2008)

- Now we are given partially labeled data: $\mathcal{D} = \{ < \mathbf{x}^i, \mathbf{y}^i, \mathbf{z}^i > \}_{i=1}^N$



  - PoMEN: learning $p(\mathbf{w}, \mathbf{z})$

$$\text{P2(PoMEN)}: \quad \min_{p(\mathbf{w},\{\mathbf{z}\}),\xi} KL(p(\mathbf{w},\{\mathbf{z}\})||p_0(\mathbf{w},\{\mathbf{z}\})) + U(\xi)$$
$$\text{s.t.} \quad p(\mathbf{w},\{\mathbf{z}\}) \in \mathcal{F}_2, \ \xi_i \geq 0, \forall i.$$

$$\mathcal{F}_2 = \{ p(\mathbf{w},\{\mathbf{z}\}) : \sum_{\mathbf{z}} \int p(\mathbf{w},\mathbf{z})[\Delta F_i(\mathbf{y},\mathbf{z};\mathbf{w}) - \Delta \ell_i(\mathbf{y})] \, d\mathbf{w} \geq -\xi_i, \ \forall i, \forall \mathbf{y} \neq \mathbf{y}^i \},$$

  - Prediction: $h_2(\mathbf{x}) = \arg \max_{\mathbf{y} \in \mathcal{Y}(\mathbf{x})} \sum_{\mathbf{z}} \int p(\mathbf{w},\mathbf{z}) F(\mathbf{x},\mathbf{y},\mathbf{z};\mathbf{w}) \, d\mathbf{w}$

Alternating Minimization Alg.

- Factorization assumption:

$$p_0(\mathbf{w}, \{\mathbf{z}\}) = p_0(\mathbf{w}) \prod_{i=1}^{N} p_0(\mathbf{z}_i) \qquad p(\mathbf{w}, \{\mathbf{z}\}) = p(\mathbf{w}) \prod_{i=1}^{N} p(\mathbf{z}_i)$$

- Alternating minimization:
  - Step 1: keep $p(\mathbf{z})$ fixed, optimize over $p(\mathbf{w})$

$$\min_{p(\mathbf{w}),\xi} \; KL(p(\mathbf{w})||p_0(\mathbf{w})) + C\sum_i \xi_i$$
$$\text{s.t.} \;\; p(\mathbf{w}) \in \mathcal{F}_1', \;\; \xi_i \geq 0, \forall i.$$

$$\mathcal{F}_1' = \{p(\mathbf{w}) : \int p(\mathbf{w}) E_{p(\mathbf{z})}[\Delta F_i(\mathbf{y}, \mathbf{z}; \mathbf{w}) - \Delta \ell_i(\mathbf{y})] \, d\mathbf{w} \geq -\xi_i, \; \forall i, \; \forall \mathbf{y}\}$$

  - Step 2: keep $p(\mathbf{w})$ fixed, optimize over $p(\mathbf{z})$

$$\min_{p(\mathbf{w}),\xi} \; KL(p(\mathbf{z})||p_0(\mathbf{z})) + C\xi_i$$
$$\text{s.t.} \;\; p(\mathbf{z}) \in \mathcal{F}_1^\star, \;\; \xi_i \geq 0.$$

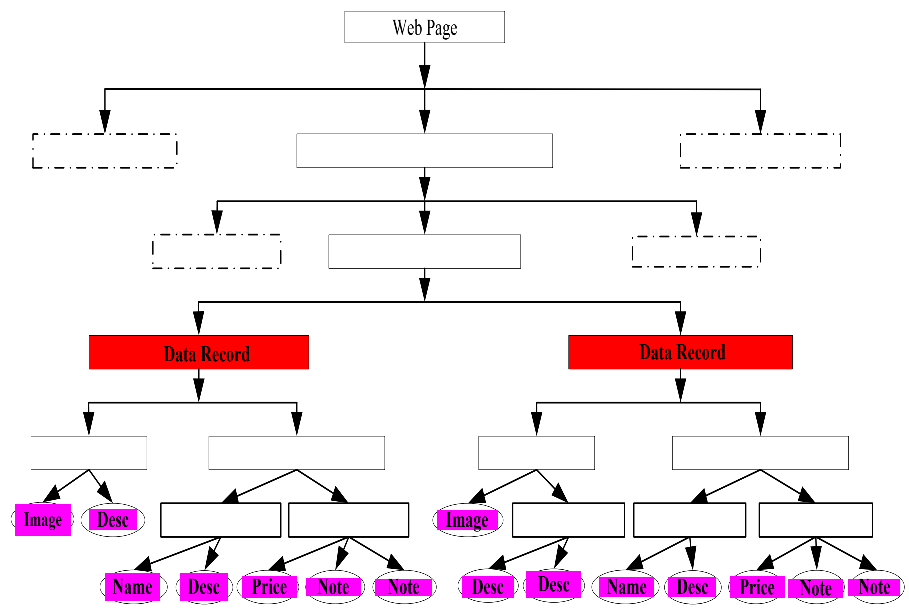
$$\mathcal{F}_1^\star = \{p(\mathbf{z}) : \sum_{\mathbf{z}} p(\mathbf{z}) \int p(\mathbf{w})[\Delta F_i(\mathbf{y}, \mathbf{z}; \mathbf{w}) - \Delta \ell_i(\mathbf{y})] \, d\mathbf{w} \geq -\xi_i, \; \forall i, \; \forall \mathbf{y}\}$$

- Normal prior
  - M^3N problem (QP)
- Laplace prior
  - Laplace M^3N problem (VB)

Equivalently reduced to an LP with a polynomial number of constraints

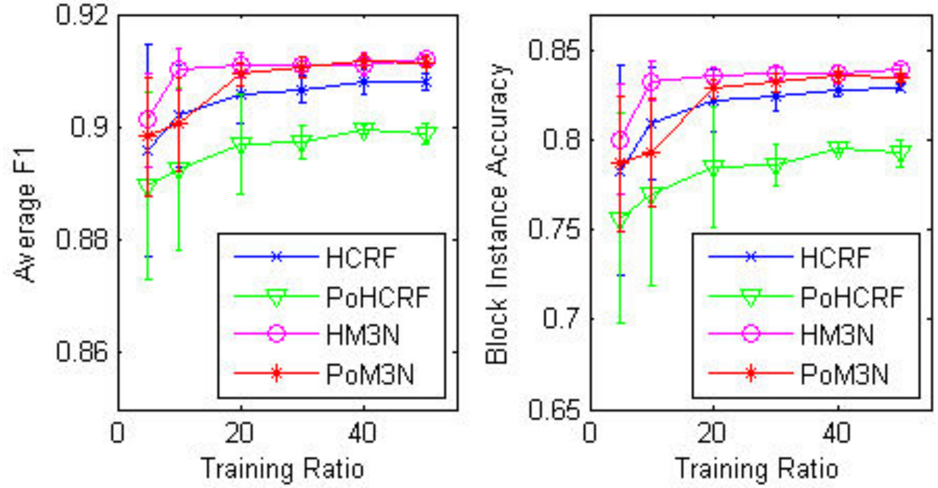
Experimental Results

- Web data extraction:
  - *Name, Image, Price, Description*
  - Methods:
    - Hierarchical CRFs, Hierarchical M^3N
    - PoMEN, Partially observed HCRFs
  - Pages from 37 templates
    - Training: 185 (5/per template) pages, or 1585 data records
    - Testing: 370 (10/per template) pages, or 3391 data records
  - Record-level Evaluation
    - Leaf nodes are labeled
  - Page-level Evaluation
    - Supervision Level 1:
      - Leaf nodes and data record nodes are labeled
    - Supervision Level 2:
      - Level 1 + the nodes above data record nodes

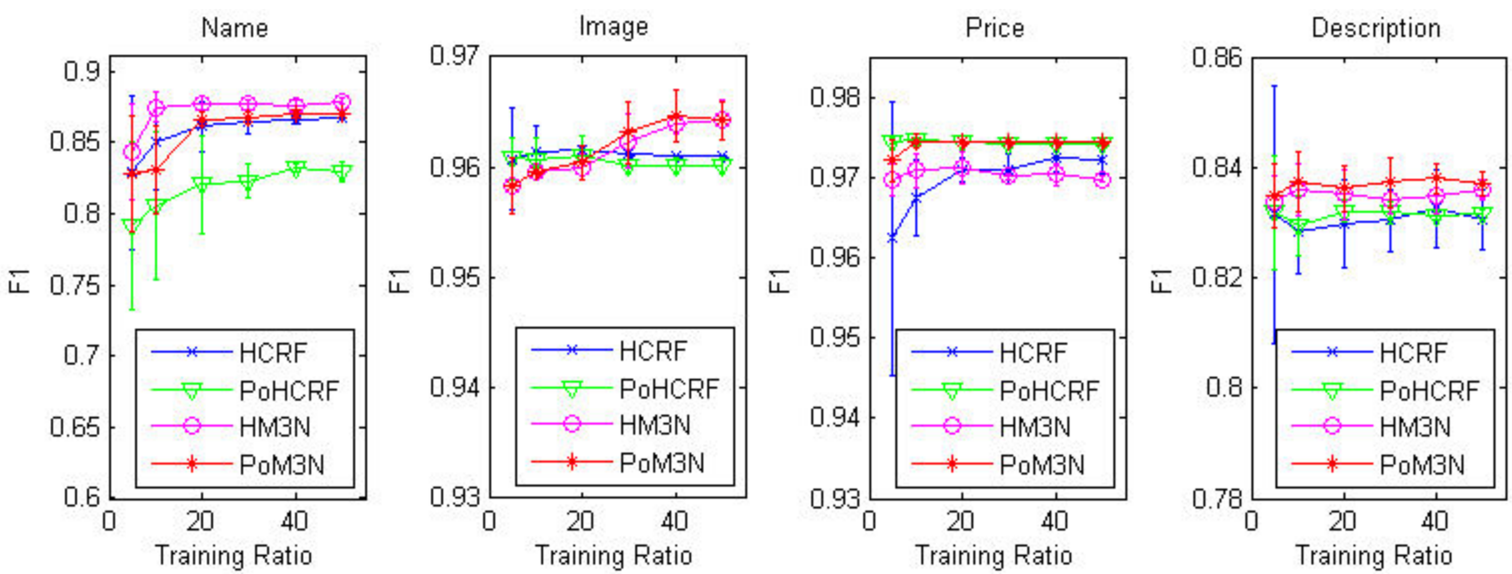


Record-Level Evaluations

- Overall performance:
  - Avg F1:
    - avg F1 over all attributes
  - Block instance accuracy:
    - % of records whose *Name*, *Image*, and *Price* are correct

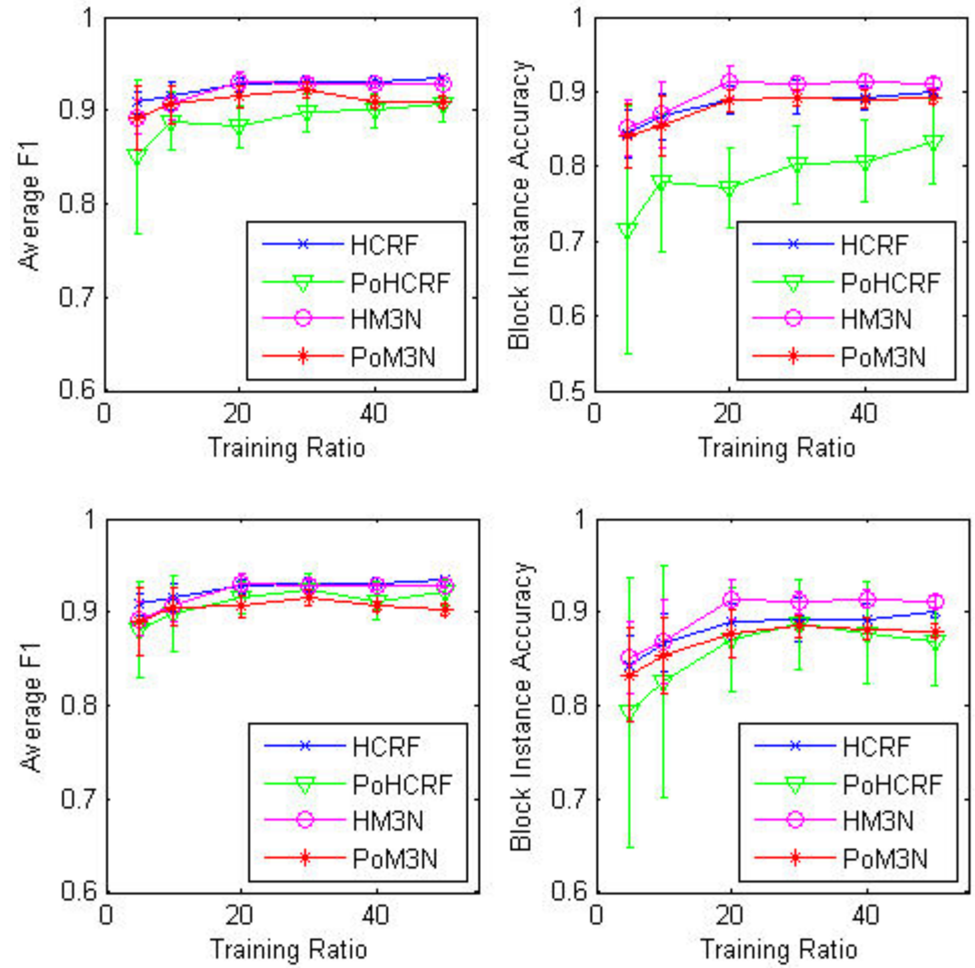


- Attribute performance:



Page-Level Evaluations

- Supervision Level 1:
  - Leaf nodes and data record nodes are labeled
- Supervision Level 2:
  - Level 1 + the nodes above data record nodes



Key message from PoMEN

- Structured MaxEnt Discrimination (SMED):

$$\mathrm{P1}: \quad \min_{p(\mathbf{w},\mathbf{z}),\xi} \; KL(p(\mathbf{w},\mathbf{z})||p_0(\mathbf{w},\mathbf{z})) + U(\xi)$$
$$, \quad \text{s.t. } p(\mathbf{w},\mathbf{z}) \in \mathcal{F}_1, \; \xi_i \geq 0, \forall i.$$

generalized maximum entropy or *regularized* KL-divergence

- Feasible subspace of weight distribution:

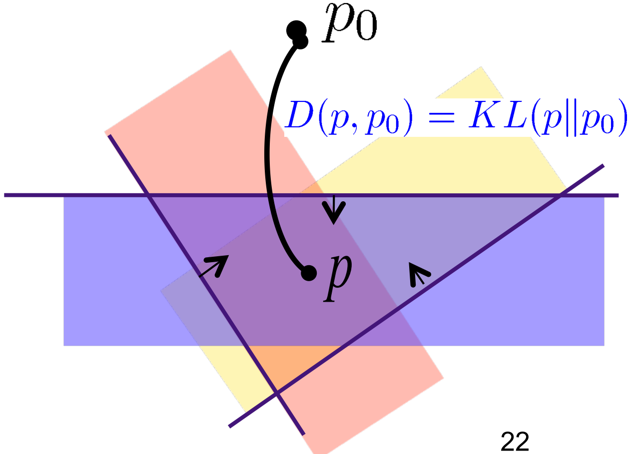
$$\mathcal{F} = \{p(\mathbf{w},\mathbf{z}) : \int\int p(\mathbf{w},\mathbf{z})[\Delta F_i(\mathbf{y};\mathbf{w},\mathbf{z}) - \Delta\ell_i(\mathbf{y})]\, d\mathbf{w}d\mathbf{z} \geq -\xi_i, \; \forall i, \forall \mathbf{y} \neq \mathbf{y}^i\},$$

expected margin constraints.

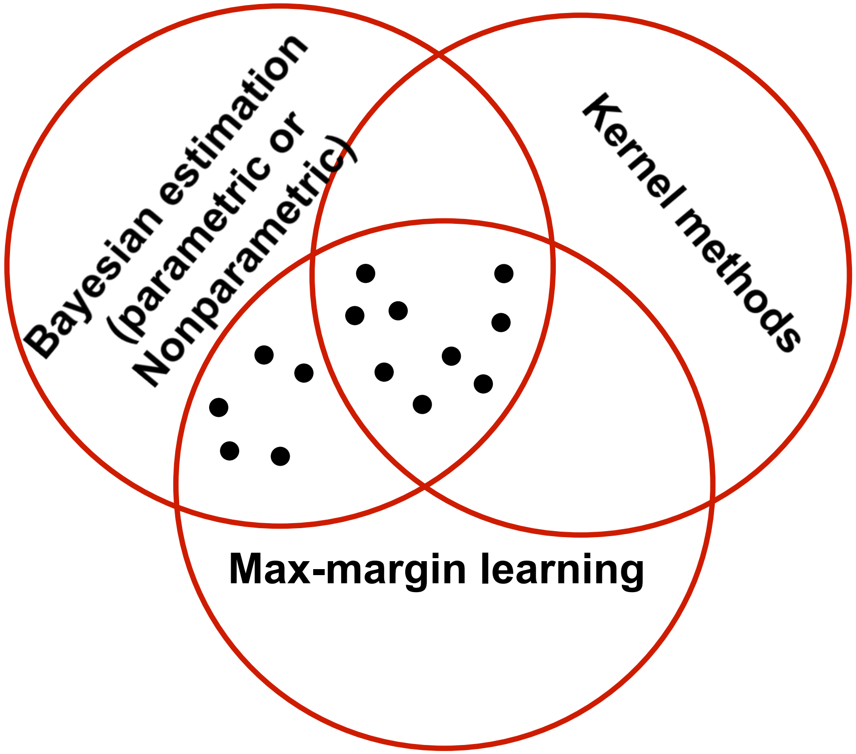
- Average from distribution of PoMENs

$$h(\mathbf{x}) = \arg\max_{\mathbf{y}\in\mathcal{Y}(\mathbf{x})} \int\int p(\mathbf{w},\mathbf{z}) F(\mathbf{x},\mathbf{y},\mathbf{z};\mathbf{w})\, d\mathbf{w}d\mathbf{z}$$

- **We can use this for any p and p_0 !**



An all inclusive paradigm for learning general GM --- RegBayes



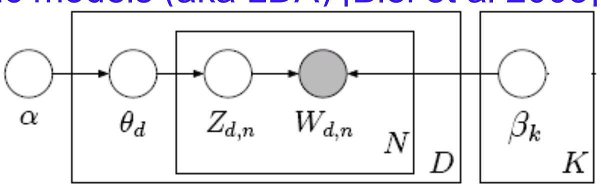
$$\inf_{q(\mathrm{M}),\xi} \mathrm{KL}(q(\mathrm{M})\|\pi(\mathrm{M})) - \int_{\mathcal{M}} \log p(\mathcal{D}|\mathrm{M})q(\mathrm{M})d\mathrm{M} + U(\xi)$$
$$\mathrm{s.t.}: q(\mathrm{M}) \in \mathcal{P}_{\mathrm{post}}(\xi),$$

Predictive Latent Subspace Learning
via a large-margin approach

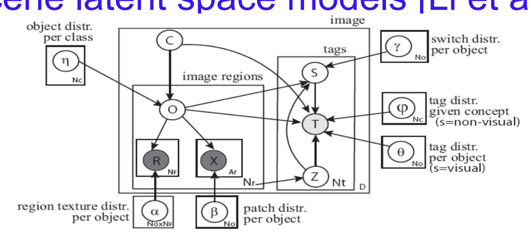
… where M is any subspace model and p is a parametric Bayesian prior

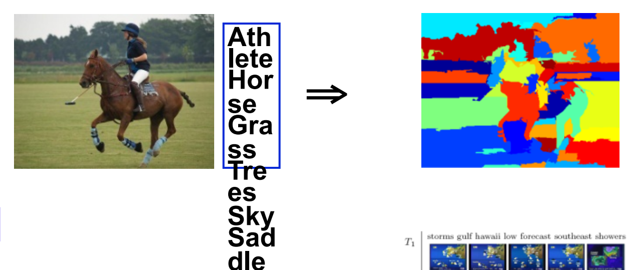
Unsupervised Latent Subspace Discovery

- Finding latent subspace representations (an old topic)
  - Mapping a high-dimensional representation into a latent low-dimensional representation, where each dimension can have some interpretable meaning, e.g., a semantic topic
- Examples:
  - Topic models (aka LDA) [Blei et al 2003]

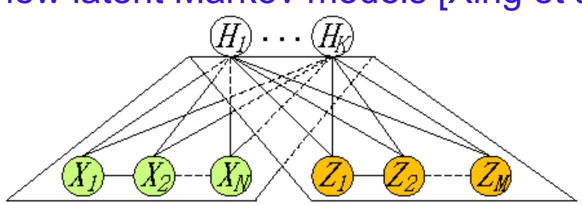


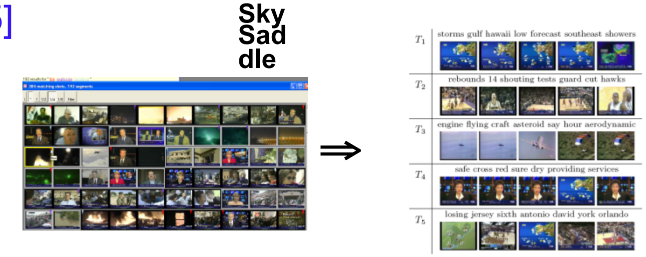
  - Total scene latent space models [Li et al 2009]





  - Multi-view latent Markov models [Xing et al 2005]



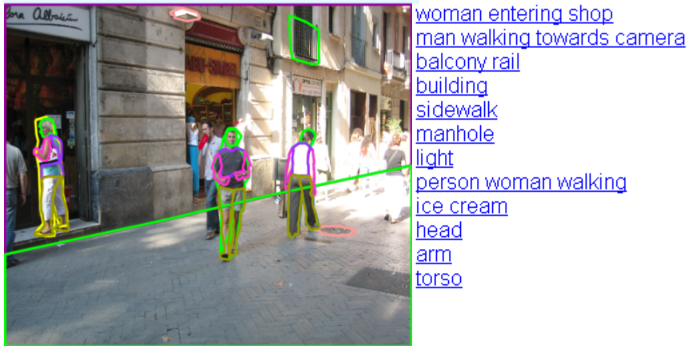


  - PCA, CCA, …

Predictive Subspace Learning with *Supervision*

- Unsupervised latent subspace representations are generic but can be sub-optimal for predictions
- Many datasets are available with supervised side information
  - Tripadvisor Hotel Review (http://www.tripadvisor.com)

  - LabelMe http://labelme.csail.mit.edu/



  - Many others

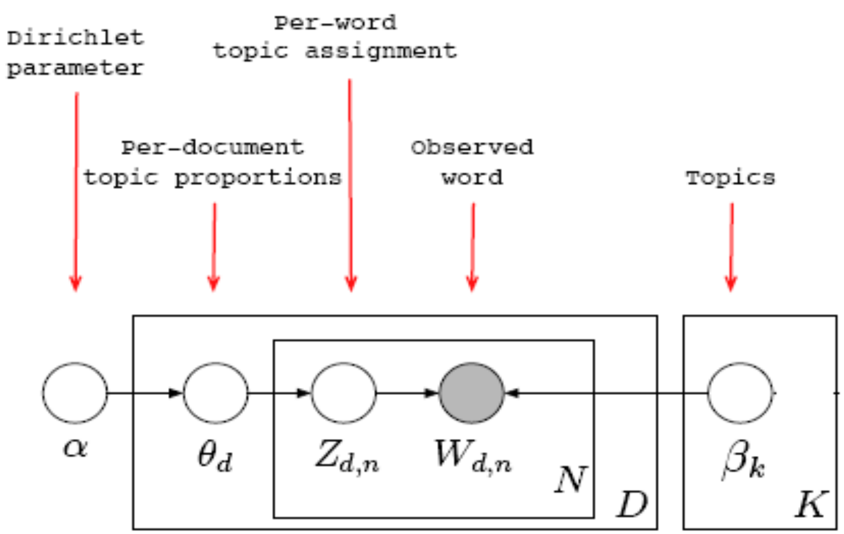
Flickr (http://www.flickr.com/)

IMAGENET

- Can be noisy, but not random noise (Ames & Naaman, 2007)
  - labels & rating scores are usually assigned based on some intrinsic property of the data
  - helpful to suppress noise and capture the most useful aspects of the data
- **Goals:**
  - **Discover latent subspace representations that are both *predictive* and *interpretable* by exploring weak supervision information**

I. LDA: Latent Dirichlet Allocation

(Blei et al., 2003)



- **Generative Procedure:**
  - **For each document *d*:**
    - **Sample a topic proportion** $\theta_d \sim \mathrm{Dir}(\alpha)$
    - **For each word:**
      - **Sample a topic** $Z_{d,n} \sim \mathrm{Mult}(\theta_d)$
      - **Sample a word** $W_{d,n} \sim \mathrm{Mult}(\beta_{z_{d,n}})$

- Joint Distribution: $p(\theta, \mathbf{z}, \mathbf{W} | \alpha, \beta) = \prod_{d=1}^{D} p(\theta_d|\alpha)(\prod_{n=1}^{N} p(z_{dn}|\theta_d)p(w_{dn}|z_{dn}, \beta))$

  exact inference intractable!

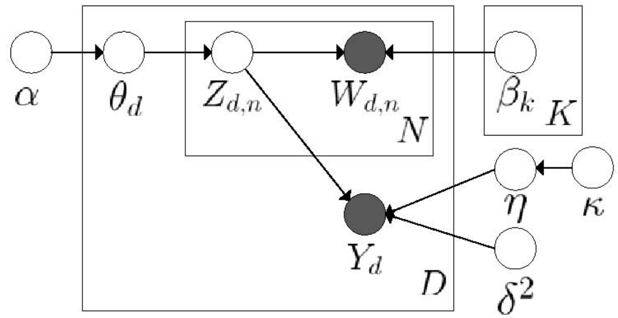
- Variational Inference with $q(\mathbf{z}, \theta) \sim p(\mathbf{z}, \theta | \mathbf{W}, \alpha, \beta)$

$$\mathcal{L}(q) \triangleq -E_q[\log p(\theta, \mathbf{z}, \mathbf{W}|\alpha, \beta)] - \mathcal{H}(q(\mathbf{z}, \theta)) \geq -\log p(\mathbf{W}|\alpha, \beta)$$

- Minimize the variational bound to estimate parameters and infer the posterior distribution

Maximum Entropy Discrimination LDA (MedLDA) (Zhu et al, ICML 2009)

- Bayesian sLDA:



- MED Estimation:
  - MedLDA Regression Model

$$\mathrm{P1(MedLDA^r)}: \min_{q,\alpha,\beta,\delta^2,\xi,\xi^\star} \mathcal{L}(q) + C\sum_{d=1}^{D}(\xi_d + \xi_d^\star)$$

$$\text{s.t. } \forall d: \begin{cases} y_d - E[\eta^\top \bar{Z}_d] \leq \epsilon + \xi_d, & \mu_d \\ -y_d + E[\eta^\top \bar{Z}_d] \leq \epsilon + \xi_d^\star, & \mu_d^\star \\ \xi_d \geq 0, & v_d \\ \xi_d^\star \geq 0, & v_d^\star \end{cases}$$

model fitting

predictive accuracy

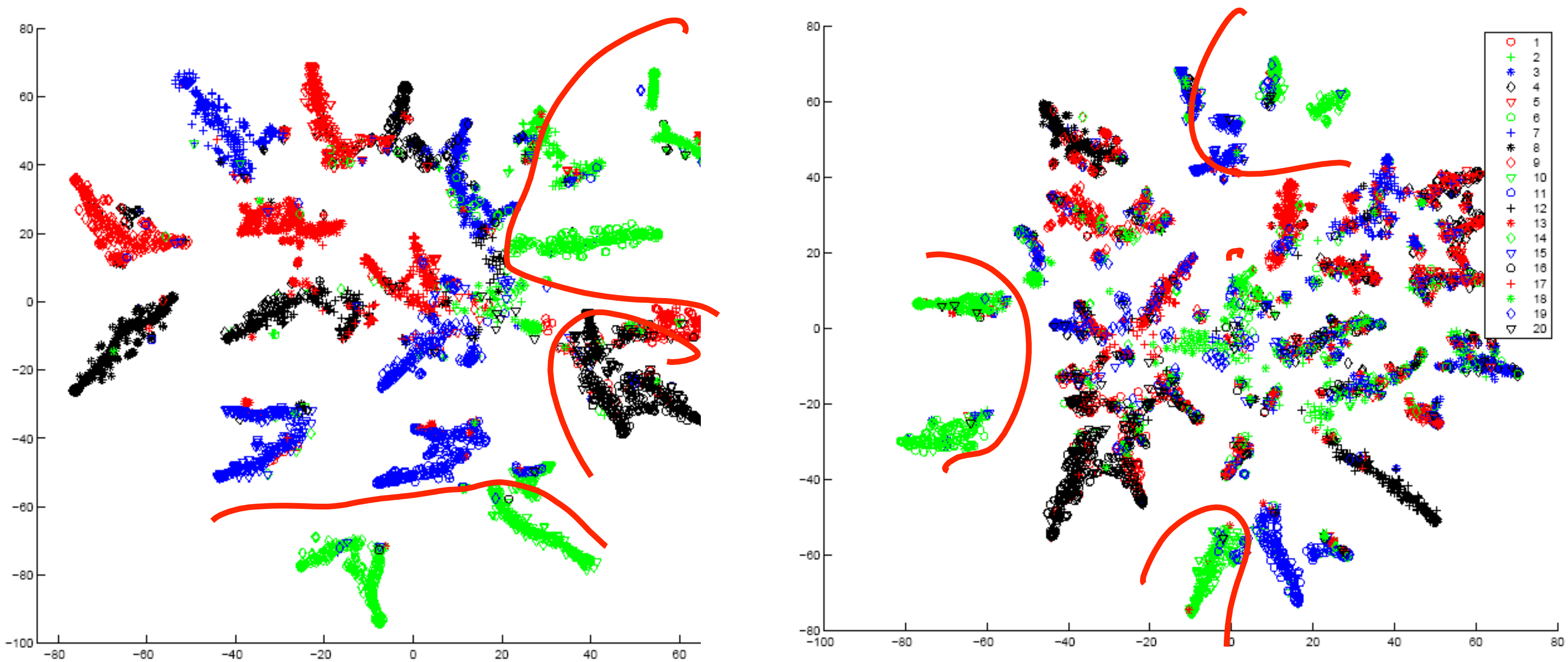
  - MedLDA Classification Model

$$\mathrm{P2(MedLDA^c)}: \min_{q,q(\eta),\alpha,\beta,\xi} \mathcal{L}(q) + C\sum_{d=1}^{D}\xi_d$$

$$\text{s.t. } \forall d,\ y \neq y_d: \quad E[\eta^\top \Delta\mathbf{f}_d(y)] \geq 1 - \xi_d;\ \xi_d \geq 0.$$

Document Modeling

- Data Set: 20 Newsgroups
- 110 topics + 2D embedding with t-SNE (var der Maaten & Hinton, 2008)

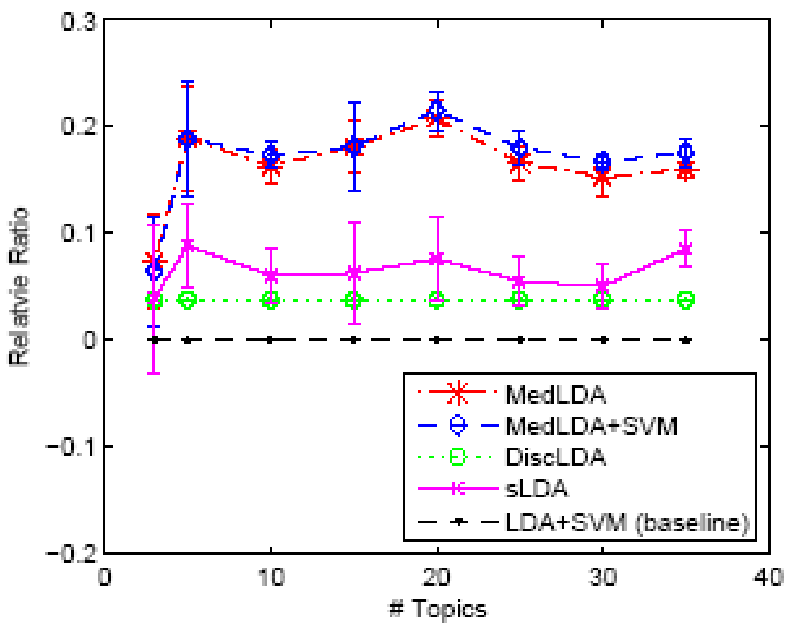


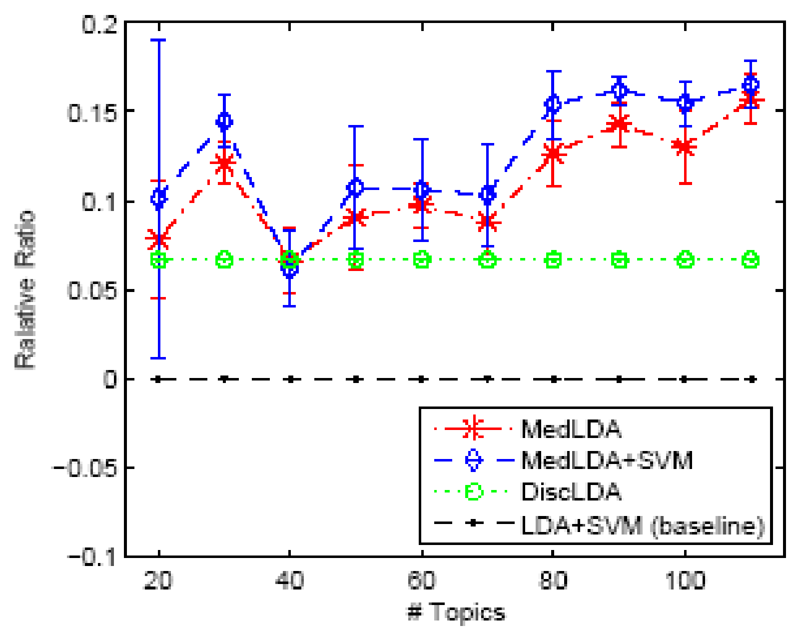
MedLDA **LDA**

Classification

- **Data Set:** 20Newsgroups
  - Binary classification: “alt.atheism” and “talk.religion.misc” (Simon et al., 2008)
  - Multiclass Classification: all the 20 categories
- **Models**: DiscLDA, sLDA (Binary ONLY! Classification sLDA (Wang et al., 2009)), LDA+SVM (baseline), MedLDA, MedLDA+SVM
- **Measure**: Relative Improvement Ratio

$$RR(\mathcal{M}) = \frac{precision(\mathcal{M})}{precision(LDA + SVM)} - 1$$

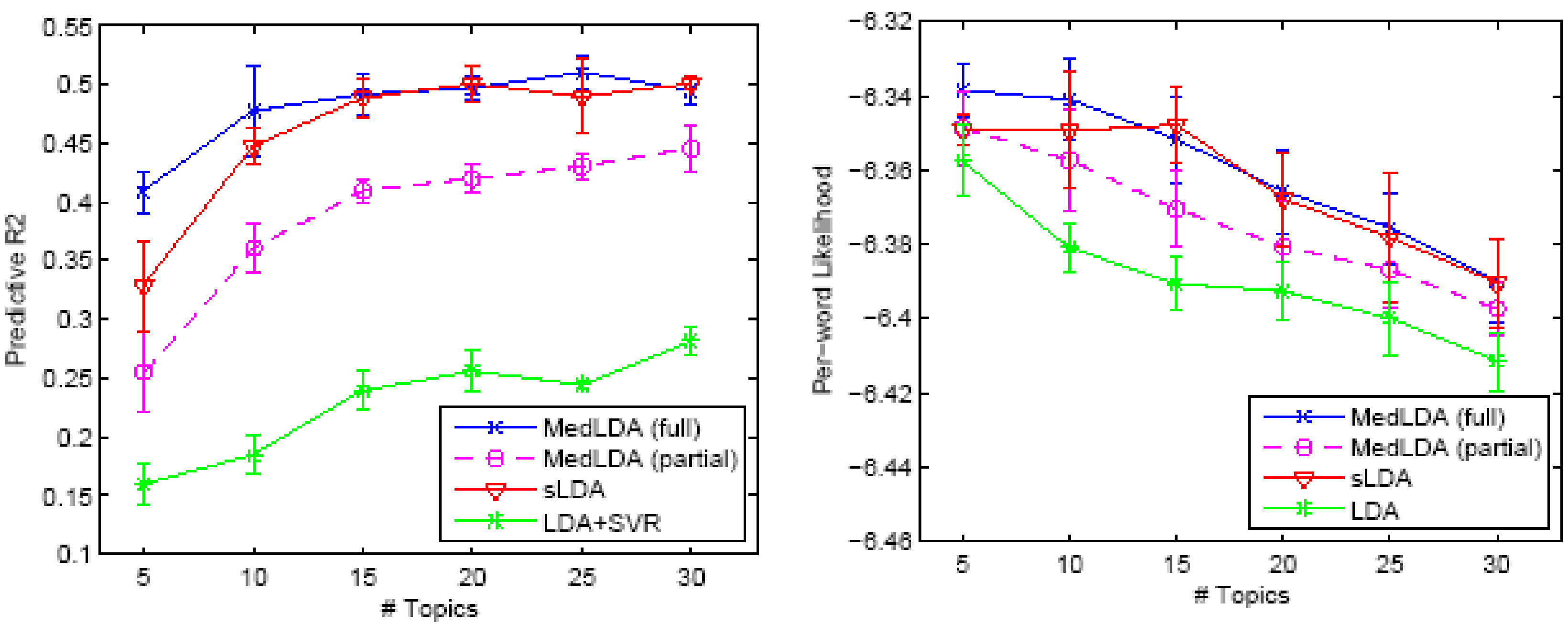




Regression

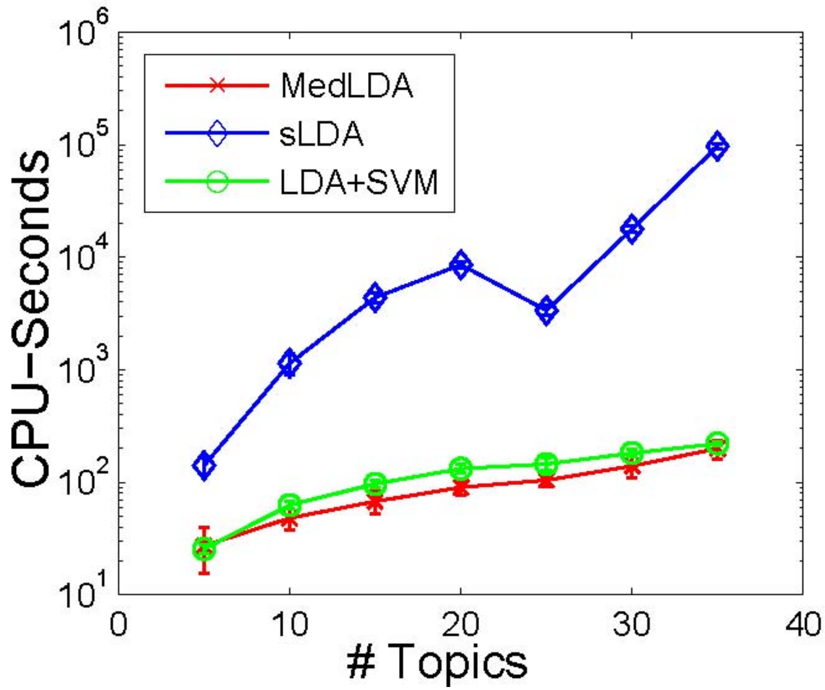
- **Data Set**: Movie Review (Blei & McAuliffe, 2007)
- **Models**: MedLDA(*partial*), MedLDA(*full*), sLDA, LDA+SVR
- **Measure**: predictive R^2 and per-word log-likelihood

$$pR^2 = 1 - \frac{\sum_d (y_d - \hat{y}_d)^2}{\sum_d (y_d - \bar{y}_d)^2}$$



Time Efficiency

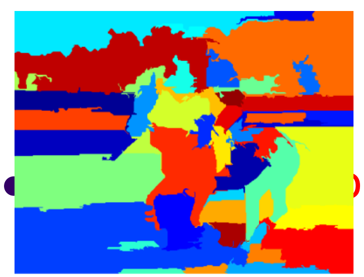
- Binary Classification



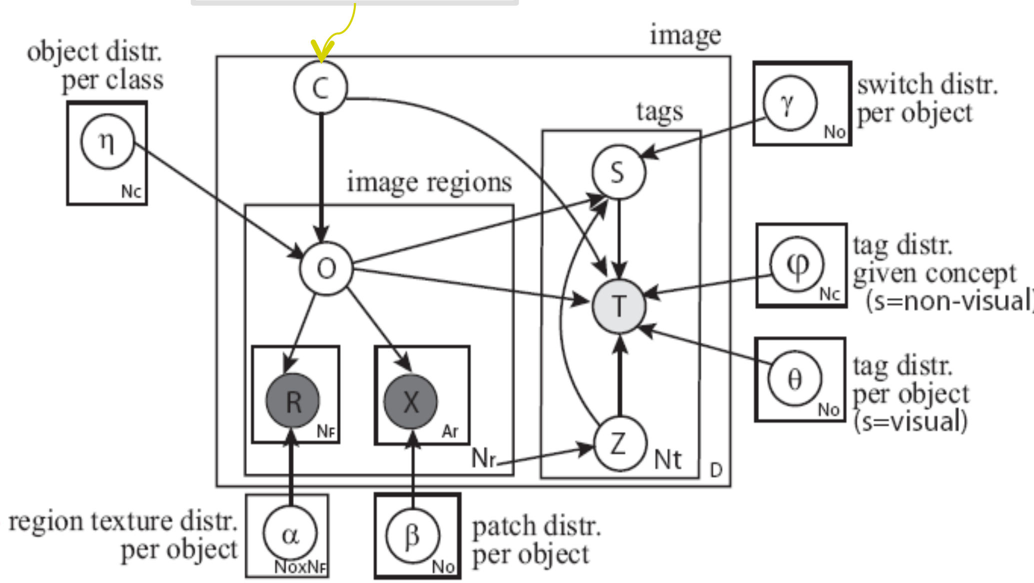
- Multiclass:
  - MedLDA is comparable with LDA+SVM
- Regression:
  - MedLDA is comparable with sLDA

II. Upstream Scene Understanding Models

- The "Total Scene Understanding" Model (Li et al, CVPR 2009)



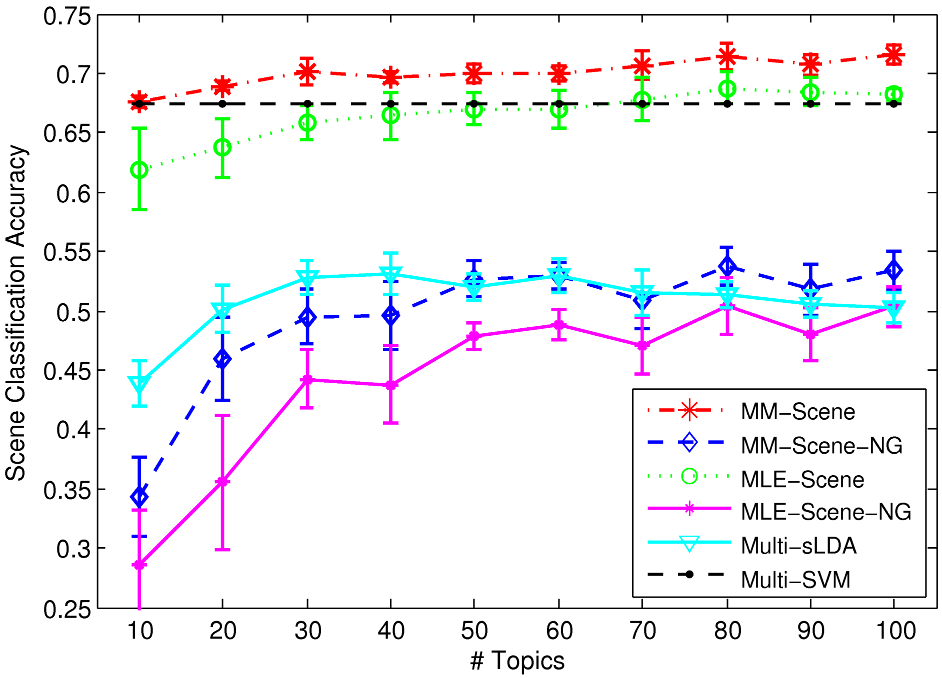
class: Polo



Athlete
Horse
Grass
Trees
Sky
Saddle

Scene Classification

- 8-category sports data set (Li & Fei-Fei, 2007):



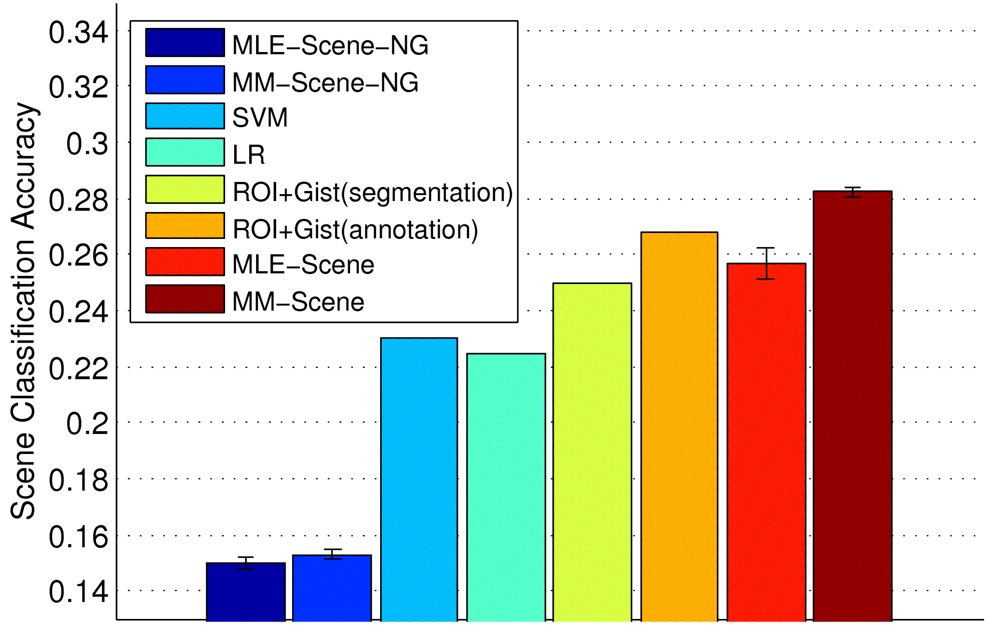
- Fei-Fei's theme model: 0.65 (different image representation)
- SVM: 0.673

•1574 images (50/50 split)
•Pre-segment each image into regions
•Region features:
  •color, texture, and location
  •patches with SIFT features
•Global features:
  •Gist (Oliva & Torralba, 2001)
  •Sparse SIFT codes (Yang et al, 2009)

MIT Indoor Scene

- Classification results:

- **67-category MIT indoor scene (Quattoni & Torralba, 2009):**
  - **~80 per-category for training; ~20 per-category for testing**
  - **Same feature representation as above**
  - **Gist global features**



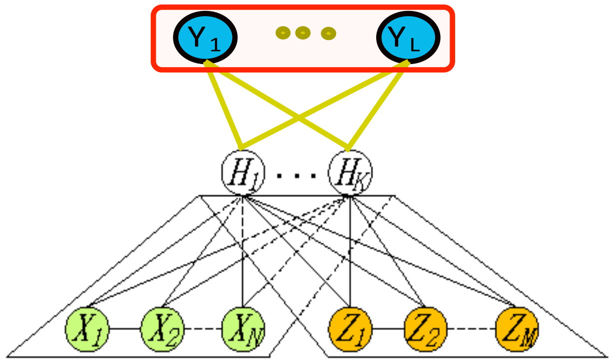
$ROI+Gist(annotation) used *human annotated* interest regions.

III. Supervised Multi-view RBMs

- A probabilistic method with an additional view of response variables *Y*

$$p(y|\mathbf{h}) = \frac{\exp\{\mathbf{V}^\top \mathbf{f}(\mathbf{h}, y)\}}{Z(V, \mathbf{h})}$$

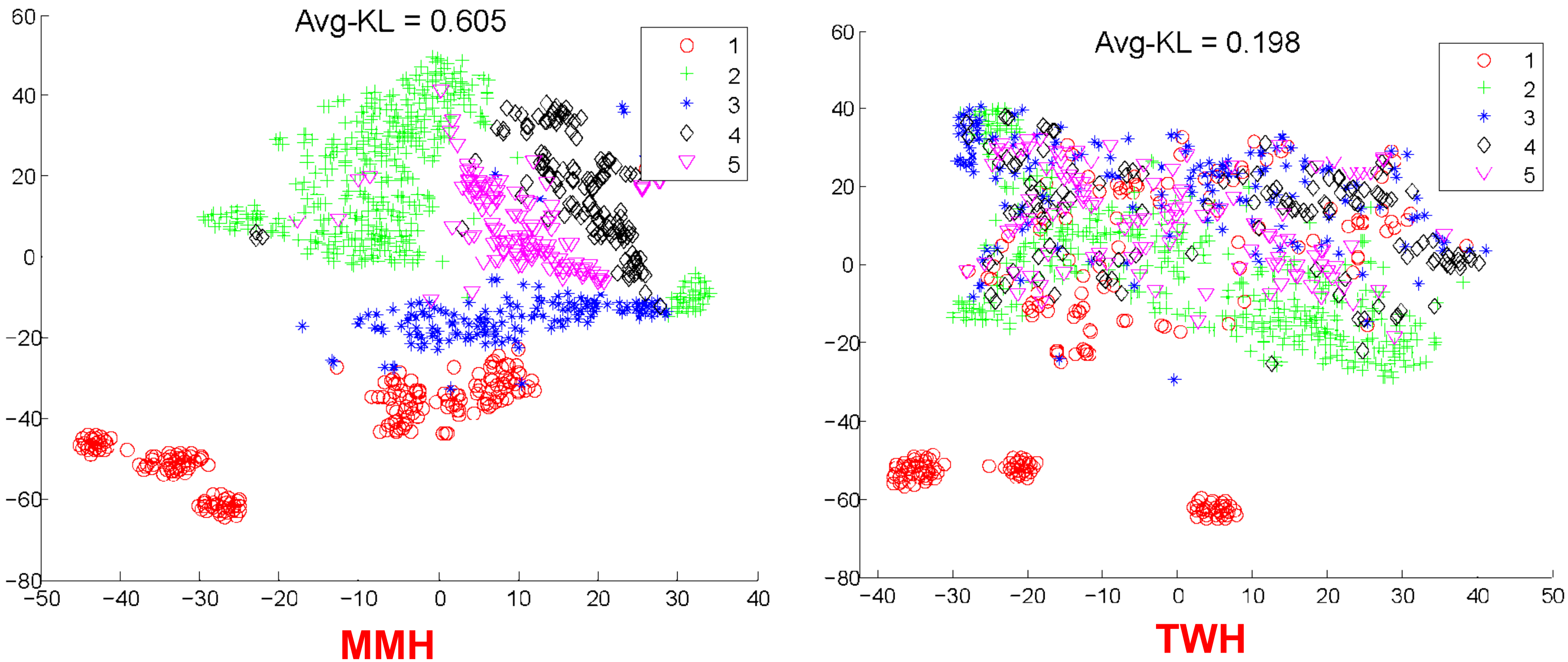
normalization factor



- Parameters can be learned with maximum likelihood estimation, e.g., special supervised Harmonium (Yang et al., 2007)
  - contrastive divergence is the commonly used approximation method in learning undirected latent variable models (Welling et al., 2004; Salakhutdinov & Murray, 2008).

Predictive Latent Representation

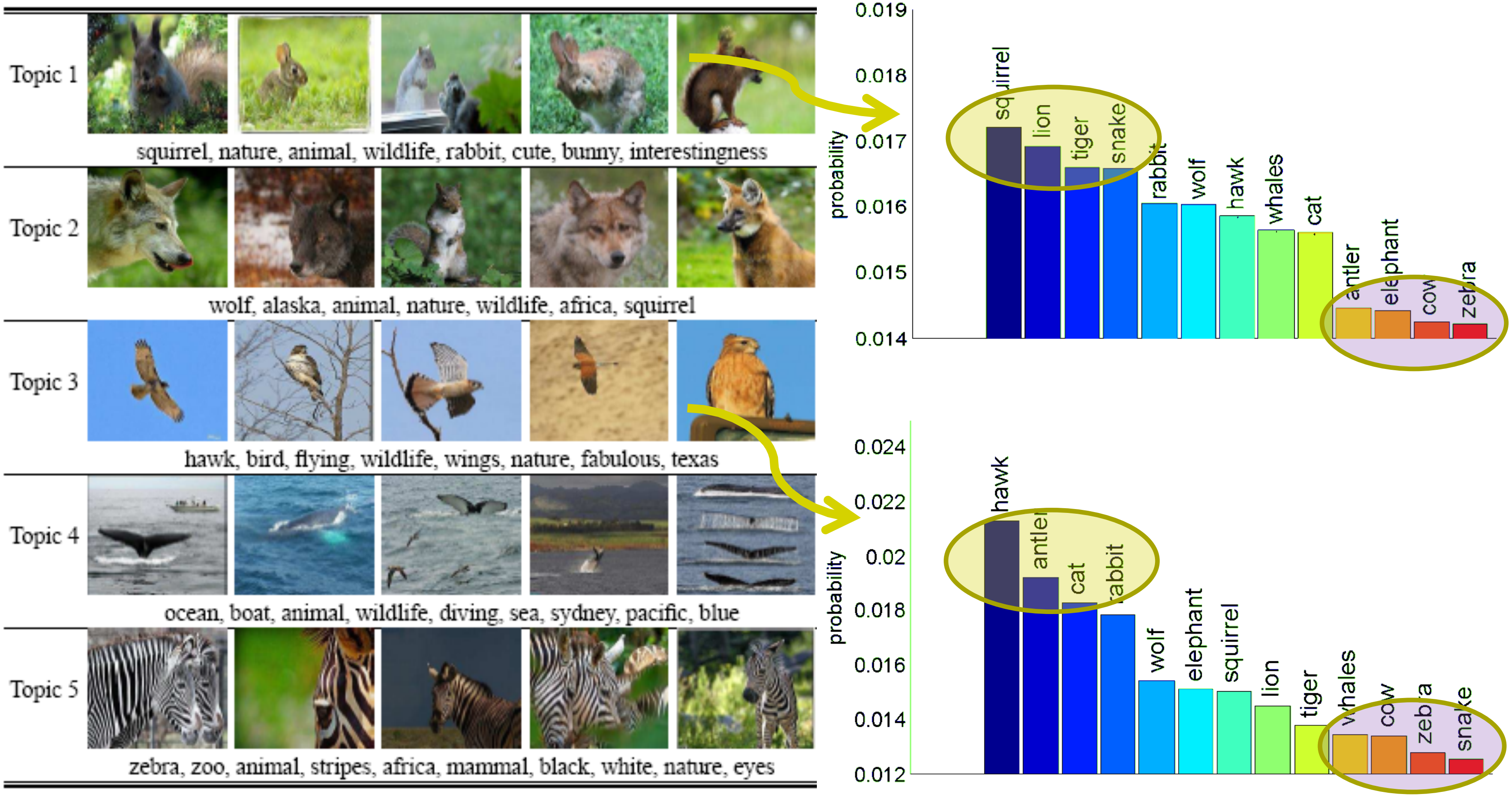
- t-SNE (van der Maaten & Hinton, 2008) 2D embedding of the discovered latent space representation on the TRECVID 2003 data



- Avg-KL: average pair-wise divergence

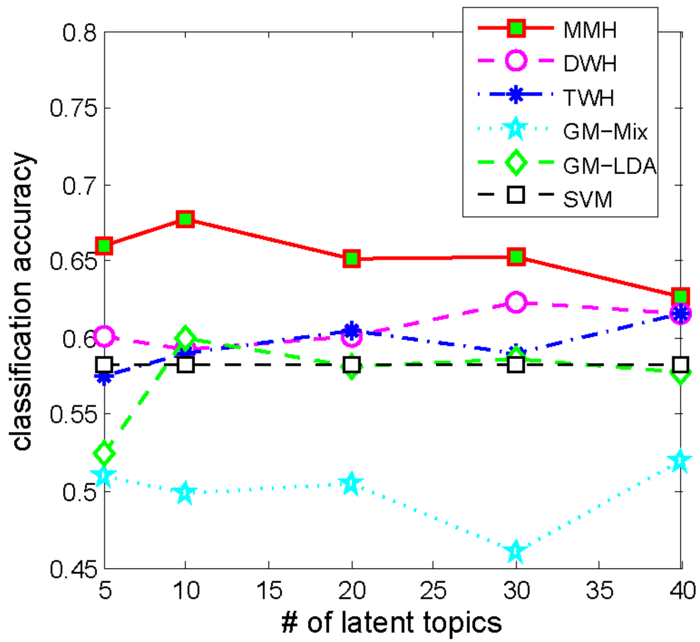
Predictive Latent Representation

- Example latent topics discovered by a 60-topic MMH on Flickr Animal Data

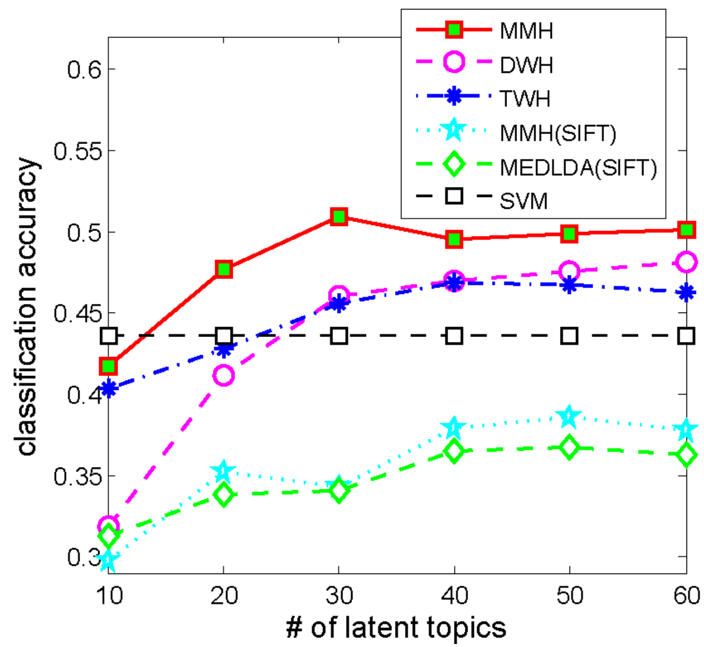


Classification Results

- **Data Sets:**
  - (Left) TRECVID 2003: (text + image features)
  - (Right) Flickr 13 Animal: (sift + image features)
- **Models**:
  - baseline(SVM),DWH+SVM, GM-Mixture+SVM, GM-LDA+SVM, TWH, MedLDA(sift only), MMH



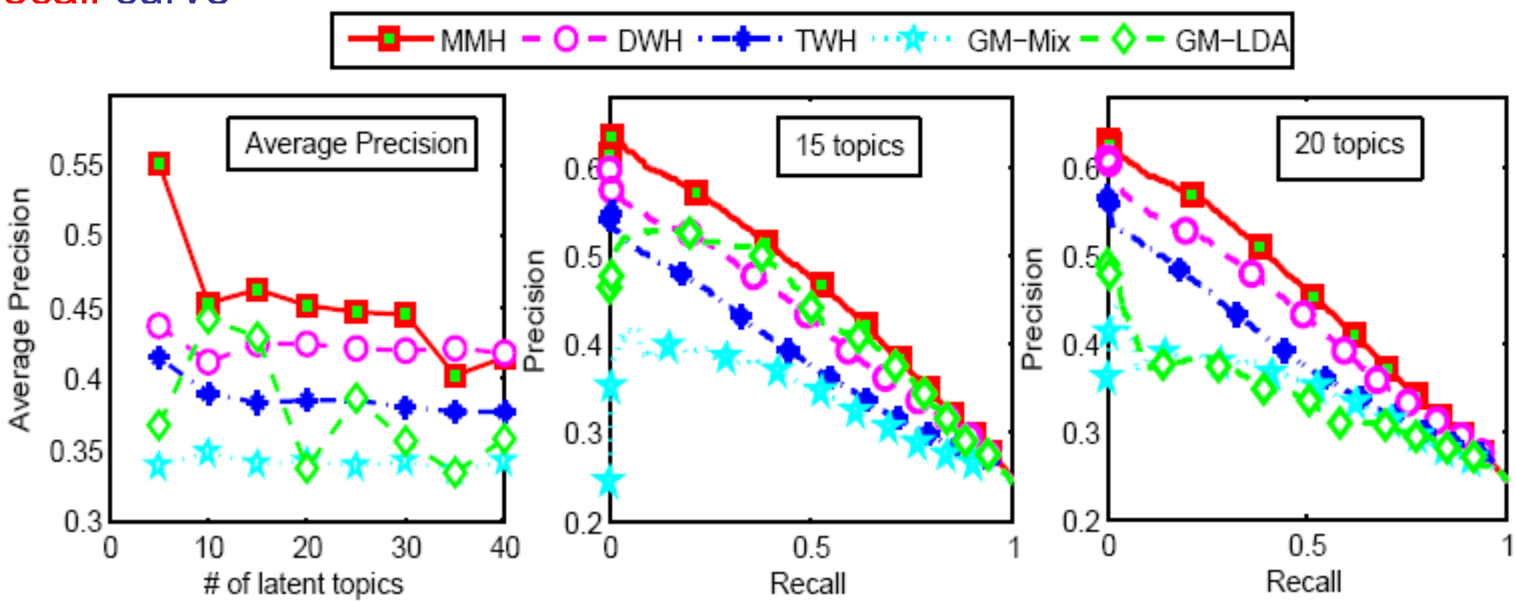
TRECVID



Flickr

Retrieval Results

- **Data Set: TRECVID 2003**
  - Each test sample is treated as a query, training samples are ranked based on the cosine similarity between a training sample and the given query
  - Similarity is computed based on the discovered latent topic representations
- **Models:** DWH, GM-Mixture, GM-LDA, TWH, MMH
- **Measure**: (Left) average precision on different topics and (Right) precision-recall curve



Infinite SVM and infinite latent SVM:

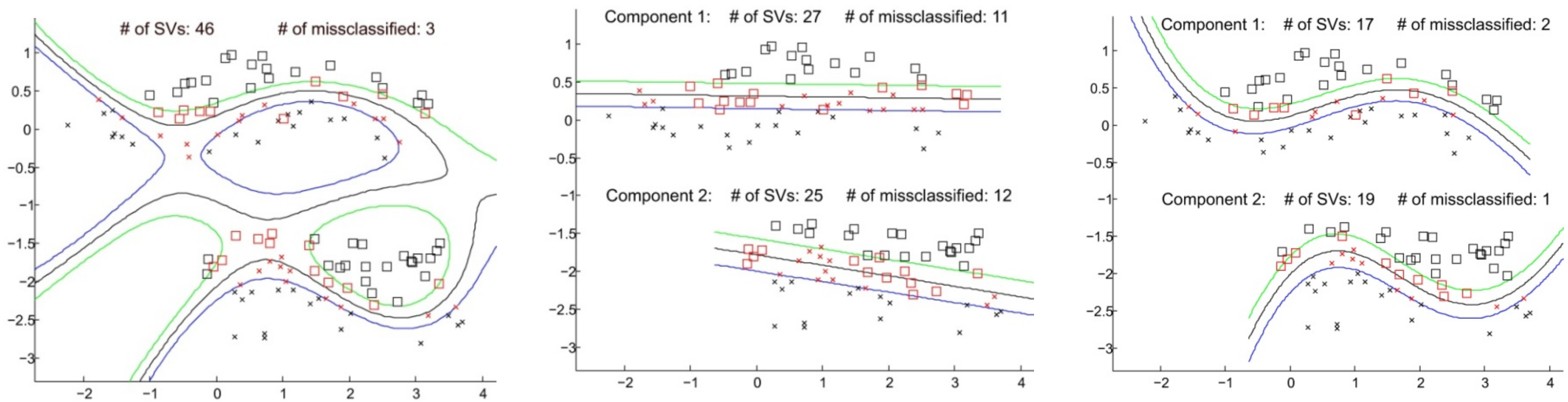
-- where SVMs meet NB for classification and feature selection

… where M is any combinations of classifiers and p is a nonparametric Bayesian prior

Mixture of SVMs

- Dirichlet process mixture of large-margin kernel machines
- Learn flexible non-linear local classifiers; potentially lead to a better control on model complexity, e.g., few unnecessary components

SVM using RBF kernel | **Mixture of 2 linear SVM** | **Mixture of 2 RBF-SVM**



- The first attempt to integrate Bayesian nonparametrics, large-margin learning, and kernel methods

Infinite SVM

- RegBayes framework:

$$\min_{p(\mathcal{M}),\xi} \quad \mathrm{KL}(p(\mathcal{M})\|\pi(\mathcal{M})) - \sum_{n=1}^{N} \int \log p(\mathbf{x}_n|\mathcal{M})p(\mathcal{M})d\mathcal{M} + U(\xi)$$

$$\text{s.t.}: \quad p(\mathcal{M}) \in \mathcal{P}_{\text{post}}(\xi),$$

convex function

direct and rich constraints on posterior distribution

- Model – latent class model
- Prior – Dirichlet process
- Likelihood – Gaussian likelihood
- Posterior constraints – max-margin constraints

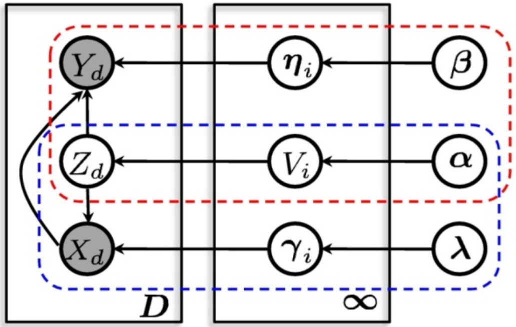
Infinite SVM

- DP mixture of large-margin classifiers

  process of determining which classifier to use:

  1. draw $V_i|\alpha \sim \text{Beta}(1, \alpha)$, $i \in \{1, 2, \cdots\}$.
  2. draw $\eta_i|G_0 \sim G_0$, $i \in \{1, 2, \cdots\}$.
  3. for the dth data point:

     (a) draw $Z_d|\{v_1, v_2, \cdots\} \sim \text{Mult}(\pi(\mathbf{v}))$



Graphical model with stick-breaking construction of DP

- Given a component classifier:

$$F(y, \mathbf{x}; z, \boldsymbol{\eta}) = \eta_z^\top \mathbf{f}(y, \mathbf{x}) = \sum_{i=1}^{\infty} \delta_{z,i} \eta_i^\top \mathbf{f}(y, \mathbf{x})$$

- Overall discriminant function:

$$F(y, \mathbf{x}) = \mathrm{E}_{q(z,\boldsymbol{\eta})}[F(y, \mathbf{x}; z, \boldsymbol{\eta})] == \sum_{i=1}^{\infty} q(z = i) \mathrm{E}_q[\eta_i]^\top \mathbf{f}(y, \mathbf{x})$$

- Prediction rule: $y^* = \arg\max_y F(y, \mathbf{x})$

- Learning problem: $\min_{q(\mathbf{z},\boldsymbol{\eta})} \mathrm{KL}(q(\mathbf{z}, \boldsymbol{\eta}) \| p_0(\mathbf{z}, \boldsymbol{\eta})) + C_1 \mathcal{R}(q(\mathbf{z}, \boldsymbol{\eta}))$,

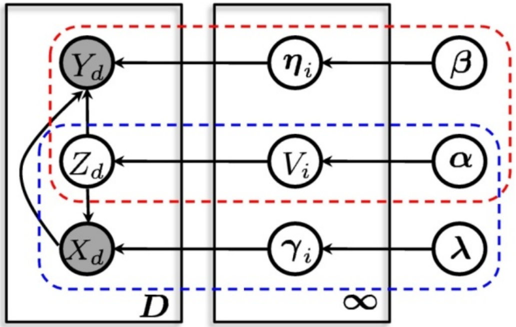
$$\mathcal{R}(q(\mathbf{z}, \boldsymbol{\eta})) = \sum_d \max_y (\ell_d^{\Delta}(y) + F(y, \mathbf{x}_d) - F(y_d, \mathbf{x}_d))$$

Infinite SVM

- Assumption and relaxation
  - Truncated variational distribution

$$q(\mathbf{z}, \boldsymbol{\eta}, \boldsymbol{\gamma}, \mathbf{v}) = \prod_{d=1}^{D} q(z_d) \prod_{t=1}^{T} q(\eta_t) \prod_{t=1}^{T} q(\gamma_t) \prod_{t=1}^{T-1} q(v_t)$$

  - Upper bound the KL-regularizer



Graphical model with stick-breaking construction of DP

- Opt. with coordinate descent
  - For $q(\boldsymbol{\eta})$, we solve an SVM learning problem
  - For $q(\mathbf{z})$, we get the closed update rule

$$q(z_d = t) \propto \exp\left\{ (\mathbb{E}[\log v_t] + \sum_{i=1}^{t-1} \mathbb{E}[\log(1 - v_i)]) + \rho(\mathbb{E}[\gamma_t]^\top \mathbf{x}_d - \mathbb{E}[A(\gamma_t)]) + (1-\rho) \sum_y \omega_d^y \mu_t^\top \mathbf{f}_d^\Delta(y) \right\}$$

    - The last term regularizes the mixing proportions to favor prediction
  - For $q(\boldsymbol{\gamma}), q(\mathbf{v})$, the same update rules as in (Blei & Jordan, 2006)

Experiments on high-dim real data

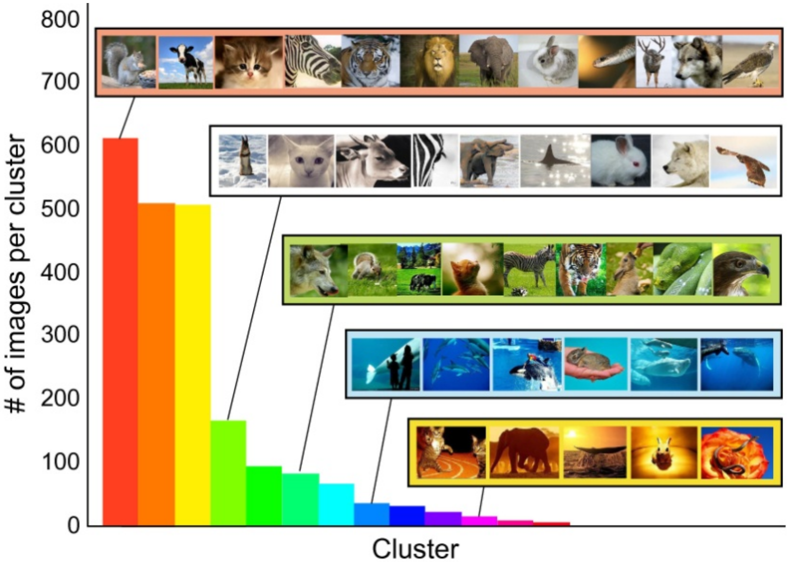
- Classification results and test time:

Table 4. Classification accuracy (%), F1 score (%), and test time (sec) for different models on the Flickr image dataset. All methods except dpMNL are implemented in C.

	ACCURACY	F1 SCORE	TEST TIME
MNL	49.8 ± 0.0	48.4 ± 0.0	**0.02** ± 0.00
MMH	51.7 ± 0.0	50.1 ± 0.0	0.33 ± 0.01
RBF-SVM	52.2 ± 0.0	48.4 ± 0.0	7.58 ± 0.06
DPMNL-EFH70	51.2 ± 0.9	49.9 ± 0.8	42.1 ± 7.39
DPMNL-PCA50	51.9 ± 0.7	49.9 ± 0.8	27.4 ± 2.08
LINEAR-ISVM	53.2 ± 0.4	51.3 ± 0.4	0.22 ± 0.01
RBF-ISVM	**54.2** ± 0.5	**51.6** ± 0.7	6.67 ± 0.05

For training, linear-iSVM is very efficient (~200s); RBF-iSVM is much slower, but can be significantly improved using efficient kernel methods (Rahimi & Recht, 2007; Fine & Scheinberg, 2001)

- Clusters:
  - simiar backgroud images group
  - a cluster has fewer categories



Learning Latent Features

- Infinite SVM is a Bayesian nonparametric latent class model
  - discover clustering structures
  - each data point is assigned to a single cluster/class

- Infinite Latent SVM is a Bayesian nonparametric latent feature/factor model
  - discover latent factors
  - each data point is mapped to a set (can be infinite) of latent factors

  - Latent factor analysis is a key technique in many fields; Popular models are FA, PCA, ICA, NMF, LSI, etc.

Infinite Latent SVM

- RegBayes framework:

$$\min_{p(\mathcal{M}),\xi} \quad \mathrm{KL}(p(\mathcal{M})\|\pi(\mathcal{M})) - \sum_{n=1}^{N} \int \log p(\mathbf{x}_n|\mathcal{M})p(\mathcal{M})d\mathcal{M} + U(\xi)$$

$$\mathrm{s.t.}: \quad p(\mathcal{M}) \in \mathcal{P}_{\mathrm{post}}(\xi),$$

convex function

direct and rich constraints on posterior distribution

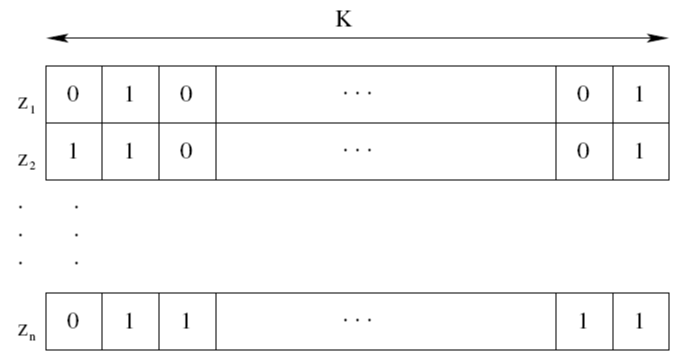
- Model – latent feature model
- Prior – Indian Buffet process
- Likelihood – Gaussian likelihood
- Posterior constraints – max-margin constraints

Beta-Bernoulli Latent Feature Model

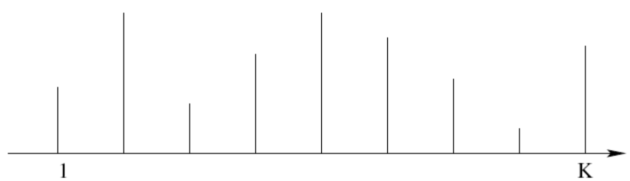
- A random finite binary latent feature models

$$\pi_k | \alpha \sim \text{Beta}(\frac{\alpha}{K}, 1)$$

$$z_{ik} | \pi_k \sim \text{Bernoulli}(\pi_k)$$



  - π_k is the relative probability of each feature being on, e.g.,

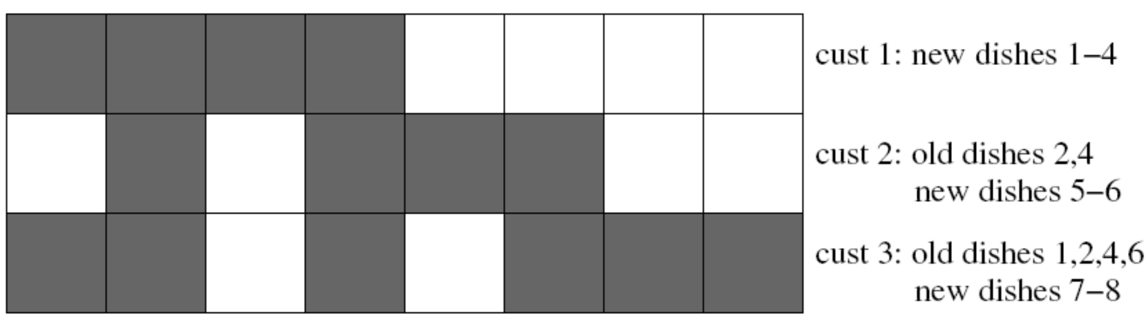


  - $z_{i\cdot}$ are binary vectors, giving the latent structure that's used to generate the data, e.g.,

$$\mathbf{x}_i \sim \mathcal{N}(\eta^\top z_{i\cdot}, \delta^2)$$

Indian Buffet Process

- A stochastic process on infinite binary feature matrices
- Generative procedure:
  - Customer 1 chooses the first K_1 dishes: $K_1 \sim \text{Poisson}(\alpha)$
  - Customer *i* chooses:
    - Each of the existing dishes with probability $\frac{m_k}{i}$
    - K_i additional dishes, where $K_i \sim \text{Poisson}(\frac{\alpha}{i})$



$$Z_{i\cdot} \sim \mathcal{IBP}(\alpha)$$

Posterior Constraints – classification

- Suppose latent features **z** are given, we define *latent discriminant function*:

$$f(y, \mathbf{x}, \mathbf{z}; \boldsymbol{\eta}) = \boldsymbol{\eta}^\top \mathbf{g}(y, \mathbf{x}, \mathbf{z})$$

- Define *effective discriminant function* (reduce uncertainty):

$$f(y, \mathbf{x}; p(\mathbf{Z}, \boldsymbol{\eta})) = \mathbb{E}_{p(\mathbf{Z}, \boldsymbol{\eta})}[f(y, \mathbf{x}, \mathbf{z}; \boldsymbol{\eta})] = \mathbb{E}_{p(\mathbf{Z}, \boldsymbol{\eta})}[\boldsymbol{\eta}^\top \mathbf{g}(y, \mathbf{x}, \mathbf{z})]$$

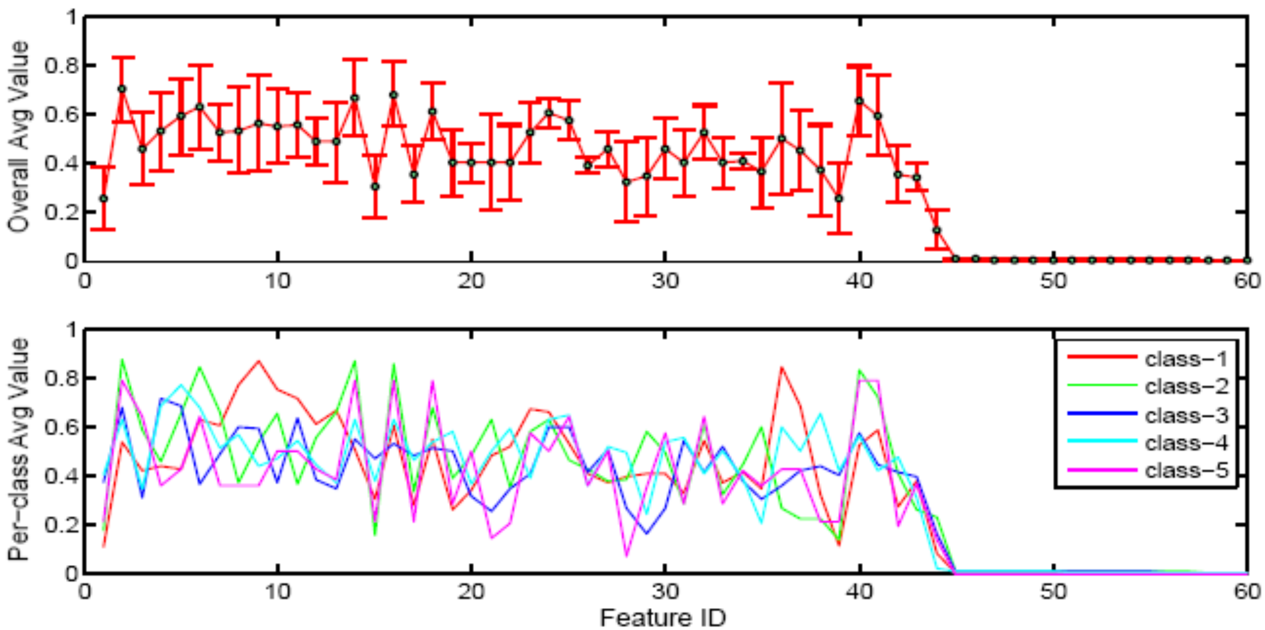
- Posterior constraints with max-margin principle

$$\forall n \in \mathcal{I}_{\mathrm{tr}}, \forall y: \quad f(y_n, \mathbf{x}_n; p(\mathbf{Z}, \boldsymbol{\eta})) - f(y, \mathbf{x}_n; p(\mathbf{Z}, \boldsymbol{\eta})) \geq \ell(y, y_n) - \xi_n$$

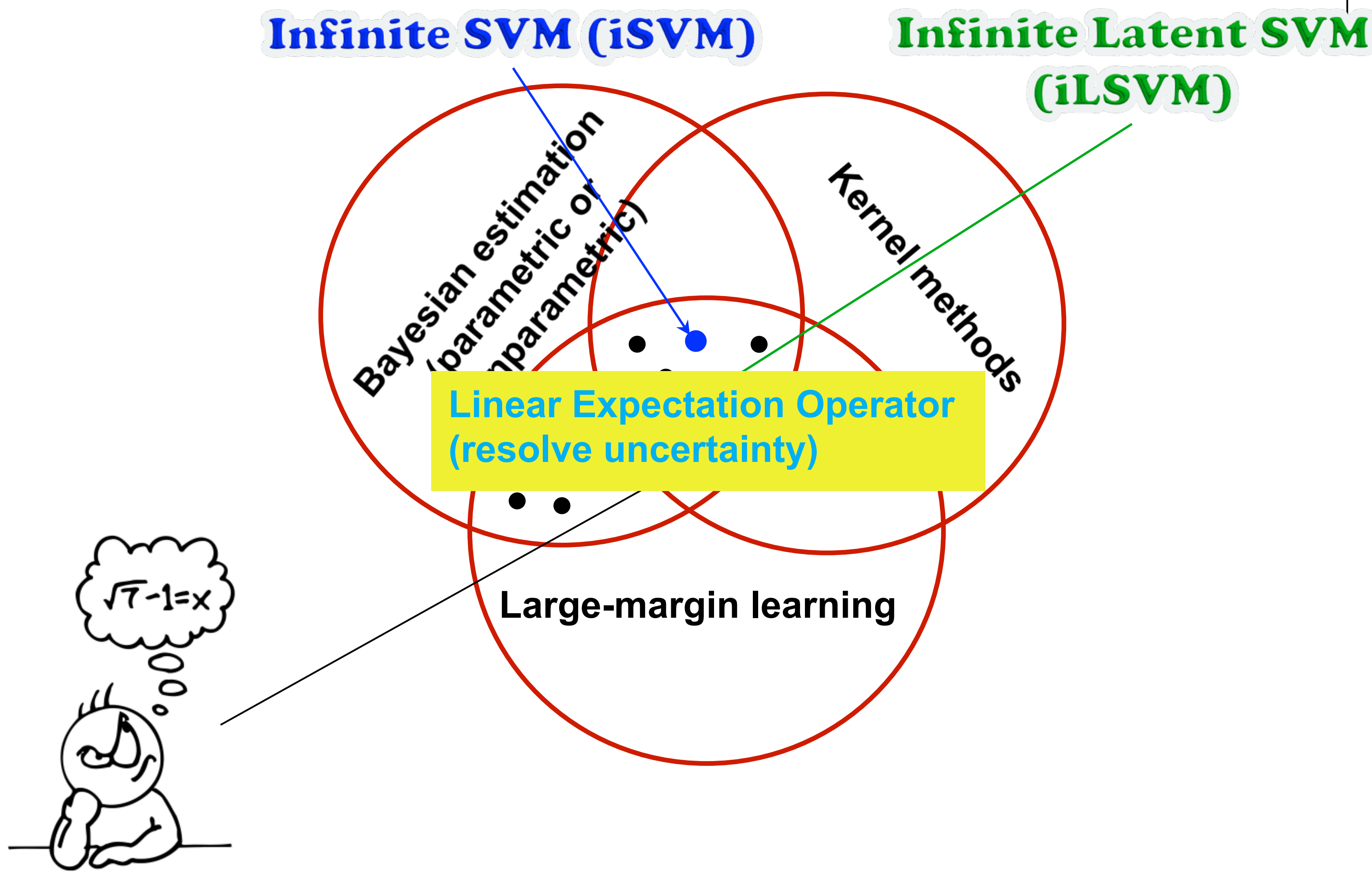
Experimental Results

- Classification
  - Accuracy and F1 scores on TRECVID2003 and Flickr image datasets

Model	TRECVID2003 Accuracy	TRECVID2003 F1 score	Flickr Accuracy	Flickr F1 score
EFH+SVM	0.565 ± 0.0	0.427 ± 0.0	0.476 ± 0.0	0.461 ± 0.0
MMH	**0.566** ± 0.0	0.430 ± 0.0	**0.538** ± 0.0	**0.512** ± 0.0
IBP+SVM	0.553 ± 0.013	0.397 ± 0.030	0.500 ± 0.004	0.477 ± 0.009
iLSVM	0.563 ± 0.010	**0.448** ± 0.011	0.533 ± 0.005	0.510 ± 0.010



Summary



Infinite SVM (iSVM)
Infinite Latent SVM (iLSVM)
Bayesian estimation
Kernel methods
Linear Expectation Operator (resolve uncertainty)
Large-margin learning
$\sqrt{7}-1=x$

Summary

- A general framework of MaxEnDNet for learning structured input/output models
  - Subsumes the standard M^3Ns
  - Model averaging: PAC-Bayes theoretical error bound
  - Entropic regularization: sparse M^3Ns
  - **Generative + discriminative: latent variables, semi-supervised learning on partially labeled data, fast inference**
- PoMEN
  - Provides an elegant approach to incorporate latent variables and structures under max-margin framework
  - Enable Learning arbitrary graphical models discriminatively
- Predictive Latent Subspace Learning
  - MedLDA for text topic learning
  - Med total scene model for image understanding
  - Med latent MNs for multi-view inference
- Bayesian nonparametrics meets max-margin learning
- Experimental results show the advantages of max-margin learning over likelihood methods in **EVERY** case.

Remember: Elements of Learning

- **Here are some important elements to consider before you start:**
  - Task:
    - Embedding? Classification? Clustering? Topic extraction? …
  - Data and other info:
    - Input and output (e.g., continuous, binary, counts, …)
    - Supervised or unsupervised, of a blend of everything?
    - Prior knowledge? Bias?
  - Models and paradigms:
    - BN? MRF? Regression? SVM?
    - Bayesian/Frequents ? Parametric/Nonparametric?
  - Objective/Loss function:
    - MLE? MCLE? Max margin?
    - Log loss, hinge loss, square loss? …
  - Tractability and exactness trade off:
    - Exact inference? MCMC? Variational? Gradient? Greedy search?
    - Online? Batch? Distributed?
  - Evaluation:
    - Visualization? Human interpretability? Perperlexity? Predictive accuracy?