# Machine Learning

10-701/15-781, Spring 2008

## Reinforcement learning 2

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Reading: Chap. 13, T.M. book

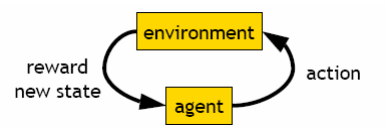


## Outline

- Defining an RL problem
  - Markov Decision Processes
- Solving an RL problem
  - Dynamic Programming
  - Monte Carlo methods
  - Temporal-Difference learning
- Miscellaneous
  - state representation
  - function approximation
  - rewards

# Markov Decision Process (MDP)

- set of states S, set of actions A, initial state $S_0$
- transition model P(s,a,s')
  - P( [1,1], up, [1,2] ) = 0.8
- reward function r(s)
  - r( [4,3] ) = +1




- goal: maximize cumulative reward in the long run
- policy: mapping from S to A
  - π(s) or π(s,a)
- reinforcement learning
  - transitions and rewards usually not available
  - how to change the policy based on experience
  - how to explore the environment



# Dynamic programming

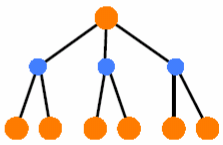
- Main idea
  - use value functions to structure the search for good policies
  - need a perfect model of the environment
- Two main components
  - policy evaluation: compute $V^\pi$ from $\pi$
  - policy improvement: improve $\pi$ based on $V^\pi$
  - start with an arbitrary policy
  - repeat evaluation/improvement until convergence

# Policy/Value iteration

- Policy iteration

$$\pi_0 \rightarrow^E V^{\pi_0} \rightarrow^I \pi_1 \rightarrow^E V^{\pi_1} \rightarrow^I \ldots \rightarrow^I \pi^* \rightarrow^E V^*$$

  - two nested iterations; too slow
  - don't need to converge to $V^{\pi_k}$
    - just move towards it



- Value iteration

$$V_{k+1}(s) = \max_a \sum_{s'} P^a_{ss'} \left[ r^a_{ss'} + \gamma V_k(s') \right]$$

  - use Bellman optimality equation as an update
  - converges to V*



# Using DP

- need complete model of the environment and rewards
  - robot in a room
    - state space, action space, transition model
- can we use DP to solve
  - robot in a room?
  - back gammon?
  - helicopter?
- DP bootstraps
  - updates estimates on the basis of other estimates

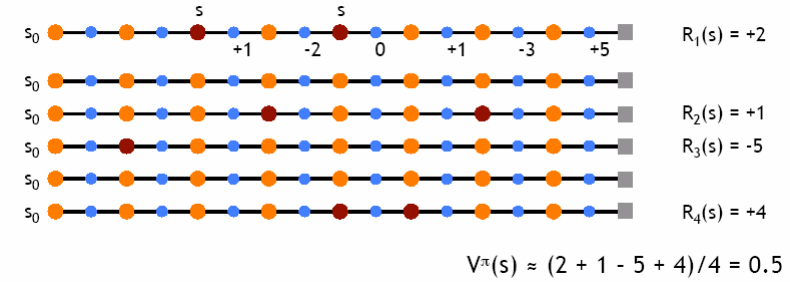
# Monte Carlo methods

- don't need full knowledge of environment
  - just experience, or
  - simulated experience
- but similar to DP
  - policy evaluation, policy improvement
- averaging sample returns
  - defined only for episodic tasks
  - episodic (vs. continuing) tasks
    - "game over" after N steps
    - optimal policy depends on N; harder to analyze



# Monte Carlo policy evaluation

- Want to estimate $V^{\pi}(s)$
  = expected return starting from s and following $\pi$
  - estimate as average of observed returns in state s
- First-visit MC
  - average returns following the first visit to state s



# Monte Carlo control

- $V^{\pi}$ not enough for policy improvement
  - need exact model of environment $\pi^*(s) = \arg\max_{a \in A} \sum_{s' \in S} P_{sa}(s') V^*(s')$
- Estimate $Q^{\pi}(s,a)$ $\pi'(s) = \arg\max_a Q^{\pi}(s,a)$ Q(s,a) V(s)
- MC control $\pi_0 \rightarrow^E Q^{\pi_0} \rightarrow^I \pi_1 \rightarrow^E Q^{\pi_1} \rightarrow^I \ldots \rightarrow^I \pi^* \rightarrow^E Q^*$
  - update after each episode
- Non-stationary environment $V(s) \leftarrow V(s) + \alpha[R - V(s)]$
- A problem
  - greedy policy won't explore all actions



# Maintaining exploration

- Deterministic/greedy policy won't explore all actions
  - don't know anything about the environment at the beginning
  - need to try all actions to find the optimal one
- Maintain exploration
  - use *soft* policies instead: $\pi(s,a)>0$ (for all $s,a$)
- $\varepsilon$-greedy policy
  - with probability 1-$\varepsilon$ perform the optimal/greedy action
  - with probability $\varepsilon$ perform a random action
  - will keep exploring the environment
  - slowly move it towards greedy policy: $\varepsilon$ -> 0

# Simulated experience

- 5-card draw poker
  - s0: A♣, A♦, 6♠, A♥, 2♠
  - a0: discard 6♠, 2♠
  - s1: A♣, A♦, A♥, A♠, 9♠ + dealer takes 4 cards
  - return: +1 (probably)
- DP
  - list all states, actions, compute P(s,a,s')
  - P( [A♣,A♦,6♠,A♥,2♠], [6♠,2♠], [A♠,9♠,4] ) = 0.00192
- MC
  - all you need are sample episodes
  - let MC play against a random policy, or itself, or another algorithm



# Temporal Difference Learning

- Combines ideas from MC and DP
  - like MC: learn directly from experience (don't need a model)
  - like DP: bootstrap
  - works for continuous tasks, usually faster than MC
- Constant-alpha MC:
  - have to wait until the end of episode to update

$$V(s_t) \leftarrow V(s_t) + \alpha[R_t - V(s_t)]$$

- simplest TD
  - update after every step, based on the successor

$$V(s_t) \leftarrow V(s_t) + \alpha[r_{t+1} + \gamma V(s_{t+1}) - V(s_t)]$$

# TD in passive learning

- TD(0) key idea:
  - adjust the estimated utility value of the current state based on its immediately reward and the estimated value of the next state.
- The updating rule

$$V(s_t) \leftarrow V(s_t) + \alpha[r_{t+1} + \gamma V(s_{t+1}) - V(s_t)]$$

  - $\alpha$ is the learning rate parameter
  - Only when $\alpha$ is a function that decreases as the number of times a state has been visited increased, then can *V(s)* converge to the correct value.



# Algorithm TD(λ)

**(not in Russell & Norvig book)**

- Idea: update from the whole epoch, not just on state transition.

$$V(s_t) \leftarrow V(s_t) + \alpha \sum_{k=t}^{\infty} \lambda^{t-k}[r_{k+1} + V(s_{k+1}) - V(s_t)]$$

  - Special cases:

    λ=1: LMS

    λ=0: TD
- Intermediate choice of λ (between 0 and 1) is best.
- Interplay with α …

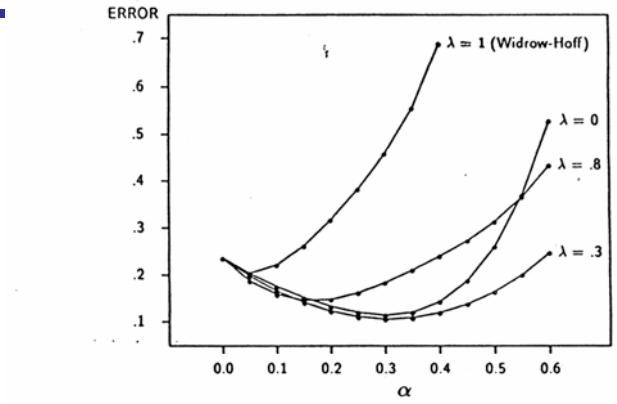
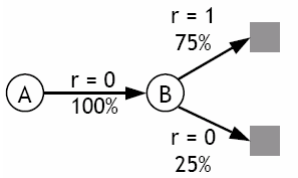



*Figure 4.* Average error on random walk problem after experiencing 10 sequences. All data are from TD(λ) with different values of $\alpha$ and $\lambda$. The dependent measure is the RMS error between the ideal predictions and those found by the learning procedure after a single presentation of a training set. This measure was averaged over 100 training sets. The $\lambda = 1$ data points represent performances of the Widrow-Hoff supervised-learning procedure.

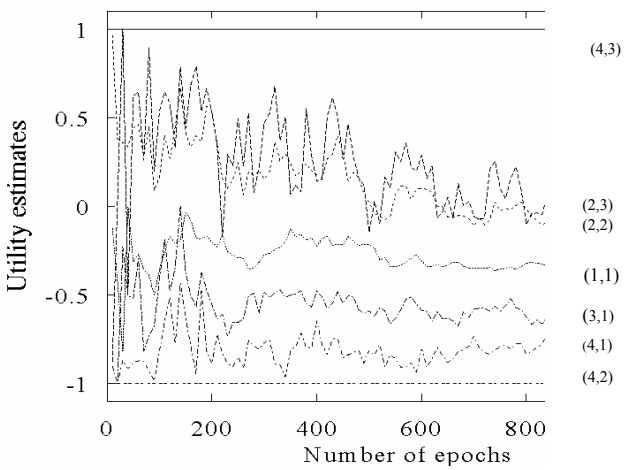


# MC vs. TD

- Observed the following 8 episodes:
  - A – 0, B – 0 B – 1 B – 1 B - 1
  - B – 1 B – 1 B – 1 B – 0
- MC and TD agree on V(B) = 3/4
- MC: V(A) = 0
  - converges to values that minimize the error on training data
- TD: V(A) = 3/4
  - converges to ML estimate of the Markov process



# The TD learning curve






# Another model free method– TD-Q learning

- Define Q-value function

$$V(s) = \max_a Q(s,a)$$

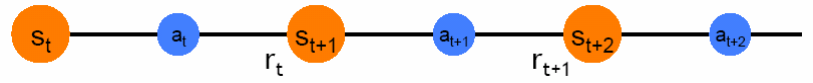
$\pi(s) = V(s) + V \operatorname{argmax}_a \sum_{s'} P_{sa}(s') V(s')$

- Q-value function updating rule
  - See subsequent slides
- Key idea of TD-Q learning
  - Combined with temporal difference approach
- Rule to chose the action to take

$$a = \arg\max_a Q(s,a)$$

# Sarsa

- Again, need Q(s,a), not just V(s)




$$Q^{\pi}(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)]$$

$$V(s_t) \leftarrow V(s_t) + \alpha[r_{t+1} + \gamma V(s_{t+1}) - V(s_t)]$$

- Control
  - start with a random policy
  - update $Q$ and $\pi$ after each step
  - again, need ε-soft policies



# Q-learning

- Before: on-policy algorithms
  - start with a random policy, iteratively improve
  - converge to optimal
- Q-learning: off-policy
  - use any policy to estimate Q

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)]$$

  - Q directly approximates Q* (Bellman optimality eqn)
  - independent of the policy being followed
  - only requirement: keep updating each (s,a) pair
- Sarsa

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)]$$

# TD-Q learning agent algorithm

For each pair (*s, a*), initialize *Q(s,a)*

Observe the current state s

Loop forever

{

Select an action ***a*** (optionally with ε-exploration) and execute it

$$a = \arg\max_a Q(s,a)$$

Receive immediate reward ***r*** and observe the new state ***s'***

Update *Q(s,a)*

$$Q(s,a) \leftarrow Q(s,a) + \alpha[r_{t+1} + \gamma \max_{a'} Q(s',a') - Q(s,a)]$$

***s=s'***

**}**



# Exploration

- Tradeoff between exploitation (control) and exploration (identification)
- Extremes: greedy vs. random acting
  (n-armed bandit models)

Q-learning converges to optimal Q-values if

- Every state is visited infinitely often (due to exploration),
- The action selection becomes greedy as time approaches infinity, and
- The learning rate a is decreased fast enough but not too fast (as we discussed in TD learning)

## Pole-balancing

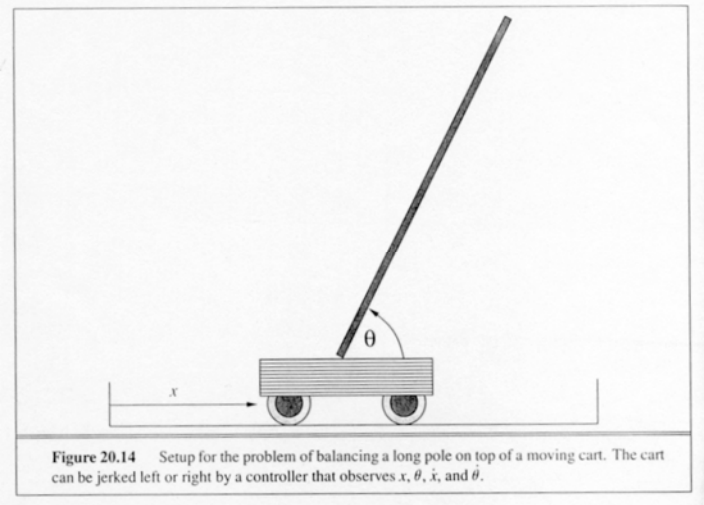
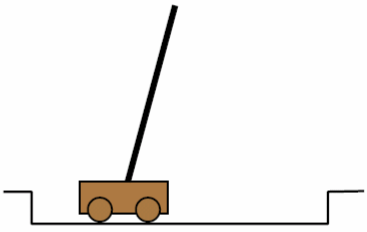



**Figure 20.14** Setup for the problem of balancing a long pole on top of a moving cart. The cart can be jerked left or right by a controller that observes $x$, $\theta$, $\dot{x}$, and $\dot{\theta}$.



## State representation

- pole-balancing
  - move car left/right to keep the pole balanced
- state representation
  - position and velocity of car
  - angle and angular velocity of pole
- what about *Markov property*?
  - would need more info
  - noise in sensors, temperature, bending of pole
- solution
  - coarse discretization of 4 state variables
    - left, center, right
  - totally non-Markov, but still works



# Function approximation

- represent $V_t$ as a parameterized function
  - linear regression, decision tree, neural net, …
  - linear regression: $V_t(s) = \vec{\theta}_t^T \vec{\phi}_s = \sum_{i=1}^{n} \theta_t(i)\phi_s(i)$
- update parameters instead of entries in a table
  - better generalization
    - fewer parameters and updates affect "similar" states as well
- TD update

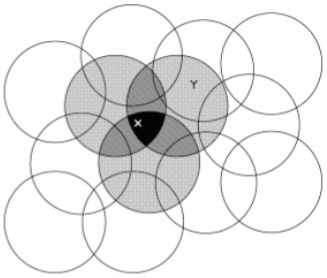
$$V(s_t) \leftarrow V(s_t) + \alpha \left[ r_{t+1} + \gamma V(s_{t+1}) - V(s_t) \right]$$

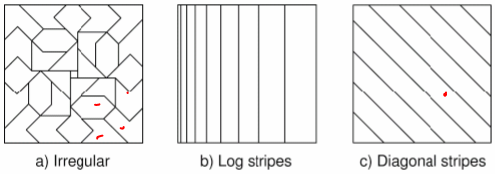
$$V(\underbrace{s_t}_{x}) \mapsto \underbrace{r_{t+1} + \gamma V(s_{t+1})}_{y}$$

  - treat as one data point for regression
  - want method that can learn on-line (update after each step)

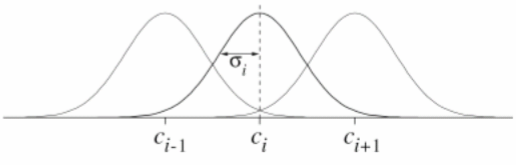
# Features

- tile coding, coarse coding
  - binary features



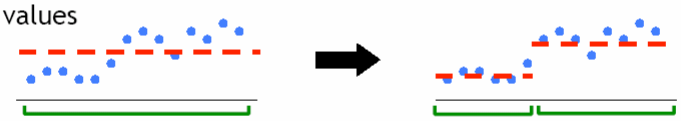

- radial basis functions
  - typically a Gaussian
  - between 0 and 1




[ Sutton & Barto, Reinforcement Learning ]

# Splitting and aggregation

- want to discretize the state space
  - learn the best discretization during training
- splitting of state space
  - start with a single state
  - split a state when different *parts of that state* have different values



- state aggregation
  - start with many states
  - merge states with similar values

# Designing rewards

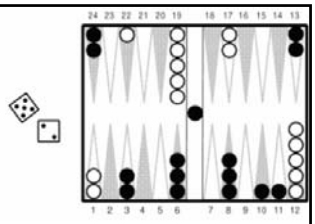
- robot in a maze
  - episodic task, not discounted, +1 when out, 0 for each step
- chess
  - GOOD: +1 for winning, -1 losing
  - BAD: +0.25 for taking opponent's pieces
    - high reward even when lose

$R(s) = \sum_w r(s)$

- rewards
  - rewards indicate what we want to accomplish
  - NOT how we want to accomplish it
- shaping

  - positive reward often very "far away"
  - rewards for achieving subgoals (domain knowledge)
  - also: adjust initial policy or initial value function

# A Success Story

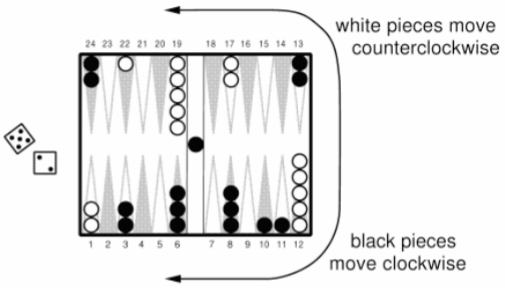


- TD Gammon (Tesauro, G., 1992)
  - A Backgammon playing program.
  - Application of temporal difference learning.
  - The basic learner is a neural network.
  - It trained itself to the world class level by playing against itself and learning from the outcome. So smart!!
  - More information: http://www.research.ibm.com/massive/tdl.html



# Case study: Back gammon

- rules
  - 30 pieces, 24 locations
  - roll 2, 5: move 2, 5
  - hitting, blocking
  - branching factor: 400
- implementation
  - use TD(λ) and neural nets
  - 4 binary features for each
  - no BG expert knowledge




- results
  - TD-Gammon 0.0: trained against itself (300,000 games)
    - as good as best previous BG computer program (also by Tesauro)
    - lot of expert input, hand-crafted features
  - TD-Gammon 1.0: add special features
  - TD-Gammon 2 and 3 (2-ply and 3-ply search)
    - 1.5M games, beat human champion

# Summary

- Reinforcement learning
  - use when need to make decisions in uncertain environment
- Solution methods
  - dynamic programming
    - need complete model
  - Monte Carlo
  - time difference learning (Sarsa, Q-learning)
- most work
  - algorithms simple
  - need to design features, state representation, rewards



# Future research in RL

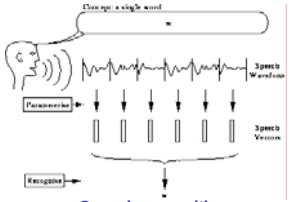
- Function approximation (& convergence results)
- On-line experience vs. simulated experience
- Amount of search in action selection
- Exploration method (safe?)
- Kind of backups
  - Full (DP) vs. sample backups (TD)
  - Shallow (Monte Carlo) vs. deep (exhaustive)
    - $\lambda$ controls this in TD($\lambda$)
- Macros
  - Advantages
    - Reduce complexity of learning by learning subgoals (macros) first
    - Can be learned by TD($\lambda$)
  - Problems
    - Selection of macro action
    - Learn models of macro actions (predict their outcome)
    - How do you come up with subgoals

# Types of Learning

- Supervised Learning
  - Training data: (X,Y). (features, label)
  - Predict Y, minimizing some loss.
  - Regression, Classification.
- Unsupervised Learning
  - Training data: X. (features only)
  - Find "similar" points in high-dim X-space.
  - Clustering.
- Reinforcement Learning
  - Training data: (S, A, R). (State-Action-Reward)
  - Develop an optimal policy (sequence of decision rules) for the learner so as to maximize its long-term reward.
  - Robotics, Board game playing programs



# Where Machine Learning is being used or can be useful?

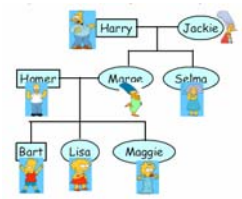

Speech recognition

Information retrieval

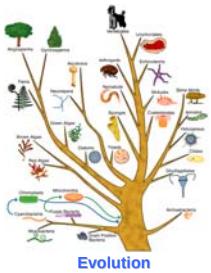

Computer vision


Games


Robotic control



Pedigree


Evolution


Planning