

# Generative Latent Variable Models of Text

Jacob Eisenstein

Machine Learning Department, CMU

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# Generative models of text

Generative models are a powerful tool for understanding document collections.

- Classification/clustering (Naive Bayes)
- Discover latent themes (LDA)
- Distinguish latent and observed factors (e.g. Topic-aspect models)

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**Unifying idea:** a probability model over text,  $P(w|z)$ ,  
where  $z$  are labels or latent variables

# Classification

Naive Bayes is a generative model for classification:

$$\begin{aligned}\log P(w^{(d)}|z^{(d)}, \beta) &= \prod_n P(w_n^{(d)}|\beta, z_n^{(d)}) \\ &= \prod_n \beta_{z_n^{(d)}, w_n^{(d)}}\end{aligned}$$

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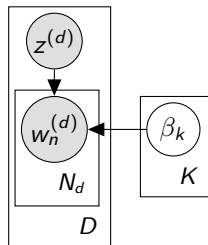
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- **training:**

$$\hat{\beta} = \arg \max_{\beta} \prod_d P(w^{(d)}|z^{(d)}, \beta)$$

- **prediction:**

$$\hat{z}^{(d)} = \arg \max_y P(w^{(d)}|z, \beta)$$



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- The conjugate prior for the multinomial is the **Dirichlet** distribution.

Conjugacy means we can do collapsed Gibbs sampling, analytically marginalizing the parameter  $\beta$ . This trick gets used **a lot**.



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  - They perform smoothing, improving performance when data is limited or the number of parameters is very large.
  - Priors also make it possible to incorporate domain knowledge.
- **Spoiler:** I'll have a lot more to say about whether the Dirichlet-Multinomial pair is the best possible choice for generative models.

# Naive Bayes

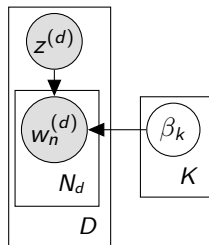
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# Example: Political ideology classification on Twitter

## Training data:

### Messages containing #p2

 **LCranston1939** Lamont Cranston  
Ain't Facism: Coordinated [iOWS](#) evictions make this clear: the military and the police exist to protect the 1% [bit.ly/lpsn5m #p2](#)  
1 hour ago

 **rhrealitycheck** RH Reality Check  
Thanks to [@BarbaraBCrane](#) (of [@lpasOrg](#)) for donating & helping us stop the right-wing effort to [#OccupyYourWomb!](#) [ow.ly/7uuej #p2...](#)  
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 **BuddyRoemer** Gov. Buddy Roemer  
Bloomberg's actions in the midnight hours against Occupy Wall Street protestors were unjust, uncalled for, and unconstitutional. [#p2 #ows](#)  
3 hours ago [♡](#) [Favorita](#) [♡](#) [Retweet](#) [♡](#) [Reply](#)

 **peterrothberg** Peter Rothberg  
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 **mmfa** Media Matters  
Don't worry, folks: [#FoxNews](#) has enough conspiracy theorists to go around for anyone willing to believe them! [bit.ly/utOthM #p2](#)  
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### Messages containing #tcot

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I miss having a president who loves America and Americans. RT if you agree, [#tcot](#)  
13 Nov  
[♡](#) [Retweeted](#) 100+ times

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- $\beta_{\#p2}$  emphasizes *protest*, *unconstitutional*, *fascism*
- $\beta_{\#tcot}$  emphasizes *nobama*, *solyndra*, *socialism*



# Naive Bayes for Ideology Prediction

Lin et al (2006) applied Naive Bayes to the “bitter lemons” corpus of text about the Palestinian-Israeli conflict:

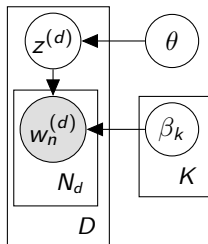
Model	Data Set	Accuracy	Reduction
Baseline		0.5	
SVM	Editors	0.9724	
NB-M	Editors	0.9895	61%
NB-B	Editors	0.9909	67%
SVM	Guests	0.8621	
NB-M	Guests	0.8789	12%
NB-B	Guests	0.8859	17%

Palestinian	palestinian, israel, state, politics, peace, international, people, settle, occupation, sharon, right, govern, two, secure, end, conflict, process, side, negotiate
Israeli	israel, palestinian, state, settle, sharon, peace, arafat, arab, politics, two, process, secure, conflict, lead, america, agree, right, gaza, govern

# Unsupervised Naive Bayes

When the label  $z$  is not observed, it can be imputed.

This is a method for probabilistic clustering:



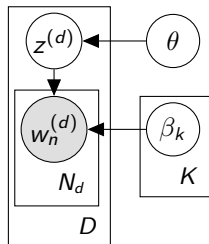
$$P(w|\theta, \beta) = \sum_z P(z|\theta) \prod_n P(w_n|\beta_z)$$

where  $\theta$  is a prior on  $z$ .

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Typically we optimize using expectation-maximization:

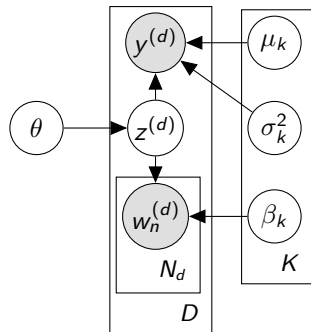
- In the **e-step** we compute the distribution  $Q(z)$
- In the **m-step** we update the parameter  $\beta$

# Latent Variable Models

- Imagine we have additional data  $y^{(d)}$ :  
for each author on Twitter,
  - $y^{(d)}$  is their geographical location,
  - $w^{(d)}$  is the set of all words in all their tweets,
  - $z^{(d)}$  is a latent variable which must explain both  $y^{(d)}$  and  $w^{(d)}$ .
- We want to learn to predict  $y$  from  $w$ .  
(Eisenstein, O'Connor, Smith, and Xing.  
EMNLP 2010)

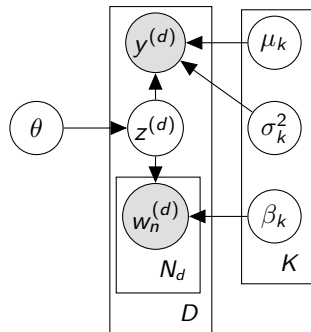
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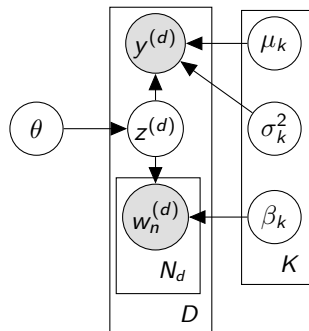


In training, we maximize:

$$P(y, w | \theta, \beta, \mu, \sigma^2) = \sum_z P(z | \theta) P(y | \mu_z, \sigma_z^2) \prod_n P(w_n | \beta_z)$$

# Latent Variable Models

- **training**: Expectation-maximization, alternating between updates to  $Q(z)$  and the parameters  $\{\beta, \theta, \mu, \sigma^2\}$



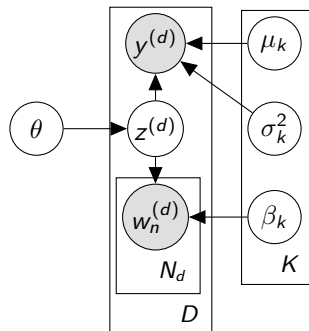
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- **prediction**:

$$\hat{y} = \arg \max_y P(y|w)$$

$$P(y|w) = \sum_z P(y|\mu_z, \sigma_z^2) P(z|w, \theta)$$

$$P(z|w, \theta) = P(w|\beta_z) P(z|\theta) / P(w)$$

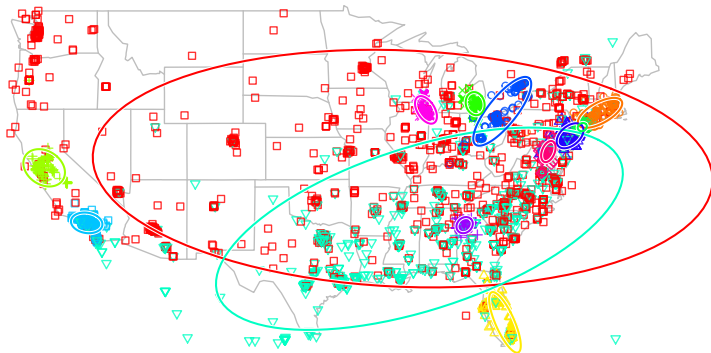




# Quantitative Results

error in km:	mean	median
mean location	1148	1018
text regression	948	712
<b>mixture model</b>	947	644

# Qualitative Results



Each author in our dataset is a point;  
cluster membership is indicated by color and shape.<sup>1</sup>

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<sup>1</sup>Figure by Brendan O'Connor

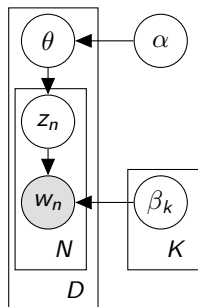
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For each cluster, we rank words by log-odds:  $\log \beta_i - \log \frac{1}{K} \sum_j \beta_j$ :

- **New York:** brib, lml, wassupp, uu, werd, deadass, flatbush, odee, dha
- **So. Cal:** disneyland, cuh, fucken, af, fasho, faded, wyd, freeway, bomb
- **No. Cal:** sac, oakland, sf, hellla, warriors, pleasure, bay, koo
- **Atlanta:** atlanta, atl, georgia, ga, \$1, waffle, af, nun, shawty
- **Cleveland/Detroit:** ctfu, detroit, foolin, .!! , cleveland, geeked, salty, ikr
- **Pac. Northwest:** seattle, portland, oregon, olympic, heh, canada, stoked

# Discovering latent themes

Topic models like latent Dirichlet allocation discover latent **themes** or **topics** in document collections:



- Each  $\beta_k$  is a topic, a distribution over words.
- Each  $\theta_d$  represents the topic proportions for document  $d$ .
- Each  $z_n$  is the latent topic which generates the word  $w_n$ .

$$P(w|\theta, \beta) = \prod_n P(z_n|\theta)P(w_n|\beta_{z_n})$$

# Topics in Twitter

“basketball”	“popular music”	“daily life”	“emoticons”	“chit chat”
PISTONS KOBE LAKERS game DUKE NBA CAVS STUCKEY JETS KNICKS	album music beats artist video #LAKERS ITUNES tour produced vol	tonight shop weekend getting going chilling ready discount waiting iam	: ) haha :d :( : ) :p xd :/ hahaha hahah	lol smh jk yea wyd coo ima wassup somethin jp

Key point is that individual authors are **admixtures** of these topics, e.g., my Twitter feed is 60% chit-chat, 30% basketball, 10% emoticons.

# Combining topics and labels

Recall the Twitter political ideology problem:

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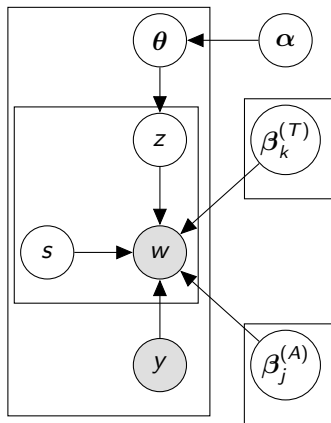


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- In **analysis**, we often want to understand topic-specific differences: e.g., how do the left-wing and right-wing perspectives differ with respect to foreign policy

# Switching models

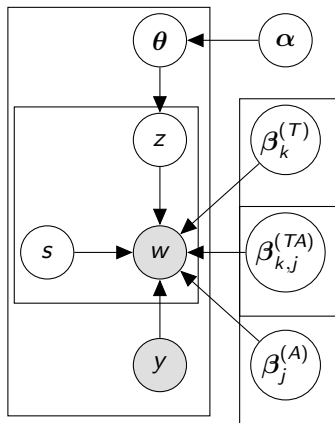
We can combine topics and labels by adding a “switch” for each word, which determines if the word is generated from a topic or the label:



- Each  $s_n$  determines whether  $w_n$  is generated by the topic  $z_n$  or the label  $y$ .
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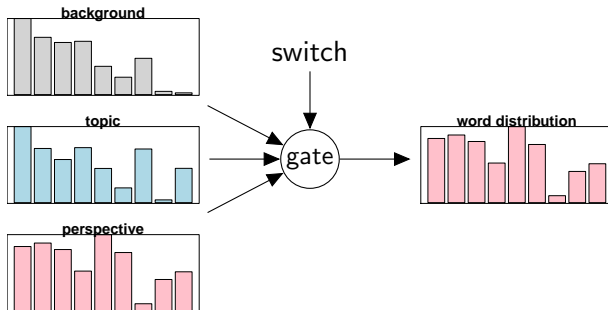
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- Each  $\beta_{k,j}^{(TA)}$  is a word distribution associated with a topic-label **interaction**.

# Switching models: a schematic

A topic-perspective-background model:



# Example output: topics and cultures

Topic 1			Topic 2		
fashion style look dress wear new collection accessories black			food add chicken recipe cooking taste rice recipes sugar soup		
UK	India	Singapore	UK	India	Singapore
shoes fashion clothing high designer style love london shirts bag	fashion women indian designer sarees leather girls china jewellery jewelry	price posted earrings length item sgd silver clothes shop code	food wine restaurant coffee cheese soup eat chef english drink	recipe recipes powder indian salt tsp rice masala oil coriander	coffee cup oil comments fried add restaurant rice tea seafood

From ccLDA (Paul and Girju, 2009)

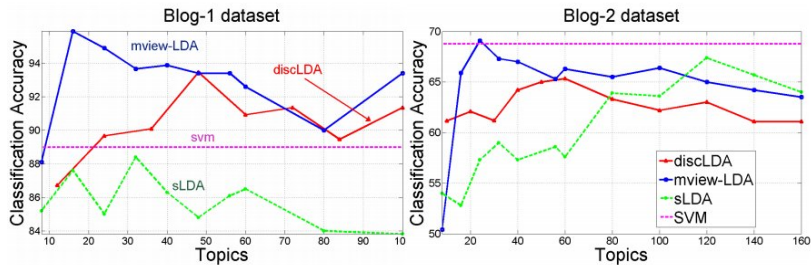
## Example output: topics and perspectives

palestinian israeli israel military civilians attacks	
Aspect A	Aspect B
war	violence
public	palestinians
government	occupation
media	resistance
society	intifada
terrorist	violent
soldiers	non
incitement	force

state israel solution palestine palestinian states borders	
Israeli	Palestinian
jewish	palestinians
arab	return
israeli	right
jews	refugees
population	problem
jordan	refugee
west	rights
south	resolution

From TAM (Paul and Girju, 2010);  
added unsupervised and semi-supervised learning to ccLDA.

# Results: ideology prediction



From Multiview-LDA (Ahmed and Xing, 2010)

## Results: geography prediction

error in km:	mean	median
mean location	1148	1018
text regression	948	712
mixture model	947	644
<b>mixture model + topics</b>	900	494



Capabilities of generative models:

- Classification and clustering (Naive Bayes)
- Discovering latent topics (LDA)
- Combining topics and labels (ccLDA, TAM, Multiview-LDA)

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We have focused on text, but there are many, many applications of these models to vision and computational biology.

Generative models have many advantages:

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But they also have problems! (Eisenstein et al., ICML 2011)

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- Heuristic solutions like stopword pruning are hard to generalize to new domains.
- It would be better to focus computation on parameters that distinguish the classes.



# Overparametrization

- An LDA **model** with  $K$  topics and  $V$  words requires  $K \times V$  parameters.
- An LDA **paper** shows 10 words per topic.

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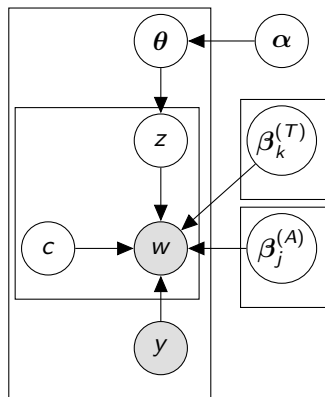
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- What about the other  $V - 10$  words per topic??
  - These parameters affect the assignment of documents...
  - But they may be unnoticed by the user.
  - And there may not be enough data to estimate them accurately.

# Inference complexity

- Latent topics may be combined with additional facets, such as sentiment and author perspective.
- “Switching” variables decide if a word is drawn from a topic or from another facet.
- Twice as many latent variables per document!



# Sparse Additive Generative Models

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- **Sparse Additive Generative models (SAGE)**: each class or latent theme is represented by its deviation from a background distribution.

$$P(w|y, \mathbf{m}) \propto \exp(\mathbf{m} + \eta_y)$$

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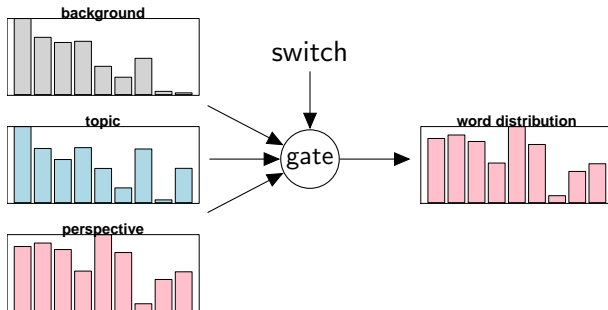
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$$P(w|y, \mathbf{m}) \propto \exp(\mathbf{m} + \boldsymbol{\eta}_y)$$

- $\mathbf{m}$  captures the background word log-probabilities
- $\boldsymbol{\eta}$  contains **sparse** deviations for each topic or class
- additional facets can be added in log-space

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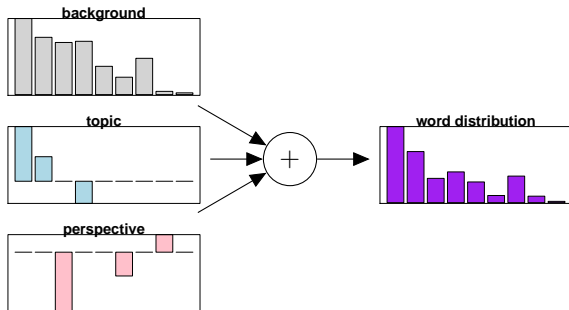
A topic-perspective-background model using Dirichlet-multinomials:





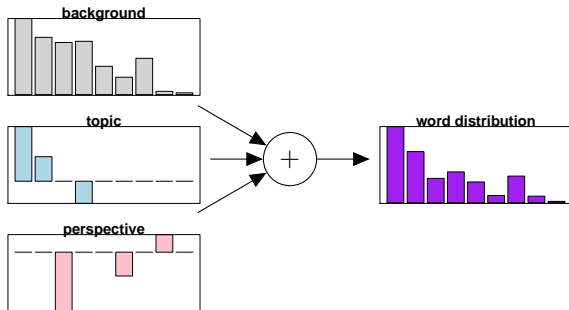
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Different notion of sparsity from sparseTM (Wang & Blei, 2009),  
which sets  $Pr(w = i|y) = 0$  for many  $i$ .

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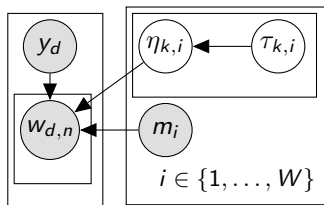
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  - The distribution  $Q(\tau)$
  - A **point estimate** of  $\eta$

# Applications

- Document classification
- Topic models
- Multifaceted topic models

# SAGE in document classification



- Each document  $d$  has a label  $y_d$
- Each token  $w_{d,n}$  is drawn from a multinomial distribution  $\beta$ , where
$$\beta_i = \frac{\exp(\eta_{y_d,i} + m_i)}{\sum_j \exp(\eta_{y_d,j} + m_j)}$$
- Each parameter  $\eta_{k,i}$  is drawn from a distribution equal to  $\mathcal{N}(0, \tau_{k,i})$ , with  $P(\tau_{k,i}) \sim 1/\tau_{k,i}$

- We maximize the variational bound

$$\begin{aligned}\ell = & \sum_d \sum_n^{N_d} \log P(w_n^{(d)} | \mathbf{m}, \boldsymbol{\eta}_{y_d}) + \sum_k \langle \log P(\boldsymbol{\eta}_k | \mathbf{0}, \boldsymbol{\tau}_k) \rangle \\ & + \sum_k \langle \log P(\boldsymbol{\tau}_k | \boldsymbol{\gamma}) \rangle - \sum_k \langle \log Q(\boldsymbol{\tau}_k) \rangle ,\end{aligned}$$



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- The gradient wrt  $\boldsymbol{\eta}$  is,

$$\frac{\partial \ell}{\partial \boldsymbol{\eta}_k} = \mathbf{c}_k - C_k \boldsymbol{\beta}_k - \text{diag}(\langle \boldsymbol{\tau}_k^{-1} \rangle) \boldsymbol{\eta}_k,$$

where

- $\mathbf{c}_k$  are the observed counts for class  $k$
- $C_k = \sum_i c_{ki}$
- $\boldsymbol{\beta}_k \propto \exp(\boldsymbol{\eta}_k + \mathbf{m})$

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- We choose  $Q(\tau_{k,i}) = \text{Gamma}(\tau_{k,i}; a_{k,i}, b_{k,i})$

# Inference

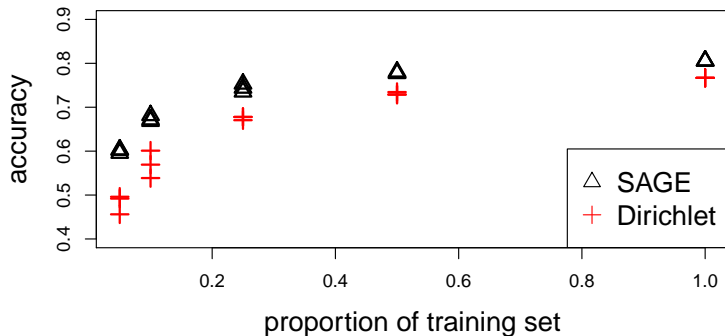
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- We choose  $Q(\tau_{k,i}) = \text{Gamma}(\tau_{k,i}; a_{k,i}, b_{k,i})$
- Iterate between a Newton update to  $a$  and a closed-form update to  $b$

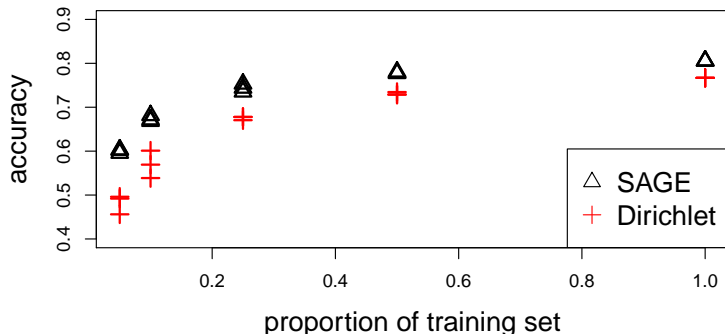
# Document classification evaluation

- 20 newsgroups data: 11K training docs, 50K vocab



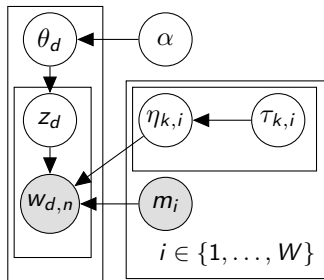
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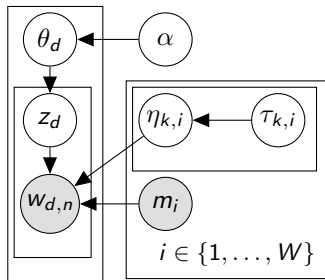


- Adaptive sparsity:
  - 10% non-zeros for full training set (11K docs)
  - 2% non-zeros for minimal training set (550 docs)

# SAGE in latent variable models



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The gradient for  $\boldsymbol{\eta}$  now includes **expected** counts:

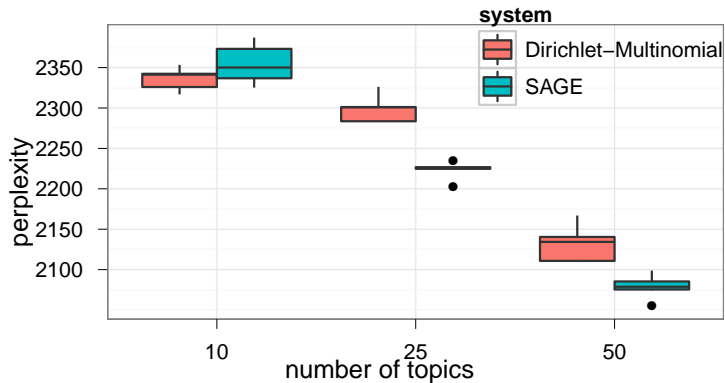
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where  $\langle c_{ki} \rangle = \sum_n Q_{z_n}(k) \delta(w_n = i)$ .



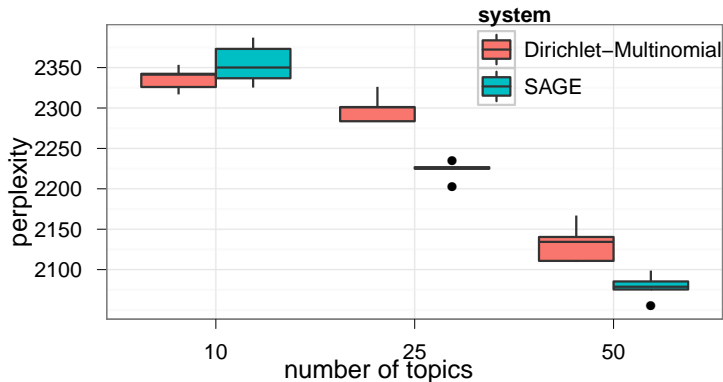
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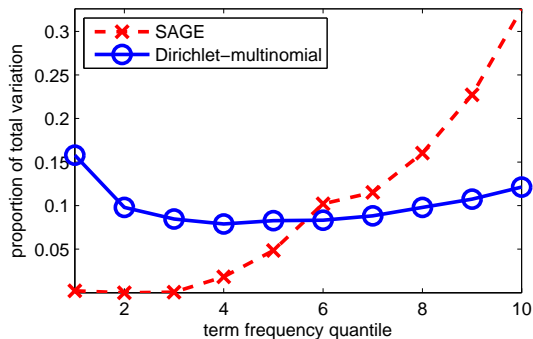
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- Adaptive sparsity:
  - 5% non-zeros for 10 topics
  - 1% non-zeros for 50 topics

# Sparse topic model analysis

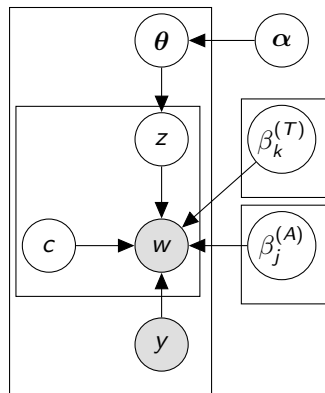
$$\text{Total variation} = \sum_i |\beta_{k,i} - \bar{\beta}_i|$$



Standard topic models assign the greatest amount of variation for the probabilities of the words with the least evidence!

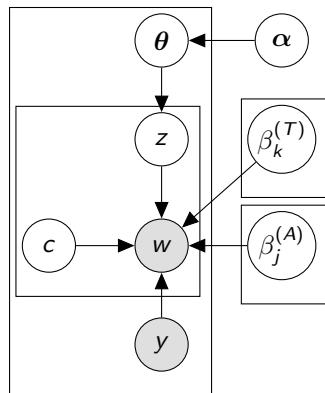
# Multifaceted generative models

- Combines latent topics  $\beta^{(T)}$  with other facets  $\beta^{(A)}$ , e.g. ideology, dialect, sentiment



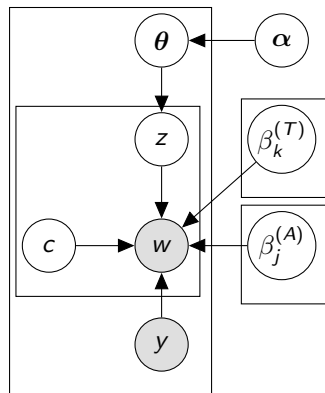
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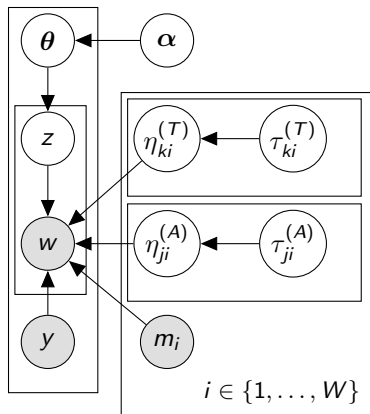
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- There is one switching variable per token, complicating inference.



# Multifaceted generative models in SAGE

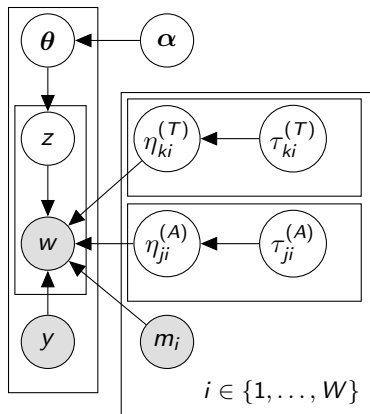
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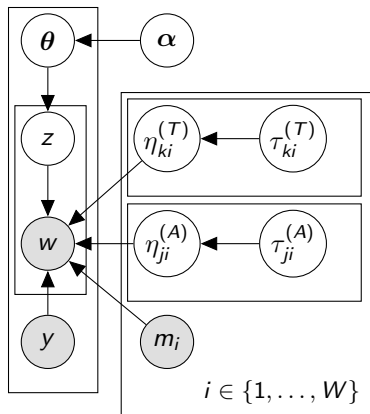
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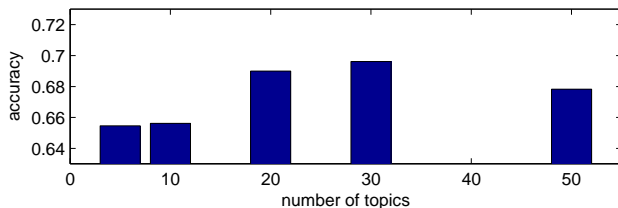


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Results match previous best of 69% for Multiview LDA and support vector machine (Ahmed & Xing, 2010).

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SAGE (22K vocab)	<b>791</b>	<b>461</b>

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- The Dirichlet-multinomial pair is computationally convenient, but does not adequately control model complexity.
- The **S**parse **A**dditive **GE**nerative model (SAGE):
  - gracefully handles extraneous parameters,
  - adaptively controls sparsity without a regularization constant,
  - facilitates inference in multifaceted models.

# Conclusion

- Generative models provide powerful tools for understanding natural language data.
- Capabilities include prediction, clustering, and discovering latent topics, as well as more exotic models that combine latent and observed aspects.
- As always, controlling model complexity is critical.
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Thanks!

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- wolverine punisher hulk mutants spiderman dy timucin bagged marvel
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## SAGE (Perplexity = 1090)

- ftp pub anonymous faq directory uk cypherpunks dcr loren
- disease msg patients candida dyer yeast vitamin infection syndrome
- car cars bike bikes miles tires odometer mavenry altcit
- jews israeli arab arabs israel objective morality baerga amehdi hossien
- god jesus christians bible faith atheism christ atheists christianity