DESIGN AND SYNTHESIS OF SYNCHRONIZATION SKELETONS USING BRANCHING TIME TEMPORAL LOGIC

Edmund M. Clarke

E. Allen Emerson

Aiken Computation Laboratory

Harvard University

Cambridge, Mass. 02138

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ABSTRACT

We present a method of constructing concurrent programs in which the *synchronization skeleton* of the program is automatically synthesized from a high-level (branching time) Temporal Logic specification. The synchronization skeleton is an abstraction of the actual program where detail irrelevant to synchronization is suppressed. Because the synchronization skeleton is in general finite state, its properties can be specified by a formula f in a propositional Temporal Logic. (The synthesis method uses a decision procedure based on the finite model property of the logic to determine satisfiability of f.) If the formula f is satisfiable, then the specification it expresses is consistent, and a model for f with a finite number of states is constructed. The synchronization skeleton of a program meeting the specification can be read from this model. If f is unsatisfiable, the specification is inconsistent.

In the traditional approach to concurrent program verification, the proof that a program meets its specification is constructed using various axioms and rules of inference in a deductive system such as temporal logic. The task of proof construction can be quite tedious, and a good deal of ingenuity may be required. We believe that this task may be unnecessary in the case of finite state concurrent systems, and can be replaced by a mechanical check that the system meets a specification expressed in a propositional temporal logic. The global system flowgraph of a finite state concurrent system may be viewed as defining a finite structure. We describe an efficient algorithm (a model checker) to decide whether a given finite structure is a model of a particular formula. We also discuss extended logics for which it is not possible to construct efficient model checkers.

1. INTRODUCTION

We propose a method of constructing concurrent programs in which the *synchronization skeleton* of the program is automatically synthesized from a high-level (branching time) Temporal Logic specification. The synchronization skeleton is an abstraction of the actual program where detail irrelevant to synchronization is suppressed. For example, in the synchronization skeleton for a solution to the critical section problem each process's critical section may be viewed as a single node since the internal structure of the critical section is unimportant. Most solutions to synchronization problems in the literature are in fact given as synchronization skeletons. Because synchronization skeletons are in general finite state, the propositional version of Temporal Logic can be used to specify their properties.

Our synthesis method exploits the (bounded) *finite model property* for an appropriate propositional Temporal Logic which asserts that if a formula of the logic is satisfiable, it is satisfiable in a finite model (of size bounded by a function of the length of the formula). Decision procedures have been devised which, given a formula of Temporal Logic, f, will decide whether f is satisfiable or unsatisfiable. If f is satisfiable, a finite model of f is constructed. In our application, unsatisfiability of f means that the specification is inconsistent (and must be reformulated). If the formula f is satisfiable, then the specification it expresses is consistent. A model for f with a finite number of states is constructed by the decision procedure. The synchronization skeleton of a program meeting the specification can be read from this model. The finite model property ensures that any program whose synchronization properties can be expressed in propositional Temporal Logic can be realized by a system of concurrently running processes, each of which is a finite state machine.

Initially, the synchronization skeletons we synthesize will be for concurrent programs running in a shared-memory environment and for monitors. However, we believe that it is also possible to extend these techniques to synthesize distributed programs. One such application would be the automatic synthesis of network communication protocols from propositional Temporal Logic specifications.

Previous efforts toward parallel program synthesis can be found in the work of [LA78] and [RK80]. [LA78] uses a specification language that is essentially predicate calculus augmented with a special predicate to define the relative order of events in time. [RK80] uses an applied linear time Temporal Logic. Both [LA80] and [RK80] use *ad hoc* techniques to construct a monitor that meets the specification. We have recently learned that [WO81] has independently developed model-theoretic synthesis techniques similar to our own. However, he uses a linear time logic for specification and generates CPS-like programs.

We also discuss how a Model Checker for Temporal Logic formulae can be used to verify the correctness of *a priori* existing programs. In the traditional approach to concurrent program verification, the proof that a program meets its specification is constructed using various axioms and rules of inference in a deductive system such as temporal logic. The task of proof construction can be quite tedious, and a good deal of ingenuity may be required. We believe that this task may be unnecessary in the case of finite state concurrent systems, and can be replaced by a mechanical check that the system meets a specification expressed in a propositional temporal logic. The global system flowgraph of a finite state concurrent system may be viewed as defining a finite structure. We describe an efficient algorithm (a model checker) to decide whether a given finite structure is a model of a particular formula. We also discuss extended logics for which it is not possible to construct efficient model checkers.

The paper is organized as follows: Section 2 discusses the model of parallel computation. Section 3 presents the branching time logic that is used to specify synchronization skeletons. Fixpoint characterizations for various temporal operators are given in Section 4. Sections 5 and 6 describe the model checker and the decision procedure, respectively. Finally, Section 7 shows how the synthesis method can be used to construct a solution to the starvation free mutual exclusion problem.

2. MODEL OF PARALLEL COMPUTATION

We discuss concurrent systems consisting of a finite number of fixed processes $P_1,\ldots,P_m$ running in parallel. The treatment of parallelism is the usual one: nondeterministic interleaving of the sequential "atomic" actions of the individual processes P_i. Each time an atomic action is executed, the system "execution" state is updated. This state may be thought of as containing the location counters and the data values for all processes. The behavior of a system starting in a particular state may be described by a computation tree. Each node of the tree is labelled with the state it represents, and each arc out of a node is labelled with a process index indicating which nondeterministic choice is made, i.e., which process's atomic action is executed next. The root is labelled with the start state. Thus, a path from the root through the tree represents a possible computation sequence of the system beginning in a given start state. Our temporal logic specifications may then be thought of as making statements about patterns of behavior in the computation trees.

Each Process P_i is represented as a flowgraph. Each node represents a region or a block of code and is identified by a unique label. For example there may be a node labelled CS_i representing "the critical section of code of process P_i." Such a region of code is uninterpreted in that its internal structure and intended application are unspecified. While in CS_i, the process P_i may simply increment variable x or it may perform an extensive series of updates on a large database. The underlying semantics of the computation performed in the various code regions are irrelevant to the synchronization skeleton. The arcs between nodes represent possible transitions between code regions. The labels on the arcs indicate under what conditions P_i can make a transition to a neighboring node. Our job is to supply the enabling conditions on the arcs so that the global system of processes $P_1,\ldots,P_k$ meets a given Temporal Logic specification.

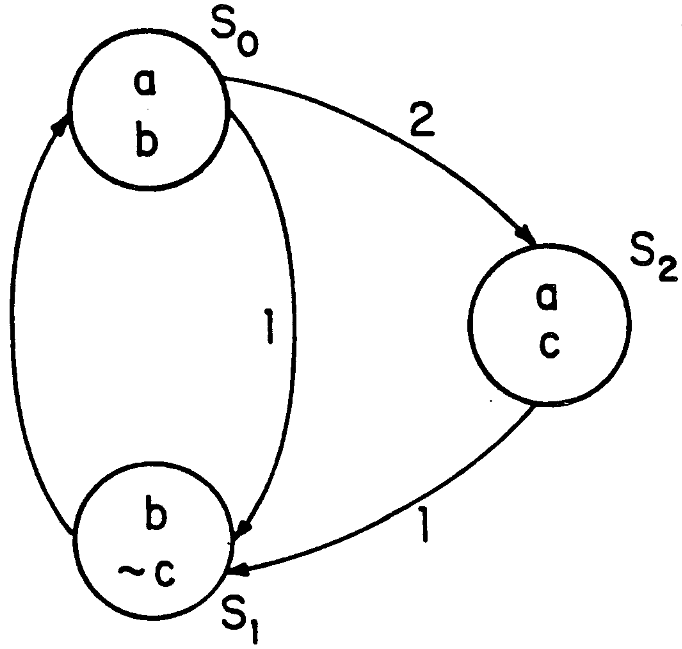
3. THE SPECIFICATION LANGUAGE

Our specification language is a (propositional) branching time temporal logic called Computation Tree Logic (CTL) and is based on the language presented in [EC80]. Our current notation is inspired by the language of "Unified Branching Time" (UB) discussed in [BM81]. UB is roughly equivalent to that subset of the language presented in [EC80] obtained by deleting the infinitary quantifiers and the arc conditions and adding an explicit next-time operator. For example, in [EC80] we write $\forall$path$\exists$node P to express the inevitability of predicate P. The corresponding formula in our UB-like notation is AFP. The language presented in [EC80] is more expressive than UB as evidenced by the formula $\forall$ path $\overset{\infty}{\forall}$node P (which is not equivalent to any formula in UB or in the language of [EC80] without infinitary quantifiers). However, the UB-like notation is more concise and is sufficiently expressive for the purposes of program synthesis.

We use the following syntax (where p denotes an atomic proposition and f_i denotes a (sub-) formula):

1. Each of p, $f_1 \wedge f_2$, and $\sim f_1$ is a formula (where the latter two constructs indicate conjunction and negation, respectively).
2. $EX_j f_1$ is a formula which intuitively means that there is an immediate successor state reachable by executing one step of process P_j in which formula f_1 holds.
3. $A[f_1 U f_2]$ is a formula which intuitively means that for every computation path, there exists an initial prefix of the path such that f_2 holds at the last state of the prefix and f_1 holds at all other states along the prefix.
4. $E[f_1 U f_2]$ is a formula which intuitively means that for some computation path, there exists an initial prefix of the path such

A structure



The corresponding tree
for start state S_0

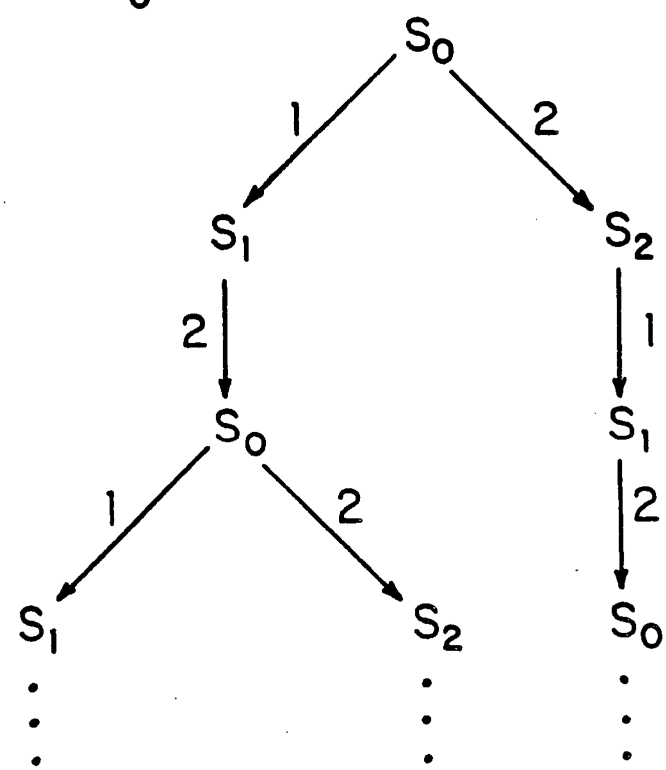


Figure 3.1

that f_2 holds at the last state of the prefix and f_1 holds at all other states along the prefix.

Formally, we define the semantics of CTL formulae with respect to a structure $M = (S, A_1, \ldots, A_k, \mathscr{L})$ which consists of

S - a countable set of states,

A_i- $\subset S \times S$, a binary relation on S giving the possible transitions by process i, and

$\mathscr{L}$- an assignment of atomic propositions true in each state.

Let $A = A_1 \cup \ldots \cup A_k$. We require that A be total, i.e., that $\forall x \in S \exists y (x,y) \in A$. A *path* is an infinite sequence of states $(s_0, s_1, s_2 \ldots) \in S^w$ such that $\forall i (s_i, s_{i+1}) \in A$. To any structure M and state $s \in S$ of M, there corresponds a computation tree with root labelled s_0 such that $s \xrightarrow{i} t$ is an arc in the tree iff $(s,t) \in A_i$. See Figure 3.1.

We use the usual notation to indicate truth in a structure: $M, s_0 \models f$ means that at state s_0 in structure M formula f holds true. When the structure M is understood, we write $s_0 \models f$. We define $\models$ inductively:

$s_0 \models p$ iff $p \in \mathscr{L}(s_0)$

$s_0 \models \sim f$ iff not $(s_0 \models f)$

$s_0 \models f_1 \wedge f_2$ iff $s_0 \models f_1$ and $s_0 \models f_2$

$s_0 \models EX_j f$ iff for some state t such that $(s_0, t) \in A_j$, $t \models f$

$s_0 \models A[f_1 U f_2]$ iff for all paths $(s_0, s_1, \ldots)$, $\exists i [i \geq 0 \wedge s_i \models f_2$
$\wedge \forall j (0 \leq j \wedge j < i \rightarrow s_j \models f_1)]$

$s_0 \models E[f_1 U f_2]$ iff for some path $(s_0, s_1, \ldots)$, $\exists i [i \geq 0 \wedge s_i \models f_2$
$\wedge \forall j (0 \leq j \wedge j < i \rightarrow s_j \models f_1)]$

We write $\models f$ to indicate that f is universally valid, i.e., true at all states in all structures. Similarly, we write $\models f$ to indicate that f is satisfiable, i.e., f is true in some state of some structure.

We introduce some abbreviations:

$f_1 \vee f_2 \equiv \sim(\sim f_1 \wedge \sim f_2)$, $f_1 \rightarrow f_2 \equiv \sim f_1 \vee f_2$, and $f_1 \leftrightarrow f_2 \equiv (f_1 \rightarrow f_2) \wedge (f_2 \rightarrow f_1)$ for logical disjunction, implication, and equivalence, respectively.

$A[f_1 V f_2] \equiv \sim E[\sim f_1 U \sim f_2]$ which means for every path, for every state s on the path, if f_1 is false at all states on the path prior to s, then f_2 holds at s.

$E[f_1 V f_2] \equiv \sim A[\sim f_1 U \sim f_2]$ which means for some path, for every state s on the path, if f_1 is false at all states on the path prior to s, then f_2 holds at s.

$AFf_1 \equiv A[\text{true } Uf_1]$ which means for every path, there exists a state on the path at which f_1 holds.

$EFf_1 \equiv E[\text{true } Uf_1]$ which means for some path, there exists a state on the path at which f_1 holds.

$AGf_1 \equiv \sim EF\sim f_1$ which means for every path, at every node on the path f_1 holds.

$EGf_1 \equiv \sim AF\sim f_1$ which means for some path, at every node on the path f_1 holds.

$AX_i f \equiv \sim EX_i \sim f$ which means at all successor states reachable by an atomic step of process P_i, f holds.

$EXf \equiv EX_1 f \vee \ldots \vee EX_k f$ which means at some successor state f holds.

$AXf \equiv \sim EX\sim f$ which means at all successor states f holds.

See Fig. 3.2 for illustrations of some of the above modalities.

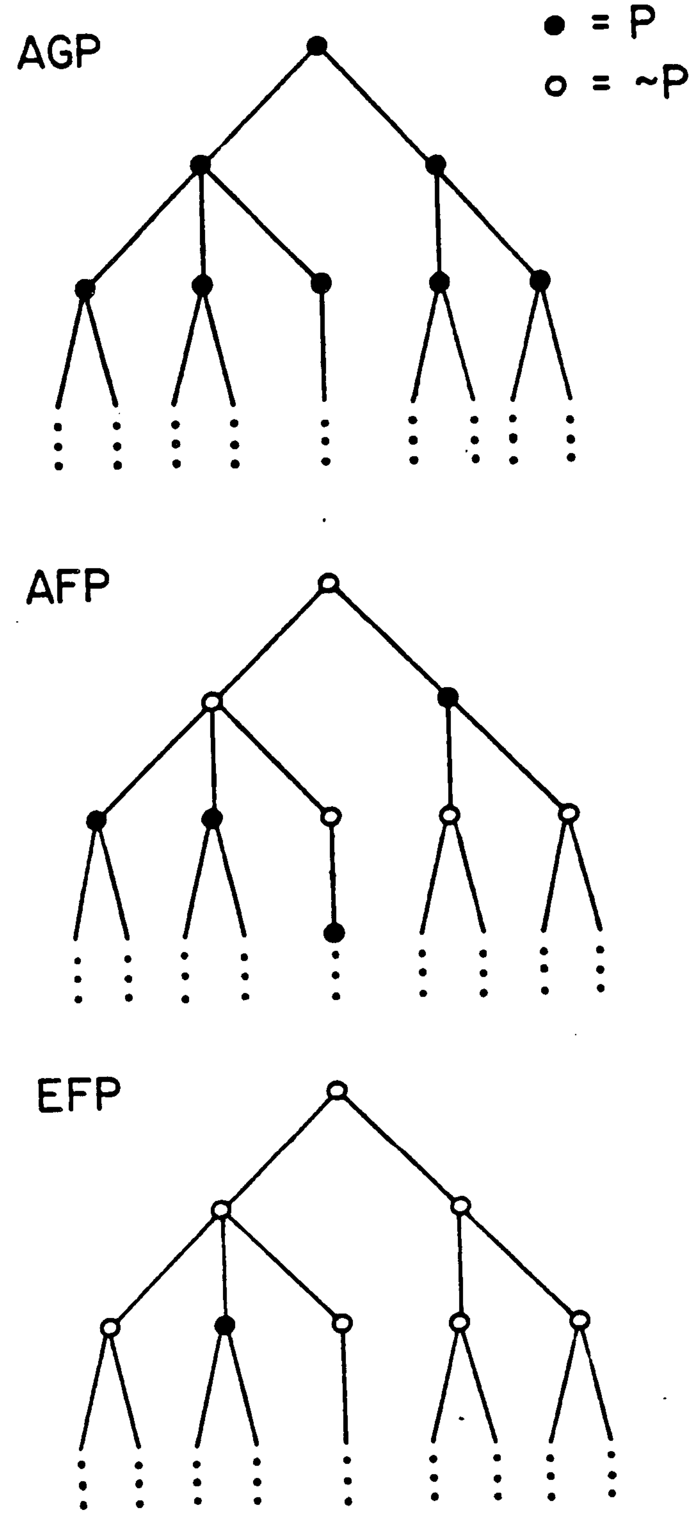


Figure 3.2

4. FIXPOINT CHARACTERIZATIONS

Each of the modal operators such as AU, EG, EF, etc., may be characterized as an extremal fixpoint of an appropriate monotonic functional. Let $M = (S, A_1, \ldots, A_k, \mathcal{L})$ be an arbitrary structure. We use PRED(S) to denote the lattice of total predicates over S where each predicate is identified with the set of states which make it true and the ordering is set inclusion. Then, each formula f defines a member of $\text{PRED}(S) = \{s : M, s \models f\}$. Let $\tau : \text{PRED}(S) \to \text{PRED}(S)$ be given; then

(1) τ is *monotonic* provided that $P \subseteq Q$ implies $\tau[P] \subseteq \tau[Q]$;

(2) τ is $\cup$-*continuous* provided that $P_1 \subseteq P_2 \subseteq \ldots$ implies $\tau[\bigcup_i P_i] = \bigcup_i \tau[P_i]$;

(3) τ is $\cap$-*continuous* provided that $P_1 \supseteq P_2 \supseteq \ldots$ implies $\tau[\bigcap_i P_i] = \bigcap_i \tau[P_i]$. []

A monotonic functional τ on PRED(S) always has both a least fixpoint, $\text{lfp}X.\tau[X]$, and a greatest fixpoint, $\text{gfp}X.\tau[X]$ (see Tarski [TA55]) : $\text{lfp}X.\tau[X] = \cap\{X : \tau[X] = X\}$ whenever τ is monotonic, and $\text{lfp}X.\tau[X] = \bigcup_i \tau^i[\text{False}]$ whenever τ is also $\cup$-continuous; $\text{gfp}X.\tau[X] = \cup\{X : \tau[X] = X\}$ whenever τ is monotonic, and $\text{gfp}X.\tau[X] = \bigcap_i \tau^i[\text{True}]$ whenever τ is also $\cap$-continuous.

The modal operators have the following fixpoint characterization:

$$\text{EF}h = \text{lfp}Z.h \vee \text{EX}Z$$

$$\text{AF}h = \text{lfp}Z.h \vee \text{AX}Z$$

$$\text{E}[g\text{U}h] = \text{lfp}Z.h \vee (g \wedge \text{EX}Z)$$

$$\text{A}[g\text{U}h] = \text{lfp}Z.h \vee (g \wedge \text{AX}Z)$$

$$\mathrm{AGh} = \mathrm{gfpZ}.h \wedge \mathrm{AXZ}$$

$$\mathrm{EGh} = \mathrm{gfpZ}.h \wedge \mathrm{EXZ}$$

$$\mathrm{E}[g\mathrm{V}h] = \mathrm{gfpZ}.h \wedge (g \vee \mathrm{EXZ})$$

$$\mathrm{A}[g\mathrm{V}h] = \mathrm{gfpZ}.h \wedge (g \vee \mathrm{AXZ})$$

If all A_i in M are of bounded nondeterminism, then each of the functional used in the fixpoint characterizations above is $\cup$-continuous and $\cap$-continuous as well as monotonic. We show that the first fixpoint characterization is correct:

PROPOSITION. 4.1. EFh *is the least fixpoint of the functional* $\tau[Z] = h \vee \mathrm{EXZ}$.

Proof. We first show that EFh is a fixpoint of $\tau[Z]$: Suppose $s_0 \models \mathrm{EFh}$. Then by definition of $\models$, there is a path $(s_0, s_1, s_2, \ldots)$ in M such that for some k, $s_k \models \mathrm{EFh}$. If $k = 0$, $s_0 \models h$. Otherwise $s_1 \models \mathrm{EFh}$ and $s_0 \models \mathrm{EXEFh}$. Thus, $\mathrm{EFh} \subseteq h \vee \mathrm{EXEFh}$. Similarly, if $s_0 \models h \vee \mathrm{EXEFh}$, then $s_0 \models h$ or $s_0 \models \mathrm{EXEFh}$. In either case, $s_0 \models \mathrm{EFh}$ and $h \vee \mathrm{EXEFh} \subseteq \mathrm{EFh}$. Thus $\mathrm{EFh} = h \vee \mathrm{EXEFh}$.

To see that EFh is the least fixpoint of $\tau[Z]$, it suffices to show that $\mathrm{EFh} = \cup_{i \geq 0} \tau^i[\mathrm{False}]$. It follows by a straightforward induction on i that $s_0 \in \tau^i[\mathrm{False}]$ iff there is a finite path $(s_0, s_1, \ldots, s_i)$ in M and a $j \leq i$ for which $s_j \models h$.

These fixpoint characterizations are helpful in proving the correctness of the model checking algorithm of Section 5 and are also used in constructing the tableau for the decision procedure of Section 6. Fixpoint characterizations have been investigated, in other contexts, by a number of researchers including [PA69], [CL77], [FS81], and [EC80].

5. MODEL CHECKER

Assume that we wish to determine whether formula f is true in the finite structure $M = (S, A_1, \ldots, A_k, \mathscr{L})$. Let $sub^+(f_0)$ denote the set subformulae of f_0 with main connective other than $\sim$. We label each state $s \in S$ with the set of positive/negative formulae f in $sub^+(f_0)$ so that

$$f \in label(s) \text{ iff } M, s \models f$$
$$\sim f \in label(s) \text{ iff } M, s \models \sim f \quad .$$

The algorithm makes $n+1$ passes where $n = length(f_0)$. On pass i, every state $s \in S$ is labelled with f or $\sim f$ for each formula $f \in sub^+(f_0)$ of length i. Information gathered in earlier passes about formulae of length less than i is used to perform the labelling. For example, if $f = f_1 \wedge f_2$, then f should be placed in the set for s precisely when f_1 and f_2 are already present in the set for s. For modalities such as $A[f_1 U f_2]$ information from the successor states of s (as well as from s itself) is used. Since $A[f_1 U f_2] = f_2 \vee (f_1 \wedge AXA[f_1 U f_2])$, $A[f_1 U f_2]$ should be placed in the set for s when f_2 is already in the set for s or when f_1 is in the set for s and $A[f_1 U f_2]$ is in the set of each immediate successor state of s.

Satisfaction of $A[f_1 U f_2]$ may be seen to "radiate" outward from states where it holds immediately by virtue of f_2 holding:

Let
$$(A[f_1 U f_2])^0 = f_2$$
$$(A[f_1 U f_2])^{k+1} = f_2 \vee AX(A[f_1 U f_2])^k \quad .$$

It can be shown that $M,s \models (A[f_1 U f_2])^k$ iff $M,s \models A[f_1 U f_2]$ and along every path starting at s, f_2 holds by the kth state following s. Thus, states where $(A[f_1 U f_2])^0$ holds are found first, then states where $(A[f_1 U f_2])^1$ holds, etc. If $A[f_1 U f_2]$ holds, then $(A[f_1 U f_2])^{card(S)}$ must hold since all loop-free paths in M are of length $\leq$card(S). Thus, if after card(S) steps of radiating outward, $A[f_1 U f_2]$ has still not been found to hold at state s, then put $\sim A[f_1 U f_2]$ in the set for s.

The algorithm for pass i is listed below in an Algol-like syntax:

```
    for every state s ∈ S do
      for every f ∈ sub⁺(f0) of length i do
        if f = A[f1Uf2] and f2 ∈ set(s) or
           f = E[f1Uf2] and f2 ∈ set(s) or
           f = EXj f1 and ∃t((s,t) ∈ Aj and f1 ∈ set(t)) or
           f = f1 ∧ f2 and f1 ∈ set(s) and f2 ∈ set(s)
        then add f to set(s)
      end
    end;
 A: for j = 1 to card(S) do
      for every state s ∈ S do
        for every f ∈ sub⁺(f0) of length i do
          if f = A[f1Uf2] and f1 ∈ set(s) and
                ∀t((s,t) ∈ A → f ∈ set(t)) or
             f = E[f1Uf2] and f1 ∈ set(s) and
                ∃t((s,t) ∈ A ∧ f ∈ set(t))
          then add f to set (s)
        end
   B: end
    end;
    for every state s ∈ S do
      for every f ∈ sub⁺(f0) of length i do
        if f ∉ set(s)
        then add ~f to set(s)
      end
 C: end
```

Figures 5.1-5.5 give snapshots of the algorithm in operation on the structure shown for the formula $AFb \wedge EGa$ (which abbreviates $AFb \wedge \sim AF \sim a$).

Suppose we extend the logic to permit $\forall$ path $\overset{\infty}{\forall}$ node p or, equivalently, its dual $\exists$ path $\overset{\infty}{\exists}$ node P which we write $E\overset{\infty}{F}p$. We can generalize the model checker to handle this case by using the following proposition:

PROPOSITION 5.1. *Let* $M = (S, A_1, \ldots, A_k, \mathcal{L})$ *be a structure and* $s \in S$. *Then* $M, s \models E\overset{\infty}{F}p$ *iff there exists a path from* s *to a node* s' *such that* $M, s' \models p$ *and either* s' *is a successor of itself or the strongly connected component of* M *containing* s' *has cardinality greater than* 1 *(see Fig. 5.6).* □

<u>Proof</u>. (only if:) Suppose $M, s \models E\overset{\infty}{F}p$. Then there is an infinite path $(s_0, s_1, s_2, \ldots)$ through M and a state $s' \in S$ such that

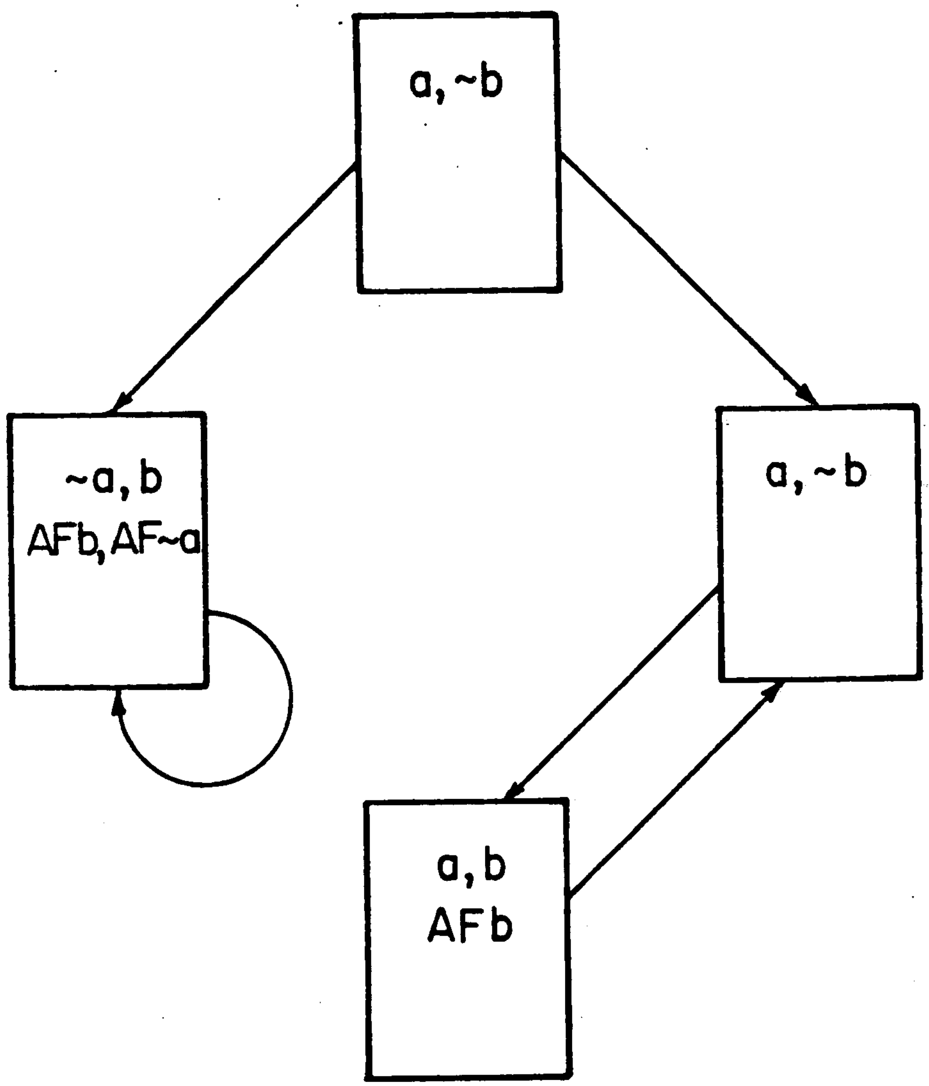
(1) $s_0 = s$;

(2) $s' = s_i$ for infinitely many distinct i;

(3) $M, s' \models p$.

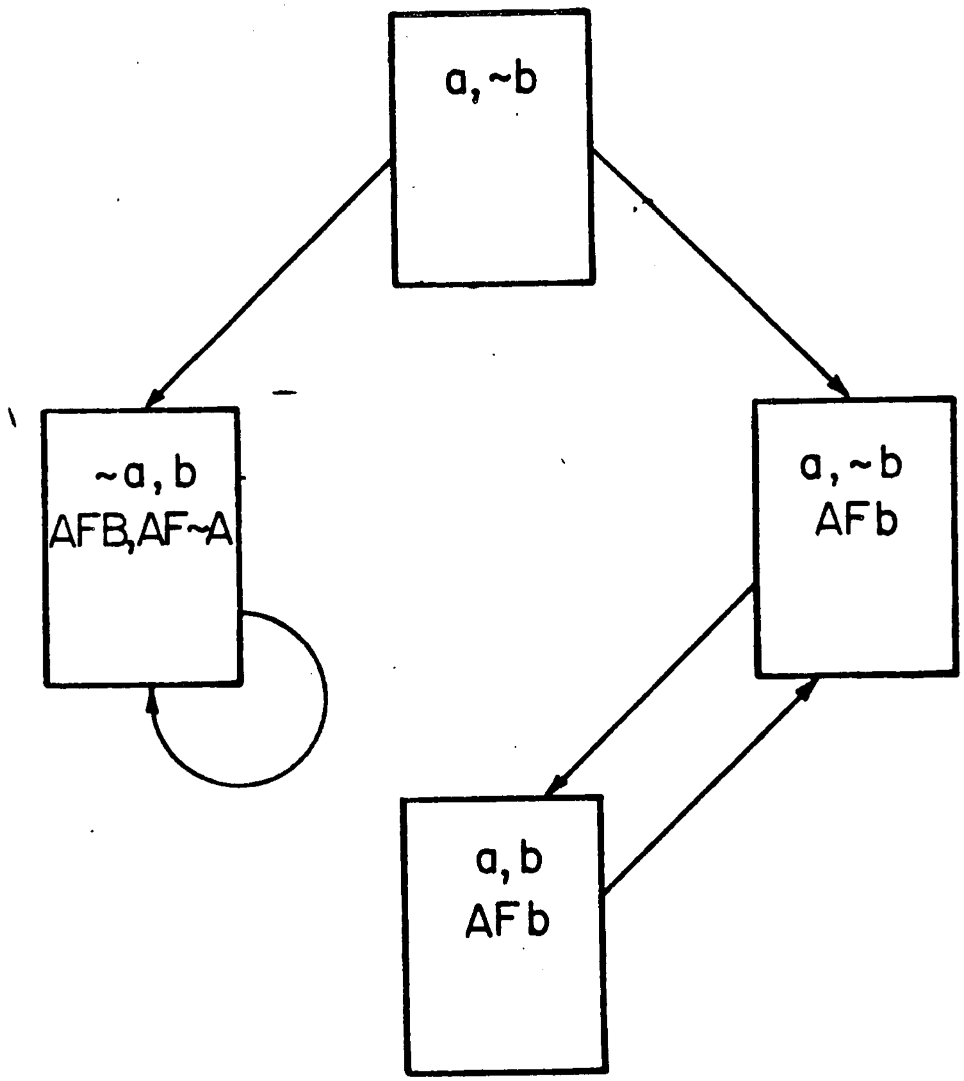
If s' is a successor of itself, we are done. Otherwise, there is a finite path (s',...,s",...s') from s' back to itself (because of (2)) which contains a state s" $\neq$ s. So, s" is reachable from s' and s' is reachable from s", and s' is in a strongly connected component of M of cardinality greater than 1.

(if:) If s' is a successor of itself, then p is true infinitely often along the path (s',s',...). Since s' is reachable from s, $M, s \models E\overset{\infty}{F}p$. If the strongly connected component of M containing s' is of cardinality greater than 1, then there is a state s" $\neq$ s' such that s' is reachable from s" and s" is reachable from s". Hence there is a finite path from s' back to itself, and an infinite path starting



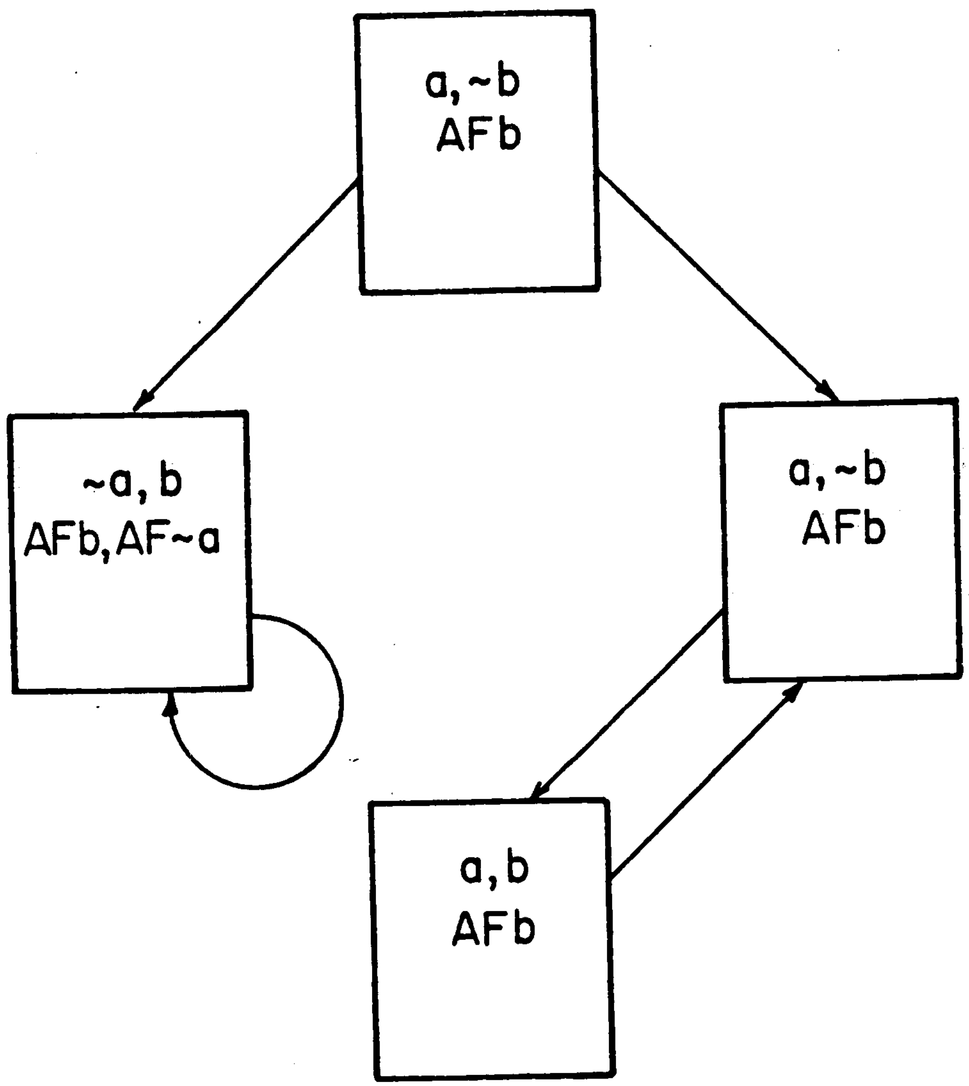
1st time at label A in pass 1

Figure 5.1



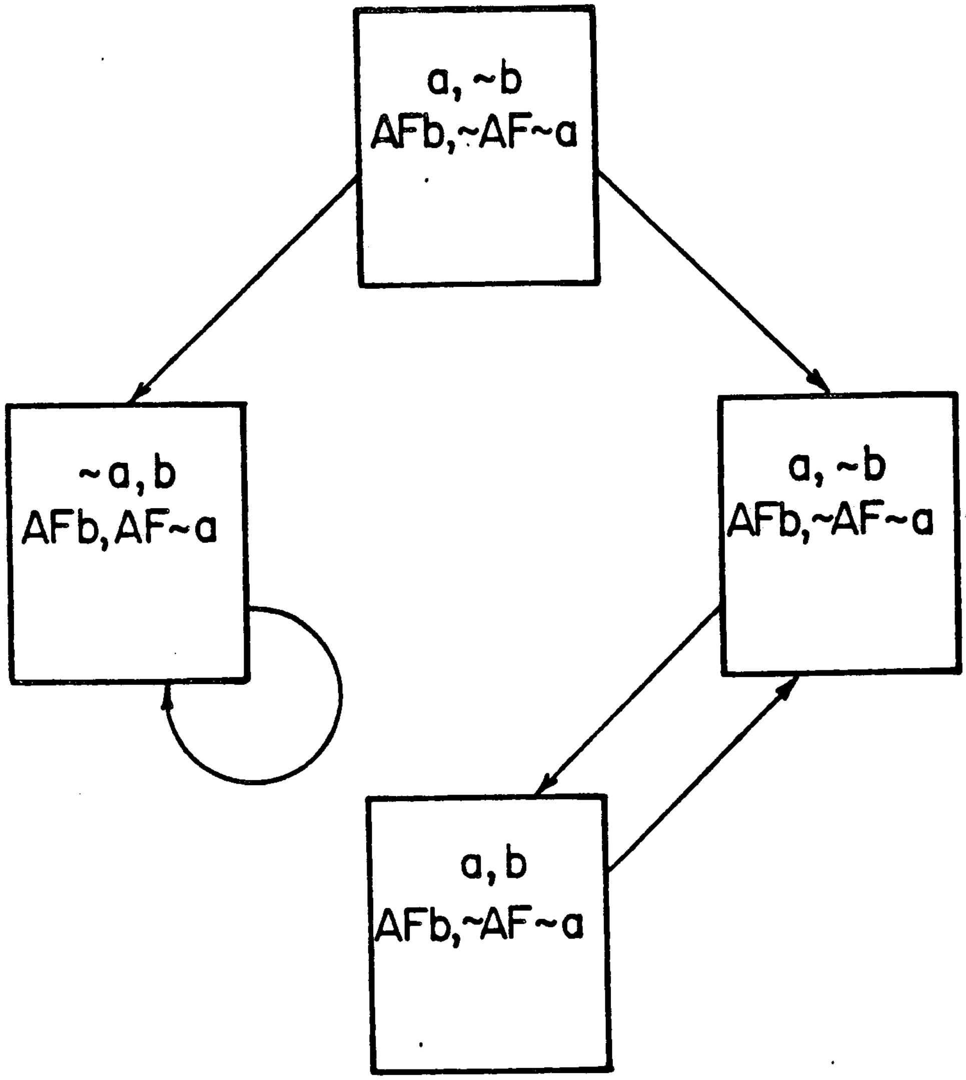
1st time at label B in pass 1

Figure 5.2



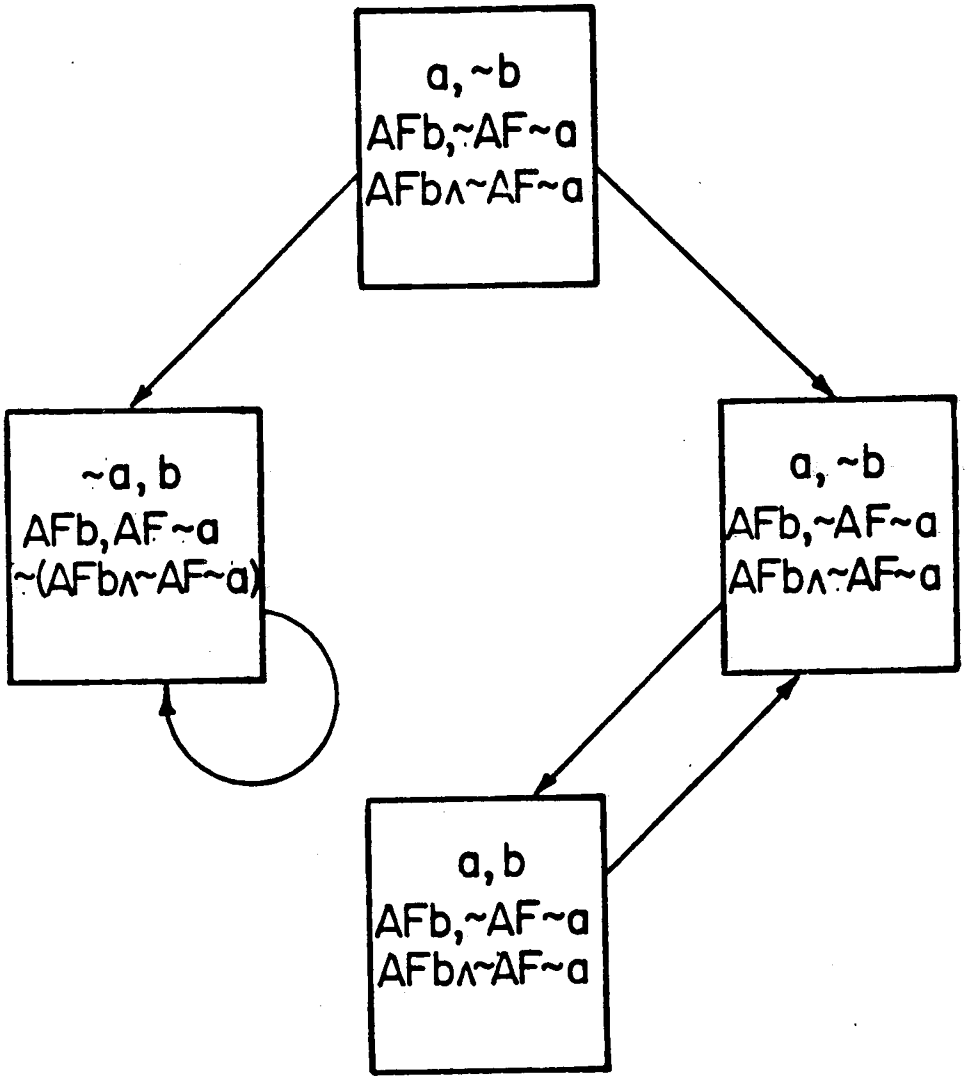
2nd time at label B in pass 1

Figure 5.3



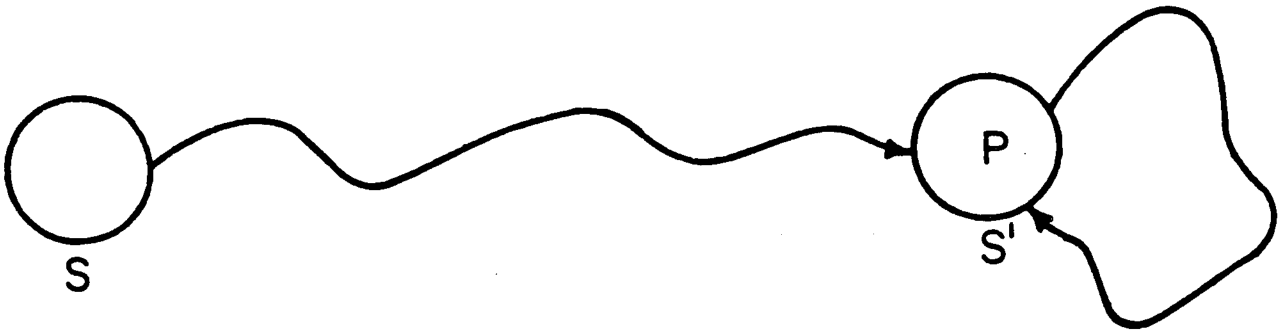
1st time at label C in pass 1

Figure 5.4



at termination

Figure 5.5



Testing for $E\overset{\infty}{F}p$

Figure 5.6

at s' which goes through s' infinitely often. Since s' is reachable from s, $M,s \models \overset{\infty}{E}Fp$.

Notice that all algorithms discussed so far run in time polynomial in the size of the candidate model and formula. The algorithm for basic CTL presented above runs in time $\text{length}(f)\cdot(\text{card}(S))^2$. Since there is a linear time algorithm for finding the strongly connected components of a graph [TA72], we can also achieve the $\text{length}(f)\cdot(\text{card}(S))^2$ time bound when we include the infinitary quantifiers.

Finally, we show that it is not always possible to obtain polynomial time algorithms for model checking. Suppose we extend our language to allow either an existential or a universal path quantifier to prefix an arbitrary assertion from linear time logic as in [LA80] and [GP80]. Thus, we can write assertions such as

$$E[Fg_1 \wedge \ldots \wedge Fg_n \wedge Gh_1 \wedge \ldots \wedge Gh_n]$$

meaning

"there exists a computation path ρ such that, along ρ
sometimes g_1 and ... and sometimes g_n and
always h_1 and ... and always h_n."

We claim that the problem of determining whether a given formula f holds in a given finite structure M is NP-hard.

THEOREM 5.2. *Directed Hamiltonian Path is reducible to the problem of determining whether* $M,s \models f$ *where*

M *is a finite structure,*

s *is a state in* M *and*

f *is the assertion (using atomic propositions* $p_1,\ldots,p_n$*):*

$E[Fp_1 \wedge \ldots \wedge Fp_n \wedge G(p_1 \rightarrow XG \sim p_1) \wedge \ldots \wedge G(p_n \rightarrow XG \sim p_n)]$. □

Proof. Consider an arbitrary directed graph $G = (V,A)$ where $V = \{v_1,\ldots,v_n\}$. We obtain a structure from G by making proposition p_i hold at node v_i and false at all other nodes (for $1 \le i \le n$), and by adding a source node u_1 from which all v_i are accessible (but not vice versa) and a sink node u_2 which is accessible from all v_i (but not vice versa).

Formally, let the structure $M = (U,B,\mathscr{L})$ consist of

$$U = V \cup \{u_1,u_2\} \quad \text{where} \quad u_1,u_2 \notin V$$

$\mathscr{L}$, on assignment of states to propositions such that

$$v_i \models p_i,\ v_i \not\models p_j,\ (1 \le i,j \le n, i \ne j)$$

$$u_1 \not\models p_i,\ u_2 \not\models p_i,\ (1 \le i \le n) \quad \text{and}$$

$$B = A \cup \{(u_1 v_i): v_i \in V\} \cup \{(v_i,u_2): v_i \in V\} \cup \{(u_2,u_2)\} \quad .$$

It follows that

$M,u_1 \models f$ <u>iff</u> there is a directed infinite path in M starting at u_1 which goes through all $v_i \in V$ exactly once and ends in the self-loop through u_2; <u>iff</u> there is a directed Hamiltonian path in G. □

We believe that the model checker may turn out to be of considerable value in the verification of certain finite state concurrent systems such as network protocols. We have developed an experimental implementation of the model checker at Harvard which is written in C and runs on the DEC 11-70.

6. THE DECISION PROCEDURE

In this section we outline a tableau-based decision procedure for satisfiability of CTL formulae. Our algorithm is similar to one proposed for UB in [BM81].* Tableau-based decision procedures for simpler program logics such as PDL and DPDL are given in [PR77] and [BH81]. The reader should consult [HC68] for a discussion of tableau-based decision procedures for classical modal logics and [SM68] for a discussion of tableau-based decision procedures for propositional logic.

We now briefly describe the decision procedure for CTL and illustrate it with a simple example. The decision procedure is described in detail in the appendix. To simplify the notation in the present discussion, we omit the labels on arcs which are normally used to distinguish between transitions by different processes.

The decision procedure takes as input a formula f_0 and returns either "YES, f_0 is satisfiable," or "NO, f_0 is unsatisfiable." If f_0 is satisfiable, a finite model is constructed. The decision procedure performs the following steps:

1. Build the initial tableau T which encodes potential models of f_0. If f_0 is satisfiable, it has a finite model that can be "embedded" in T.
2. Test the tableau for consistency by deleting inconsistent portions. If the "root" of the tableau is deleted, f_0 is unsatisfiable. Otherwise, f_0 is satisfiable.

---

*The [BM81] algorithm is incorrect and will erroneously claim that certain satisfiable formulae are unsatisfiable. Correct tableau-based and filtration-based decision procedures for UB are given in [EH81]. In addition, Ben-Ari [BA81] states that a corrected version, using different techniques, of [BM81] is forthcoming.

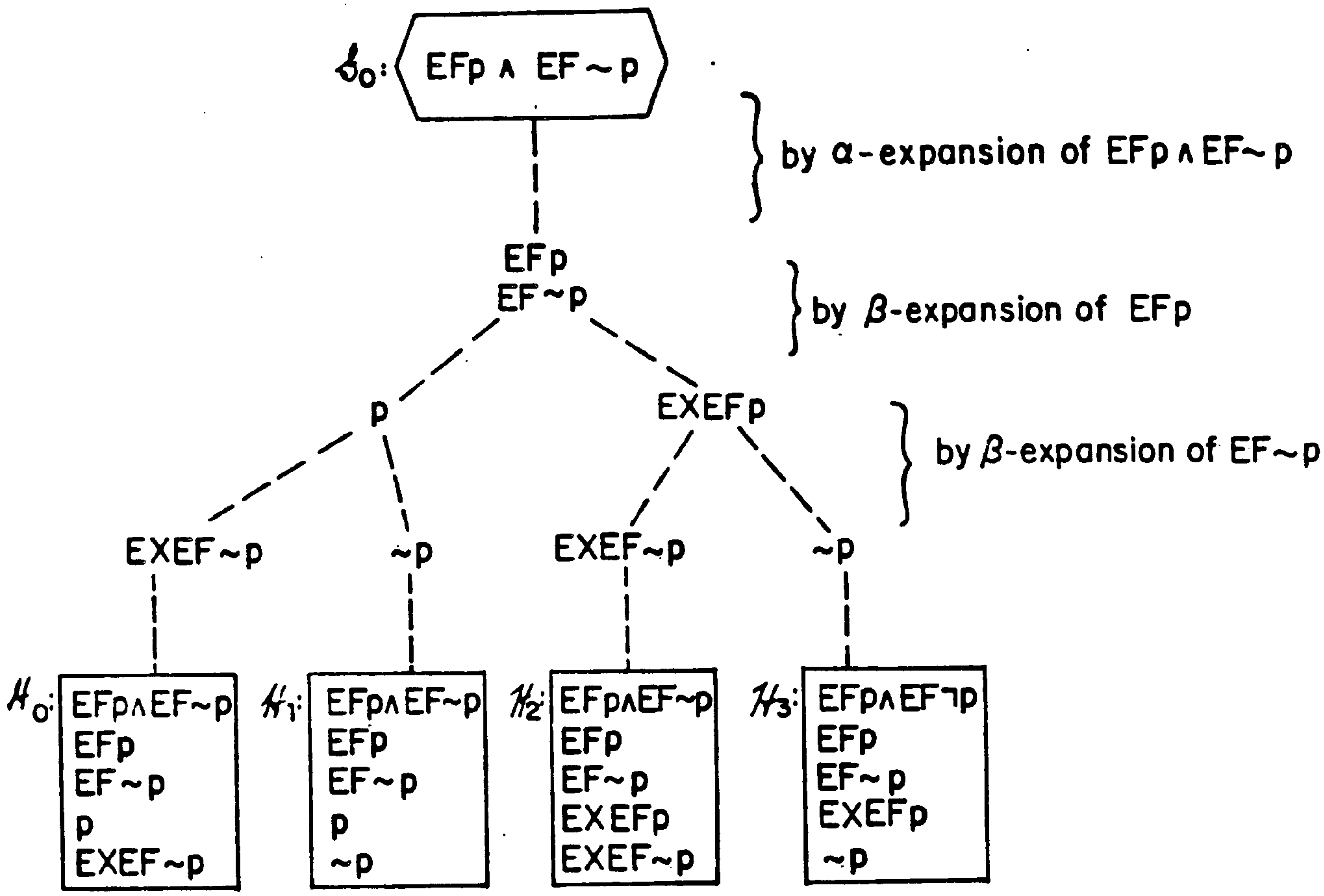
3. Unravel the tableau into a model of f_0.

The decision procedure begins by building a tableau T which is a finite directed AND/OR graph. Each node of T is either an AND-node or an OR-node and is labelled by a set of formulae. We use $G_1, G_2, \ldots$ to denote the labels of OR-nodes, $H_1, H_2, \ldots$ to denote the labels of AND-nodes, and $F_1, F_2, \ldots$ to denote the labels of arbitrary nodes of either type. No two AND-nodes have the same label, and no two OR-nodes have the same label. The intended meaning is that, when node F is considered as a state in an appropriate structure, $F \models f$ for all $f \in F$. The tableau T has a "root" node $G_0 = \{f_0\}$ from which all other nodes in T are accessible.

The set of successors of an OR-node G, $\mathrm{Blocks}(G) = \{H_1, H_2, \ldots, H_k\}$ has the property that

$$\models G \text{ iff } \models H_1 \text{ or } \ldots \text{ or } \models H_k .$$

We can explain the construction of Blocks(G) as follows: Each formula in G may be viewed as a conjunctive formula $\alpha \equiv \alpha_1 \wedge \alpha_2$ or a disjunctive formula $\beta \equiv \beta_1 \vee \beta_2$. Clearly, $f \wedge g$ is an α formula and $f \vee g$ is a β formula. A modal formula may be classified as α or β based on its fixpoint characterization; thus, $EFp = p \vee EXEFp$ is a β formula and $AGp = p \wedge AXAGp$ is an α formula. A formula that involves no modalities or has main connective one of EX or AX is both α and β and is called an *elementary* formula. Any other formula is *nonelementary*. We say that a set of formulae F is *downward closed* provided that (i) if $\alpha \in F$ then $\alpha_1, \alpha_2 \in F$, and (ii) if $\beta \in F$ then $\beta_1 \in F$ or $\beta_2 \in F$. We construct the members H_i of Blocks(G) by repeatedly expanding each nonelementary formula in G into its α or β components. Each β expansion results in two blocks, one which will contain β_1 and the other which will contain β_2. Expansion stops when all H_i are downward closed.



Blocks$(\mathcal{B}_0) = \{\mathcal{H}_0, \ldots, \mathcal{H}_3\}$. Each $\mathcal{H}_i$ is a downward closed set containing $\mathcal{B}_0$ and is obtained by taking the union of all formulae occuring along the path from the root to the ith leaf of the α-β expansion tree of EFp∧EF∼p.

Figure 6.1

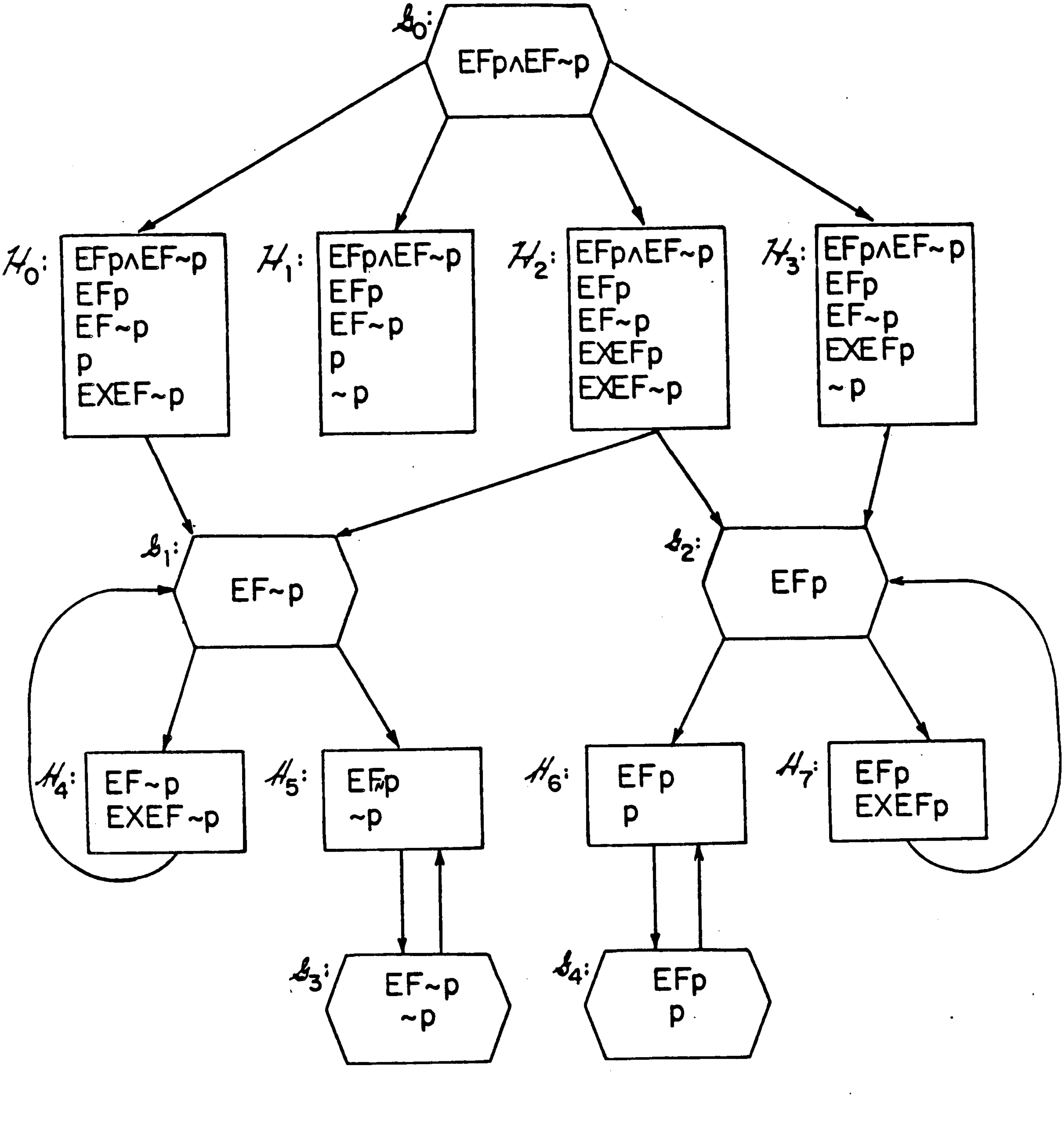
The set of successors of an AND-node H, $\mathrm{Tiles}(H) = \{G_1, G_2, \ldots, G_k\}$ has the property that, if H contains no propositional inconsistencies, then

$$\models H \text{ iff } \models G_1 \text{ and } \ldots \text{ and } \models G_k .$$

To construct $\mathrm{Tiles}(H)$ we use the information supplied by the elementary formulae in H. For example, if $\{AXh_1, AXh_2, EXg_1, EXg_2, EXg_3\}$ is the set of all elementary formulae in H, then $\mathrm{Tiles}(G) = \{\{h_1, h_2, g_1\}, \{h_1, h_2, g_2\}, \{h_1, h_2, g_3\}\}$.

To build T, we start out by letting $G_0 = \{f_0\}$ be the root node. Then we create $\mathrm{Blocks}(G_0) = \{H_1, H_2, \ldots, H_k\}$ and attach each H_i as a successor of G_0. For each H_i we create $\mathrm{Tiles}(H_i)$ and attach its members as the successors of H_i. For each $G_j \in \mathrm{Tiles}(H_i)$ we create $\mathrm{Blocks}(G_1)$, etc. Whenever we encounter two nodes of the same type with identical labels we identify them. This ensures that no two AND-nodes will have the same label, and that no two OR-nodes will have the same label. The tableau construction will eventually terminate since there are only $2^{\mathrm{length}(f_0)}$ possible labels each of which can occur at most twice.

Suppose, for example, that we want to determine whether $EFp \wedge EF{\sim}p$ is satisfiable. We build the tableau T starting with root node $G_0 = \{EFp \wedge EF{\sim}p\}$. We construct $\mathrm{Blocks}(G_0) = \{H_0, H_1, H_2, H_3\}$ as shown in Figure 6.1. Each H_i is attached as a successor of G_0. Next, $\mathrm{Tiles}(H_i)$ is determined for each H_i (except H_1 which is immediately seen to contain a propositional inconsistency) and its members are attached as successors of H_i. (Note that two copies of $G_1 = \{EF{\sim}p\}$ are created, one in $\mathrm{Tiles}(H_0)$ and the other in $\mathrm{Tiles}(H_2)$; but they are then merged into a single node.) Similarly, $G_2 \in \mathrm{Tiles}(H_2) \cap \mathrm{Tiles}(H_3)$. Continuing in this fashion we obtain the complete tableau shown in Fig. 6.2.



$\mathcal{G}_0$:
EFp∧EF~p
$\mathcal{H}_0$:
EFp∧EF~p
EFp
EF~p
p
EXEF~p
$\mathcal{H}_1$:
EFp∧EF~p
EFp
EF~p
p
~p
$\mathcal{H}_2$:
EFp∧EF~p
EFp
EF~p
EXEFp
EXEF~p
$\mathcal{H}_3$:
EFp∧EF~p
EFp
EF~p
EXEFp
~p
$\mathcal{G}_1$:
EF~p
$\mathcal{G}_2$:
EFp
$\mathcal{H}_4$:
EF~p
EXEF~p
$\mathcal{H}_5$:
EF~p
~p
$\mathcal{H}_6$:
EFp
p
$\mathcal{H}_7$:
EFp
EXEFp
$\mathcal{G}_3$:
EF~p
~p
$\mathcal{G}_4$:
EFp
p

Figure 6.2

Next we must test the tableau for consistency. Note that H_1 is inconsistent because it contains both p and ~p. We must also check that it is possible for eventuality formulae such as AFh or EFh to be *fulfilled*: e.g., if $EFh \in F$, then there must be some node F' reachable from F such that $h \in F'$. If any node fails to pass this test, it is marked inconsistent. In this example, all nodes pass the test. Since the root is not marked inconsistent, $EFp \wedge EF{\sim}p$ is satisfiable.

Finally, we construct a model M of $EFp \wedge EF{\sim}p$. The states in M will be (copies of) the AND-nodes in the tableau. The model will have the property that for each state H, $M,H \models f$ for all $f \in M$. The root of M can be any consistent state $H_i \in \mathrm{Blocks}(G_0)$. We choose H_0. Now H_0 contains the eventualities EFp and EF ~p. We must ensure that they are actually fulfilled in M. EFp is immediately fulfilled in H_0, but EF~p is not. So when we choose a successor state to H_0, which must be one of H_4 or H_5, we want to ensure that EF~p is fulfilled. Thus, we choose H_5. Finally, the only possible successor state of H_5 is H_5 itself. We obtain the model shown in Fig. 6.3 which is embedded in the tableau.

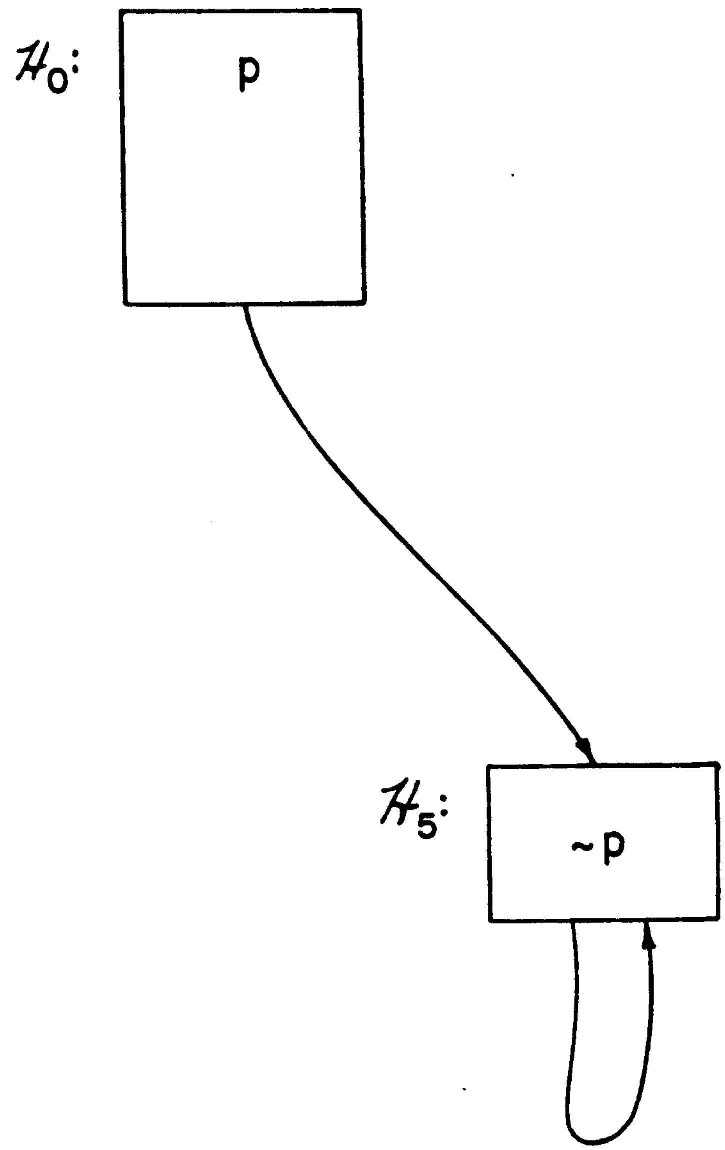


Figure 6.3

7. SYNTHESIS ALGORITHM

We now present our method of synthesizing synchronization skeletons from a CTL description of their intended behavior. We identify the following steps:

1. Specify the desired behavior of the concurrent system using CTL.
2. Apply the decision procedure to the resulting CTL formula in order to obtain a finite model of the formula.
3. Factor out the synchronization skeletons of the individual processes from the global system flowgraph defined by the model.

We illustrate the method by solving a mutual exclusion problem for processes P_1 and P_2. Each process is always in one of three regions of code:

NCS_i	the NonCritical Section
TRY_i	the TRYing Section
CS_i	the Critical Section

which it moves through as suggested in Fig. 7.1.

When it is in region NCS_i, process P_i performs "noncritical" computations which can proceed in parallel with computations by the other process P_j. At certain times, however, P_i may need to perform certain "critical" computations in the region CS_i. Thus, P_i remains in NCS_i as long as it has not yet decided to attempt critical section entry. When and if it decides to make this attempt, it moves into the region TRY_i. From there it enters CS_i as soon as possible, provided that the mutual exclusion constraint $\sim(CS_1 \wedge CS_2)$ is not violated. It remains in CS_i as long as necessary to perform its "critical" computations and then re-enters NCS_i.

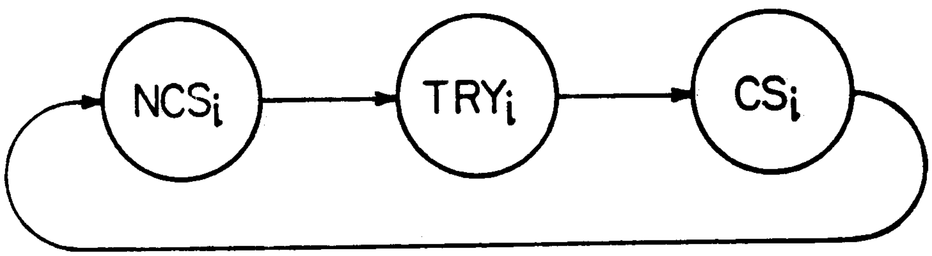


Figure 7.1

Note that in the synchronization skeleton described, we only record transitions between different regions of code. Moves entirely within the same region are not considered in specifying synchronization. Listed below are the CTL formulae whose conjunction specifies the mutual exclusion system:

1. start state

   $NCS_1 \wedge NCS_2$

2. mutual exclusion

   $AG(\sim(CS_1 \wedge CS_2))$

3. absence of starvation for P_i

   $AG(TRY_i \rightarrow AFCS_i)$

4. each process P_i is always in exactly one of the three code regions

   $AG(NCS_i \vee TRY_i \vee CS_i)$

   $AG(NCS_i \rightarrow \sim(TRY_i \vee CS_i))$

   $AG(TRY_i \rightarrow \sim(NCS_i \vee CS_i))$

   $AG(CS_i \rightarrow \sim(NCS_i \vee TRY_i))$

5. it is always possible for P_i to enter its trying region from its noncritical region

   $AG(NCS_i \rightarrow EX_i TRY_i)$

6. it is always the case that any move P_i makes from its trying region is into the critical region

   $AG(TRY_i \wedge EX_i True \rightarrow AX_i CS_i)$

7. it is always possible for P_i to re-enter its noncritical region from its critical region

   $AG(CS_i \rightarrow EX_i NCS_i)$

8. a transition by one process cannot cause a move by the other

   $AG(NCS_i \rightarrow AX_j NCS_i)$

   $AG(TRY_i \rightarrow AX_j TRY_i)$

   $AG(CS_i \rightarrow AX_j CS_i)$

9. some process can always move

   $AG(EX\ True)$

We must now construct the initial AND/OR graph tableau. In order to reduce the recording of inessential or redundant information in the node labels we observe the following rules:

(1) Automatically convert a formula of the form $f_1 \wedge \ldots \wedge f_n$ to the set of formulae $\{f_1, \ldots, f_n\}$. (Recall that the set of formulae $\{f_1, \ldots, f_n\}$ is satisfiable iff $f_1 \wedge \ldots \wedge f_n$ is satisfiable.)

(2) Do not physically write down an invariance assertion of the form AGf because it holds everywhere as do its consequences f and AXAGf (obtained by α-expansion). The consequence AXAGf serves only to propagate forward the truth of AGf to any "descendent" nodes in the tableau. Do that propagation automatically but without writing down AGf in any of the descendent nodes. The consequence f may be written down if needed.

(3) An assertion of the form $f \vee g$ need not be recorded when f is already present. Since any state which satisfies f must also satisfy $f \vee g$, $f \vee g$ is redundant.

(4) If we have TRY_i present, there is no need to record $\sim NCS_i$ and $\sim CS_i$. If we have NCS_i present, there is no need to record $\sim TRY_i$ and $\sim CS_i$. If we have CS_i present, there is no need to record $\sim NCS_i$ and $\sim TRY_i$.

By the above conventions, the root node of the tableau will have the two formulae NCS_1 and NCS_2 recorded in its label which we now write as $\langle NCS_1\ NCS_2 \rangle$. In building the tableau, it will be helpful to have constructed Blocks(G) for the following OR-nodes: $\langle NCS_1\ NCS_2 \rangle$, $\langle TRY_1\ NCS_2 \rangle$, $\langle CS_1\ NCS_2 \rangle$, $\langle TRY_1\ TRY_2 \rangle$, and $\langle CS_1\ TRY_2 \rangle$. For all other OR-nodes G' appearing in the tableau, Blocks(G') will be identical to or can be obtained by symmetry from Blocks(G) for some G in the above list. Figures 7.2-7.6 show the abbreviated construction of Blocks(G) for these OR-nodes as well as Tiles(H) for each $H \in$ Blocks(G). We then build the tableau using the

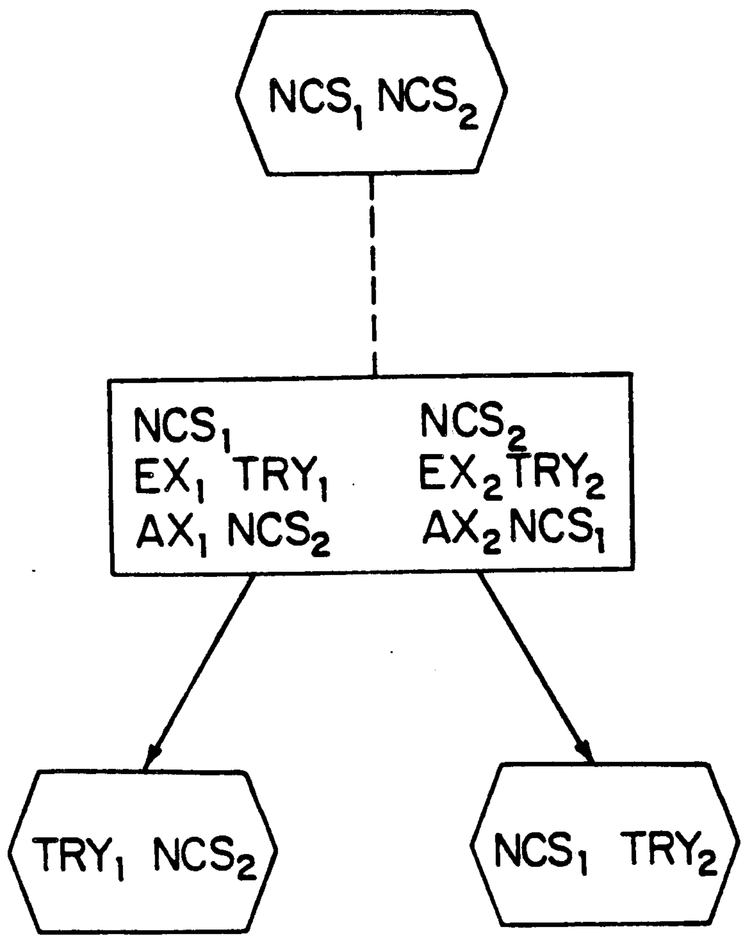

NCS_1 NCS_2
NCS_1
EX_1 TRY_1
AX_1 NCS_2
NCS_2
EX_2 TRY_2
AX_2 NCS_1
TRY_1 NCS_2
NCS_1 TRY_2

Figure 7.2

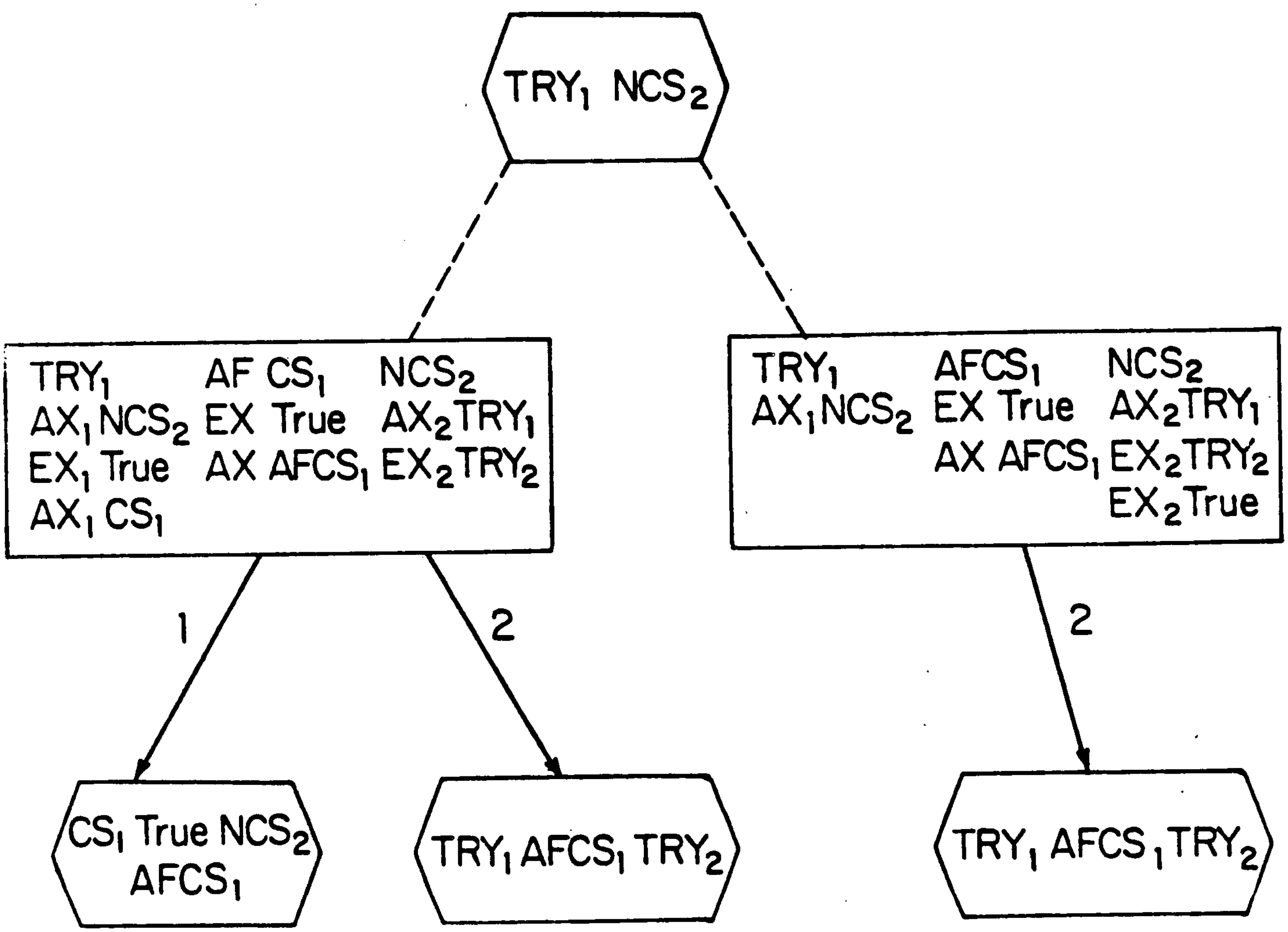

TRY1 NCS2
TRY1 AF CS1 NCS2
AX1 NCS2 EX True AX2 TRY1
EX1 True AX AFCS1 EX2 TRY2
AX1 CS1
TRY1 AFCS1 NCS2
AX1 NCS2 EX True AX2 TRY1
AX AFCS1 EX2 TRY2
EX2 True
1
2
2
CS1 True NCS2
AFCS1
TRY1 AFCS1 TRY2
TRY1 AFCS1 TRY2

Figure 7.3

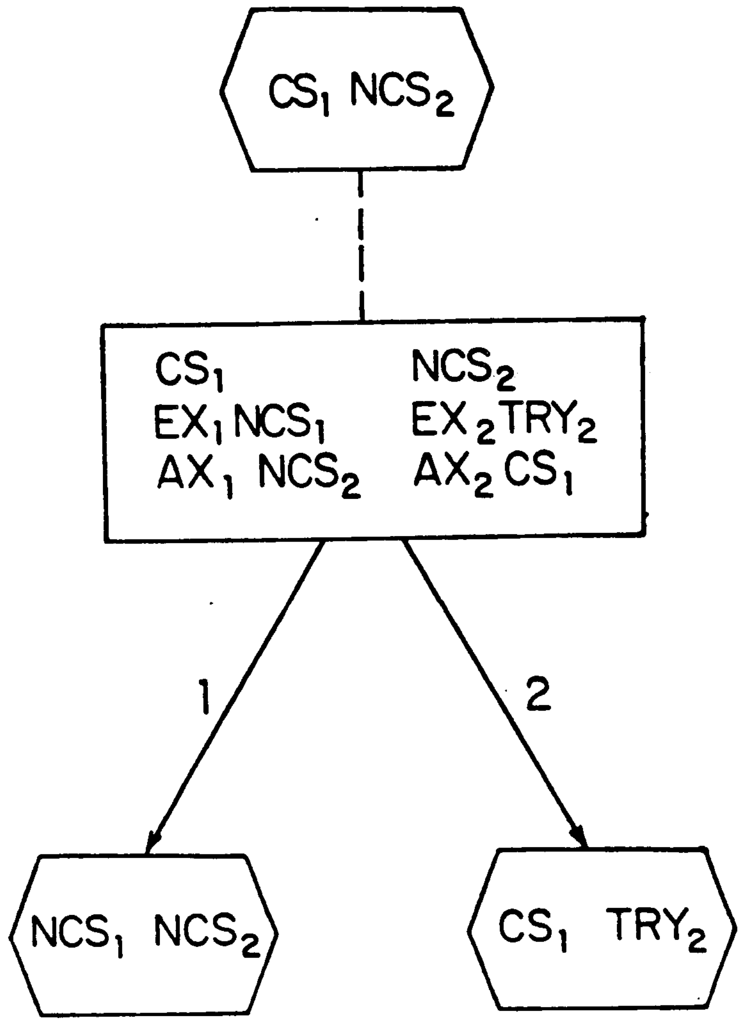

CS_1 NCS_2
CS_1
$EX_1 NCS_1$
AX_1 NCS_2
NCS_2
$EX_2 TRY_2$
$AX_2 CS_1$
1
2
NCS_1 NCS_2
CS_1 TRY_2

Figure 7.4

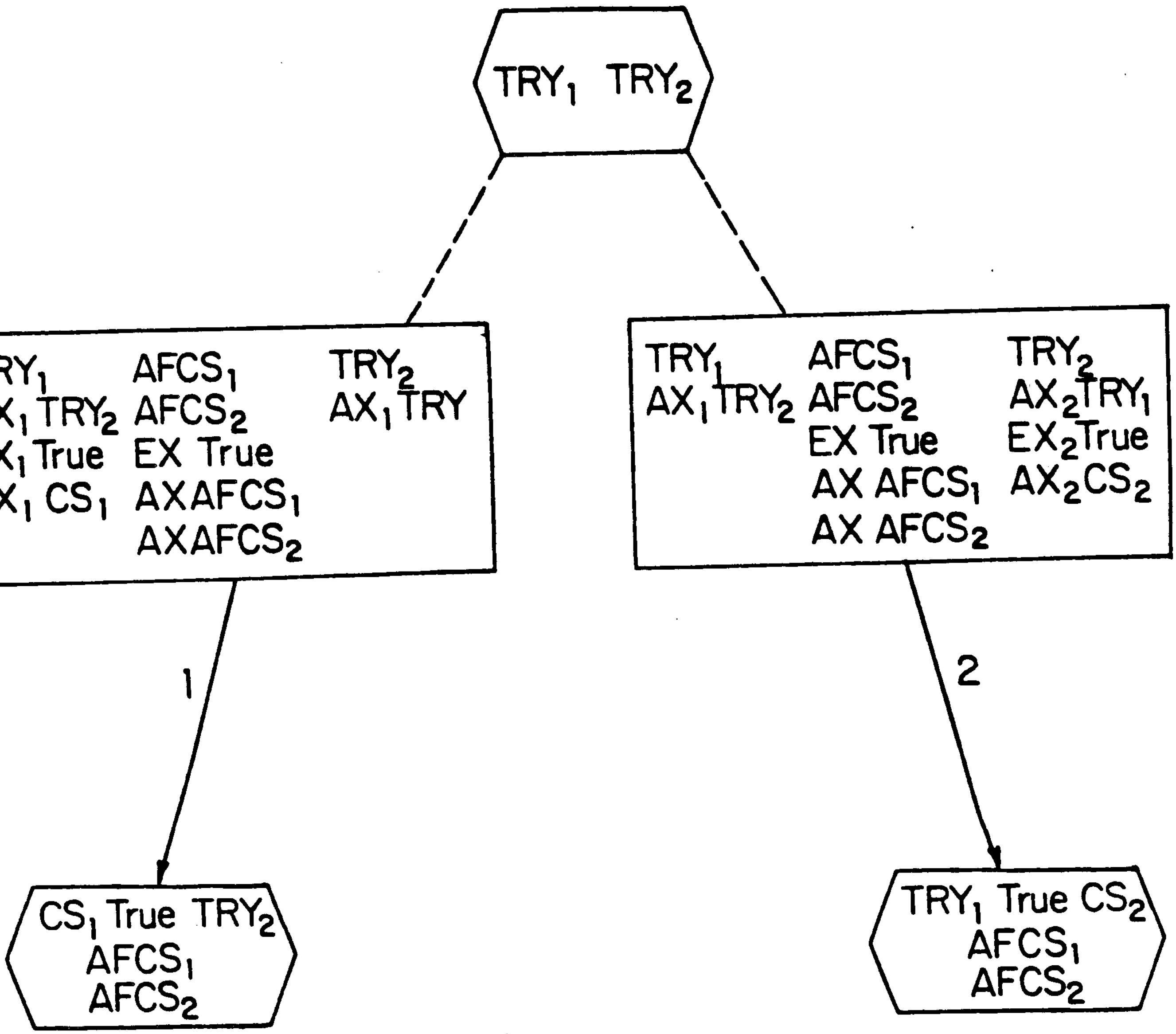
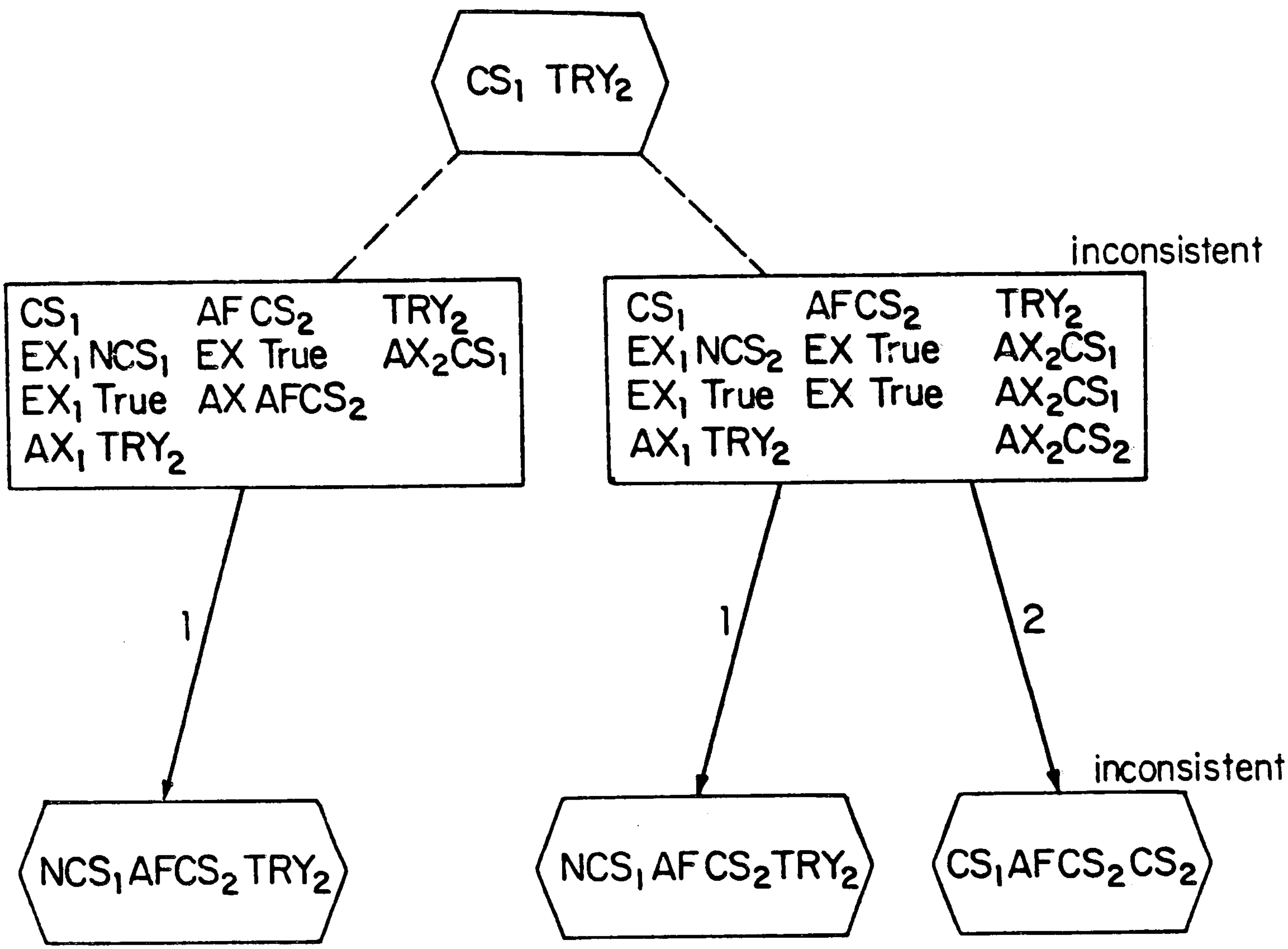

TRY1 TRY2
TRY1 AFCS1 TRY2
AX1 TRY2 AFCS2 AX1 TRY
EX1 True EX True
AX1 CS1 AXAFCS1
AXAFCS2
TRY1 AFCS1 TRY2
AX1 TRY2 AFCS2 AX2 TRY1
EX True EX2 True
AX AFCS1 AX2 CS2
AX AFCS2
1
2
CS1 True TRY2
AFCS1
AFCS2
TRY1 True CS2
AFCS1
AFCS2

Figure 7.5



CS1 TRY2
inconsistent
CS1 AF CS2 TRY2
EX1 NCS1 EX True AX2CS1
EX1 True AX AFCS2
AX1 TRY2
CS1 AFCS2 TRY2
EX1 NCS2 EX True AX2CS1
EX1 True EX True AX2CS1
AX1 TRY2 AX2CS2
1
1
2
inconsistent
NCS1 AFCS2 TRY2
NCS1 AF CS2 TRY2
CS1 AFCS2 CS2

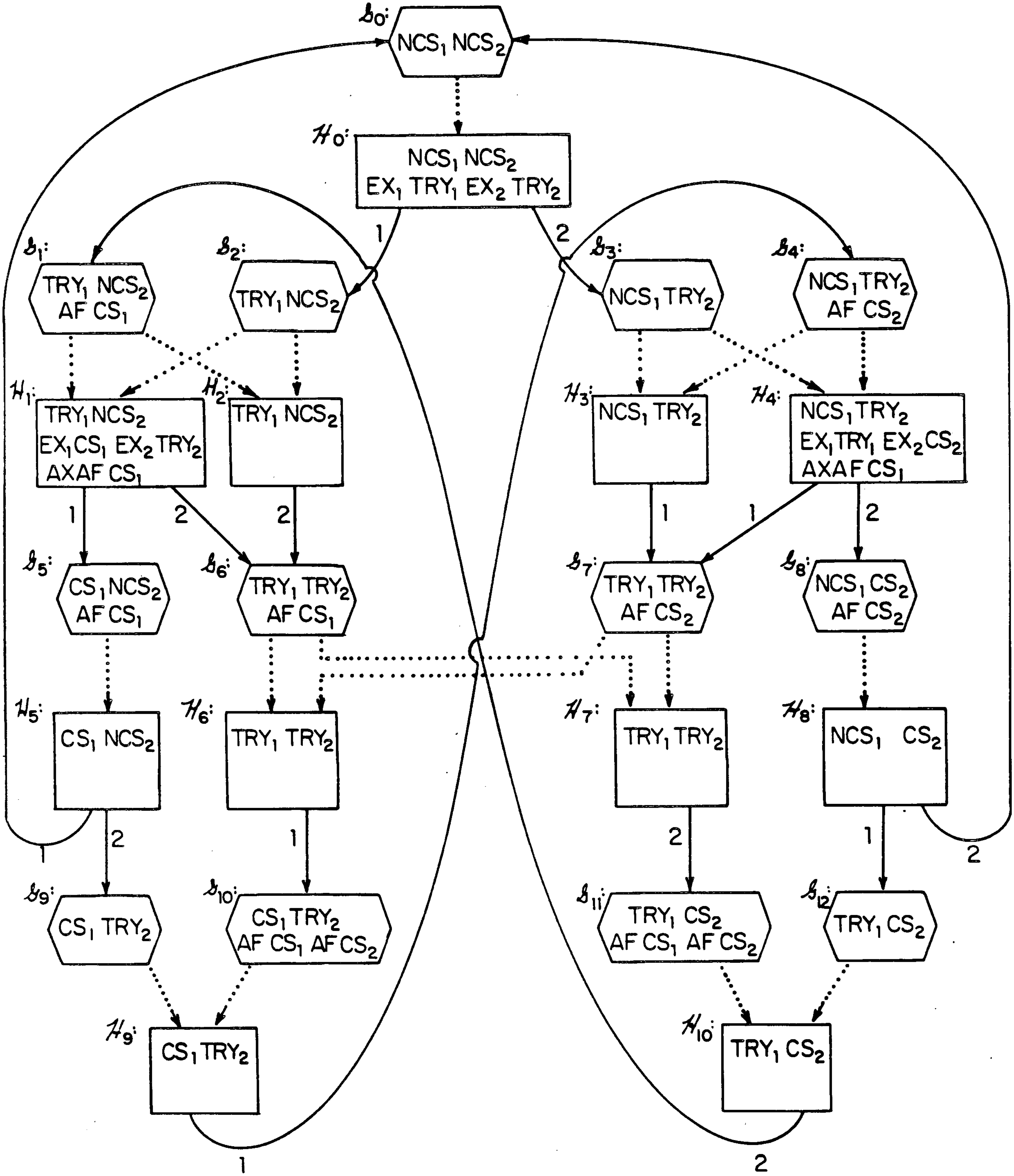
Figure 7.6

information about Blocks and Tiles contained in Figures 7.2-7.6. We next apply the marking rules to delete inconsistent nodes. Note that the OR-node $<CS_1\ CS_2\ AFCS_2>$ is marked as deleted because of a propositional inconsistency (with $\sim(CS_1 \wedge CS_2)$, a consequence of the unwritten invariance $AG(\sim(CS_1 \wedge CS))$. This, in turn, causes the AND-node that is the predecessor of $<CS_1\ CS_2\ AFCS_2>$ to be marked. The resulting tableau is shown in Fig. 7.7. Each node in Fig. 7.7 is labelled with a minimal set of formulae sufficient to distinguish it from any other node.

We construct a model M from T by pasting together model fragments for the AND-nodes using local structure information provided by T. Intuitively, a fragment is a rooted dag of AND-nodes embeddable in T such that all eventuality formulae in the label of the root node are fulfilled in the fragment. Fragments are described in detail in the appendix.

The root node of the model is H_0, the unique successor of G_0. From the tableau we see that H_0 must have two successors, one of H_1 or H_2 and one of H_3 or H_4. Each candidate successor state contains an eventuality to fulfill, so we must construct and attach its fragment. Using the method described in the appendix, we choose the fragment rooted at H_1 to be the left successor and the fragment rooted at H_4 to be the right successor (see Fig. 7.8). This yields the portion of the model shown in Fig. 7.9.

We continue the construction by finding successors for each of the leaves: H_5, H_9, H_{10} and H_8. We start with H_5. By inspection of T, we see that the only successors H_5 can have are H_0 and H_9. Since H_0 and H_9 already occur in the structure built so far, we add the arcs $H_5 \xrightarrow{1} H_0$ and $H_5 \xrightarrow{2} H_9$ to the structure. Note that this introduces a cycle $(H_0 \xrightarrow{1} H_1 \xrightarrow{1} H_5 \xrightarrow{1} H_0)$. In general, a cycle can be dangerous because it might



G0: NCS1 NCS2
H0: NCS1 NCS2 EX1 TRY1 EX2 TRY2
G1: TRY1 NCS2 AF CS1
G2: TRY1 NCS2
G3: NCS1 TRY2
G4: NCS1 TRY2 AF CS2
H1: TRY1 NCS2 EX1 CS1 EX2 TRY2 AXAF CS1
H2: TRY1 NCS2
H3: NCS1 TRY2
H4: NCS1 TRY2 EX1 TRY1 EX2 CS2 AXAF CS1
G5: CS1 NCS2 AF CS1
G6: TRY1 TRY2 AF CS1
G7: TRY1 TRY2 AF CS2
G8: NCS1 CS2 AF CS2
H5: CS1 NCS2
H6: TRY1 TRY2
H7: TRY1 TRY2
H8: NCS1 CS2
G9: CS1 TRY2
G10: CS1 TRY2 AF CS1 AF CS2
G11: TRY1 CS2 AF CS1 AF CS2
G12: TRY1 CS2
H9: CS1 TRY2
H10: TRY1 CS2
1
2

Figure 7.7

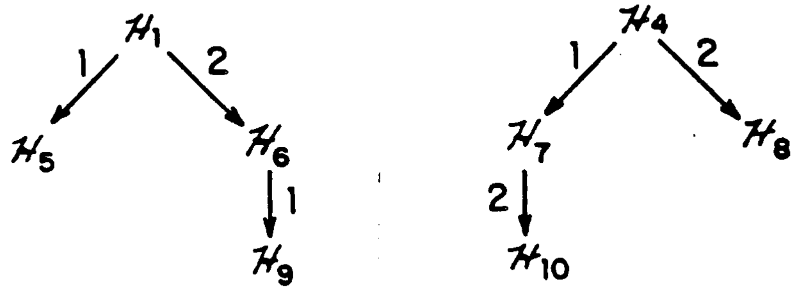


Figure 7.8

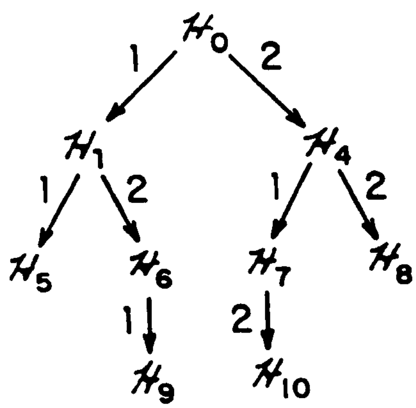


Figure 7.9

form a path along which some eventuality is never fulfilled; however, there is no problem this time because the root of a fragment, H_1, occurs along the cycle. A fragment root serves as a checkpoint to ensure that all eventualities are fulfilled. By symmetry between the roles of 1 and 2, we add in the arcs $H_8 \xrightarrow{1} H_{10}$ and $H_8 \xrightarrow{2} H_0$. The structure now has the form shown in Fig. 7.10.

We now have two leaves remaining: H_9 and H_{10}. We see from the tableau that H_4 is a possible successor to H_9. We add in the arc $H_9 \xrightarrow{1} H_4$. Again a cycle is formed but since H_4 is a fragment root no problems arise. Similarly, we add in the arc $H_{10} \xrightarrow{2} H_1$. The decision procedure thus yields a model M such that $M,s_0 \models f_0$ where f_0 is the conjunction of the mutual exclusion system specifications. The model is shown in Fig. 7.11 where only the propositions true in a state are retained in the label.

We may view the model as a flowgraph of global system behavior. For example, when the system is in state H_1, process P_1 is in its trying region and process P_2 is in its noncritical section. P_1 may enter its critical section or P_2 may enter its trying region. No other moves are possible in state H_1. Note that all states except H_6 and H_7 are distinguished by their propositional labels. In order to distinguish H_6 from H_7, we introduce a variable TURN which is set to 1 upon entry to H_6 and to 2 upon entry to H_7. If we introduce TURN's value into the labels of H_6 and H_7 then, the labels uniquely identify each node in the global system flowgraph. See Fig. 7.12.

We describe how to obtain the synchronization skeletons of the individual processes from the global system flowgraph. In the sequel we will refer to these global system states by the propositional labels.

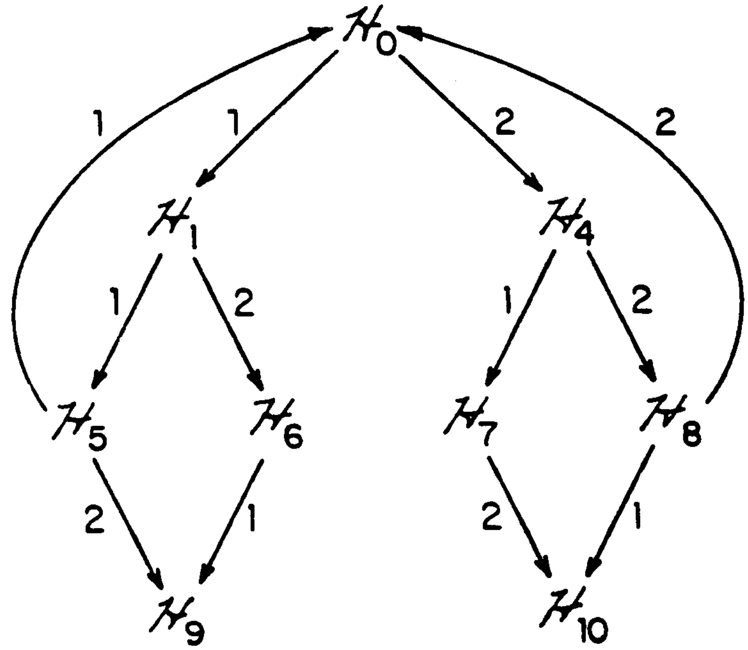
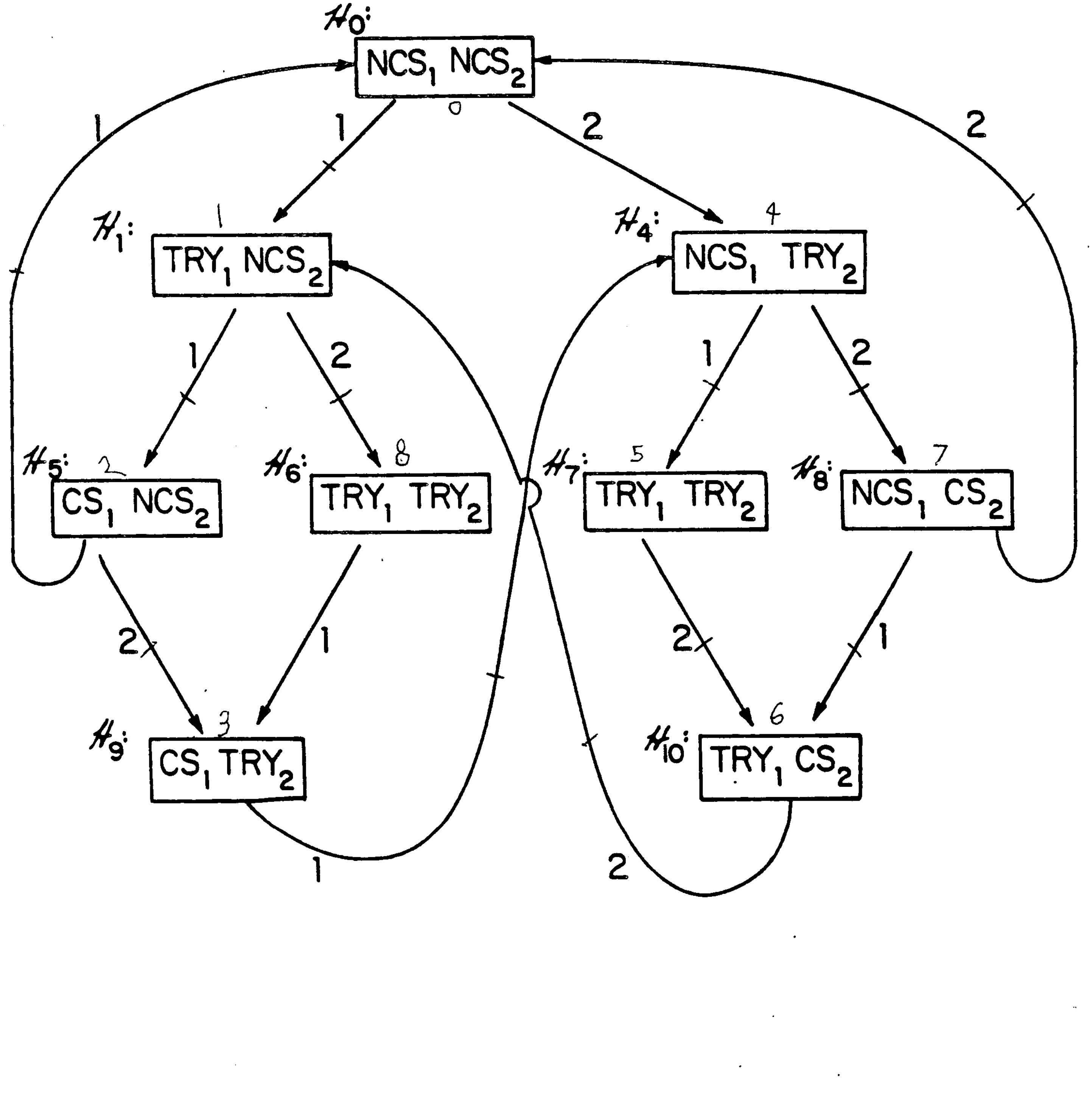
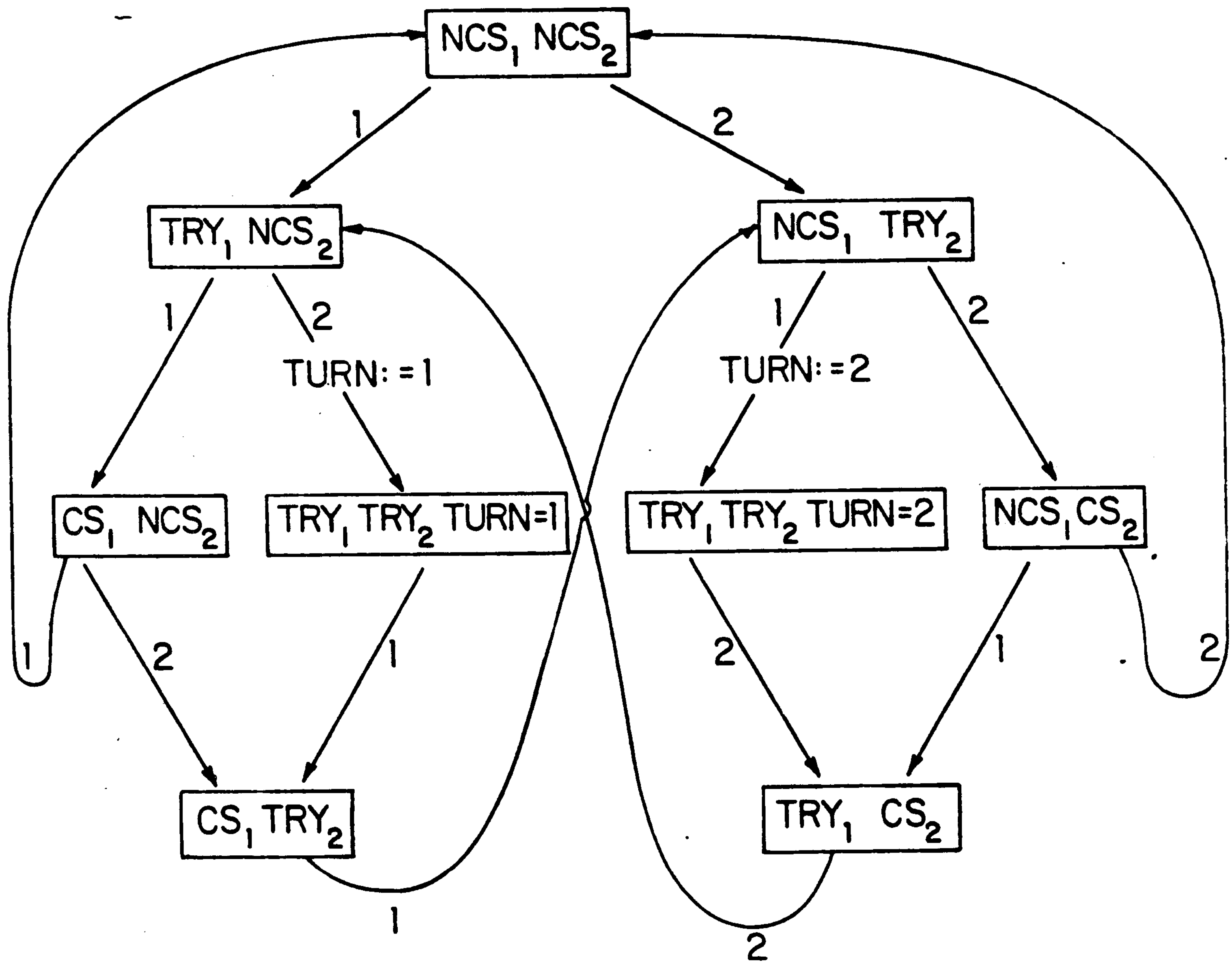

$\mathcal{H}_0$
1
1
2
2
$\mathcal{H}_1$
$\mathcal{H}_4$
1
2
1
2
$\mathcal{H}_5$
$\mathcal{H}_6$
$\mathcal{H}_7$
$\mathcal{H}_8$
2
1
2
1
$\mathcal{H}_9$
$\mathcal{H}_{10}$

Figure 7.10



H0:
NCS1 NCS2
H1:
TRY1 NCS2
H4:
NCS1 TRY2
H5:
CS1 NCS2
H6:
TRY1 TRY2
H7:
TRY1 TRY2
H8:
NCS1 CS2
H9:
CS1 TRY2
H10:
TRY1 CS2

Figure 7.11



NCS1 NCS2
1
2
TRY1 NCS2
NCS1 TRY2
1
2
TURN: =1
1
2
TURN: =2
CS1 NCS2
TRY1 TRY2 TURN=1
TRY1 TRY2 TURN=2
NCS1 CS2
1
2
1
2
1
2
CS1 TRY2
TRY1 CS2
1
2

Figure 7.12

When P_1 is in NCS_1, there are three possible global states $[NCS_1\ NCS_2]$ $[NCS_1\ TRY_2]$ $[NCS_1\ CS_2]$. In each case it is always possible for P_1 to make a transition into TRY_1 by the global transitions $[NCS_1\ NCS_2] \xrightarrow{1} [TRY_1\ NCS_2]$, $[NCS_1\ TRY_2] \xrightarrow{1,TURN:=2} [TRY_1\ TRY_2]$, and $[NCS_1\ CS_2] \xrightarrow{1} [TRY_1\ CS_2]$. From each global transition by P_1, we obtain a transition in the synchronization skeleton of P_1. The P_2 component of the global state provides enabling conditions for the transitions in the skeleton of P_1. If along a global transition, there is an assignment to TURN, the assignment is copied into the corresponding transition of the synchronization skeleton. Thus we have the transitions shown in Fig. 7.13(a) in the synchronization skeleton of P_1. We merge the transitions which lack assignments to obtain the portion of the synchronization skeleton of P_1 shown in Fig. 7.13(b).

Now when P_1 is in TRY_1, there are four possible global states: $[TRY_1\ NCS_2]$, $[TRY_1\ TRY_2\ TURN=1]$, $[TRY_1\ TRY_2\ TURN=2]$, and $[TRY_1\ CS_2]$ and their associated global transitions by P_1:
$[TRY_1\ NCS_2] \xrightarrow{1} [CS_1\ NCS_2]$ and $[TRY_1\ TRY_2\ TURN=1] \xrightarrow{1} [CS_1\ TRY_2]$.
(No transitions by P_1 are possible in $[TRY_1\ TRY_2\ TURN=2]$ or $[TRY_1\ CS_2]$.) Thus we obtain the portion of the synchronization skeleton for P_1 shown in Fig. 7.14(a). When P_1 is in CS_1 the associated global states and transitions are:

$[CS_1\ NCS_2]$, $[CS_1\ TRY_2]$, $[CS_1\ NCS_2] \xrightarrow{1} [NCS_1\ NCS_2]$, and $[CS_1\ TRY_2] \xrightarrow{1} [NCS_1\ TRY_2]$

from which we obtain the portion of the synchronization skeleton for P_1 shown in Fig. 7.14(b). Altogether, the synchronization skeleton for P_1 is shown in Fig. 7.15(a). By symmetry in the global state diagram we obtain the synchronization skeleton for P_2 as shown in Fig. 7.15(b).

The general method of factoring out the synchronization skeletons of the individual processes may be described as follows: Take the model of the

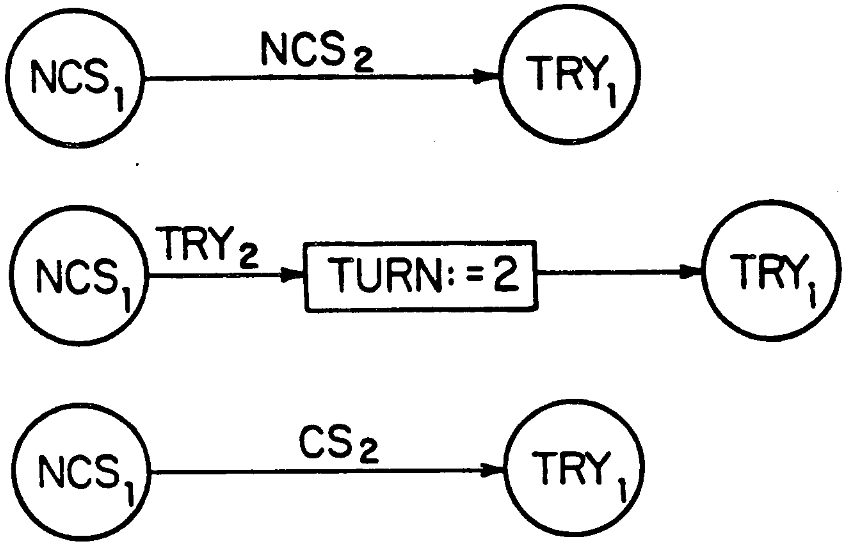


Figure 7.13 (a)

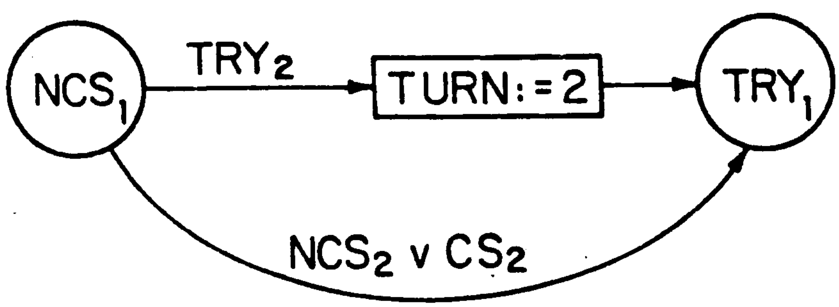


Figure 7.13 (b)

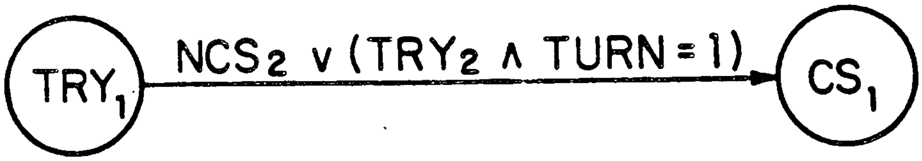


Figure 7.14 (a)

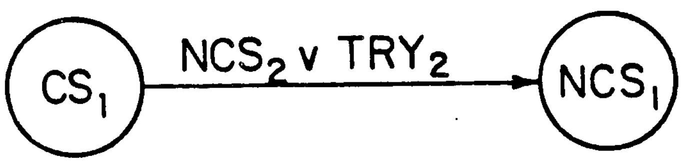


Figure 7.14 (b)

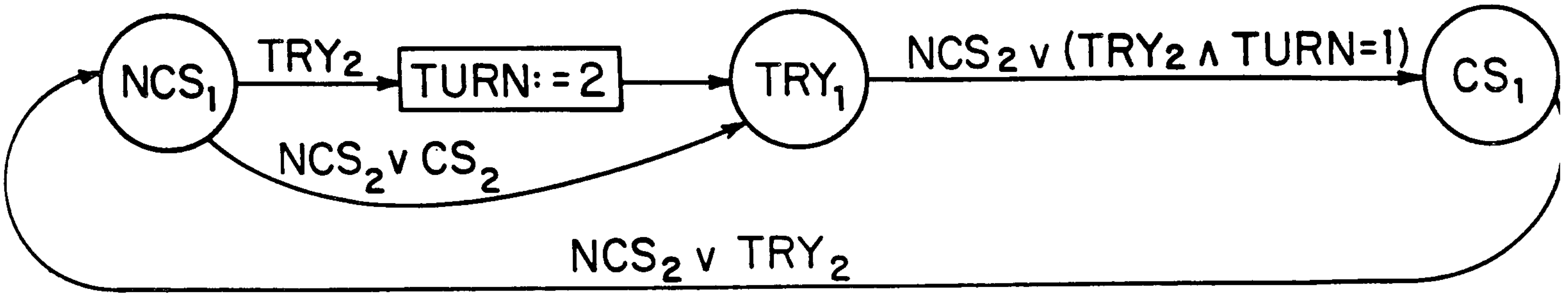


Figure 7.15(a)

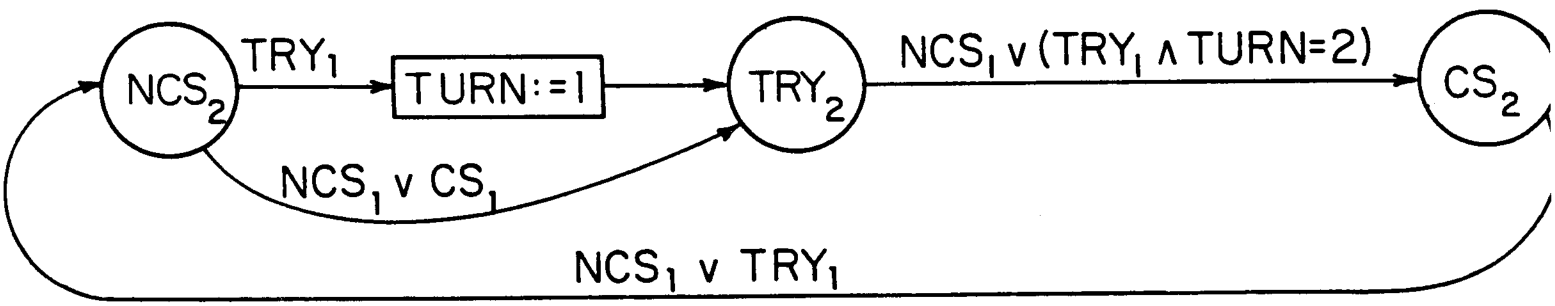


Figure 7.15(b)

specification formula and retain only the propositional formulae in the labels of each node. There may now be distinct nodes with the same label. Auxiliary variables are introduced to ensure that each node gets a distinct label: if label L occurs at $n > 1$ distinct nodes $v_1, \ldots, v_n$, then for each v_i, set $L:=i$ on all arcs coming into v_i and add $L = i$ as an additional component to the label of v_i. The resulting newly labelled graph is the global system flowgraph.

We now construct the synchronization skeleton for process P_i which has m distinct code regions $R_1, \ldots, R_m$. Initially, the synchronization skeleton for P_i is a graph with m distinct nodes $R_1, \ldots, R_m$ and no arcs. Draw an arc from R_j to R_k if there is at least one arc of the form $L_j \to L_k$ in the global system flowgraph where R_j is a component of the label L_j and R_k is a component of the label L_k. The arc $R_j \to R_k$ is a transition in the synchronization skeleton and is labelled with the enabling condition

$$\bigvee \{(S_1 \wedge \ldots \wedge S_p): [R_j\ S_1 \ldots S_p] \xrightarrow{i} [R_k\ S_1 \ldots S_p] \text{ is an arc in the global system flowgraph}\}.$$

Add $L:=n$ to the label of $R_j \to R_k$ if some arc $[R_j\ S_1 \ldots S_p] \xrightarrow{i, L:=n} [R_k\ S_1 \ldots S_p]$ also occurs in the flowgraph.

8. CONCLUSION

We have shown that it is possible to automatically synthesize the synchronization skeleton of a concurrent program from a Temporal Logic specification. We believe that this approach may in the long run turn out to be quite practical. Since synchronization skeletons are, in general, quite small, the potentially exponential behavior of our algorithm need not be an insurmountable obstacle. Much additional research may be needed, however, to make the approach feasible in practice.

We have also described a model checking algorithm which can be applied to mechanically verify that a finite state concurrent program meets a particular Temporal Logic specification. We believe that practical software tools based on this technique could be developed in the near future. Indeed, we have already programmed an experimental implementation of the model checker on the DEC 11/70 at Harvard.* Certain applications seem particularly suited to the model checker approach to verification: One example is the problem of verifying the correctness of existing network protocols many of which are coded as finite state machines. We encourage additional work in this area.

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*We would like to acknowledge Marshall Brinn who did the actual programming for our implementation of the model checker.

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10. APPENDIX

In this appendix, we describe the decision procedure for CTL in detail. We assume that the reader is familiar with the overview of the decision procedure given in Section 6. The proofs of correctness of the decision procedure and of the finite model property for CTL are similar to the corresponding proofs for UB. See [EH81].

10.1 Construction of the Initial AND/OR graph

We construct the initial AND/OR graph T in stages by the method below:

1. Initally, let the "root" node of T be the OR-node $G_0 = \{f_0\}$.

2. If all nodes in T have successors, halt. Otherwise, let F be any node without successors in T. If F is an OR-node G, construct $\mathrm{Blocks}(G) = \{H_1,\ldots,H_k\}$ and attach each H_i as an immediate successor of G in T. If any H_i has the same label as another AND-node H already present in T, then merge H_i and H. If F is an AND-node H, construct $\mathrm{Tiles}(H) = \{G_1,\ldots,G_k\}$ and attach each G_i as an immediate successor of G in T. Label the arc (H,G_i) in T with each j such that $G_i \in \mathrm{Tiles}_j(H)$. If any G_i has the same label as some other OR-node G already present in T, then merge G_i and G. Repeat this step.

10.2 Construction of Blocks(G)

For convenience, we assume that every formula in G has been placed in *standard* form with all negations driven inside so that only atomic propositions appear negated. (This can be done using duality: $\sim(f \wedge g) \leftrightarrow \sim f \vee \sim g$,

$\sim$AFh $\leftrightarrow$ EG $\sim$h, etc.) We say that a formula is *elementary* provided that it is a proposition, the negation of a proposition, or has main connective AX_j or EX_j. Any other formula is *nonelementary*.

We classify nonelementary formulae as either α or β as discussed in Section 6. The following table summarizes the classification:

$\alpha = f \wedge g$	$\alpha_1 = f$	$\alpha_2 = g$
$\alpha = A[f \vee g]$	$\alpha_1 = g$	$\alpha_2 = f \vee AXA[f \vee g]$
$\alpha = E[f \vee g]$	$\alpha_1 = g$	$\alpha_2 = f \vee EXA[f \vee g]$
$\beta = f \vee g$	$\beta_1 = f$	$\beta_2 = g$
$\beta = A[fUg]$	$\beta_1 = g$	$\beta_2 = f \wedge AXA[fUg]$
$\beta = E[fUg]$	$\beta_1 = g$	$\beta_2 = f \wedge EXE[fUg]$

To construct Blocks(G) we first build a finitely branching tree whose nodes are labelled with sets of formulae. (This tree is essentially a propositional logic tableau as described in [Smullyan].) Initially, let the root = G. In general, let F be a leaf in the tree constructed so far for which there exists a nonelementary formula $f \in F$. Add one or two sons to F as appropriate according to the rules shown in Fig. 10.1. Eventually, this construction must halt because all leaves $F_1, \ldots, F_m$ will contain only elementary formulae. (This can be proved by induction of the length of the longest formula in G.) Then let Blocks(G) = $\{H_1, \ldots, H_m\}$ where H_i is the set of all formulae appearing in some node on the path from F_i back to the root of the tree.

10.3 Construction of Tiles(H)

For each $j \in [1:k]$, we must determine the set $\text{Tiles}_j(H)$ of successors associated with process j. Let

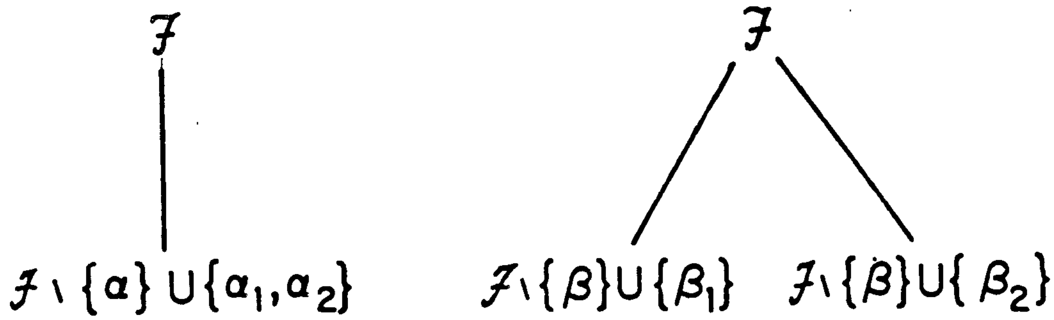

$\mathcal{F}$
$\mathcal{F} \setminus \{\alpha\} \cup \{\alpha_1, \alpha_2\}$
$\mathcal{F}$
$\mathcal{F} \setminus \{\beta\} \cup \{\beta_1\}$
$\mathcal{F} \setminus \{\beta\} \cup \{\beta_2\}$

Figure 10.1

$$HA_j = \{f: AX_j f \in H\} \quad \text{and}$$

$$HE_j = \{g: EX_j g \in H\} \quad .$$

If $HE_j \neq \emptyset$ then write HE_j as $\{g_1,\ldots,g_n\}$ and define

$$\text{Tiles}_j(H) = \{G_j^1,\ldots,G_j^n\} \quad \text{where}$$

$$G_j^i = HA_j \cup \{g_i\} \quad \text{for} \quad i \in [1:n] \quad .$$

Now define

$$\text{Tiles}(H) = \cup\{\text{Tiles}_j(H): j \in [1:k]\} \quad .$$

If $G_i \in \text{Tiles}(H)$ then the arc from H to G_i in T is labelled with $j_1,\ldots,j_m$ where $G_i \in \text{Tiles}_{j_1}(H),\ldots,\text{Tiles}_{j_m}(H)$. Figure 10.2 gives an example.

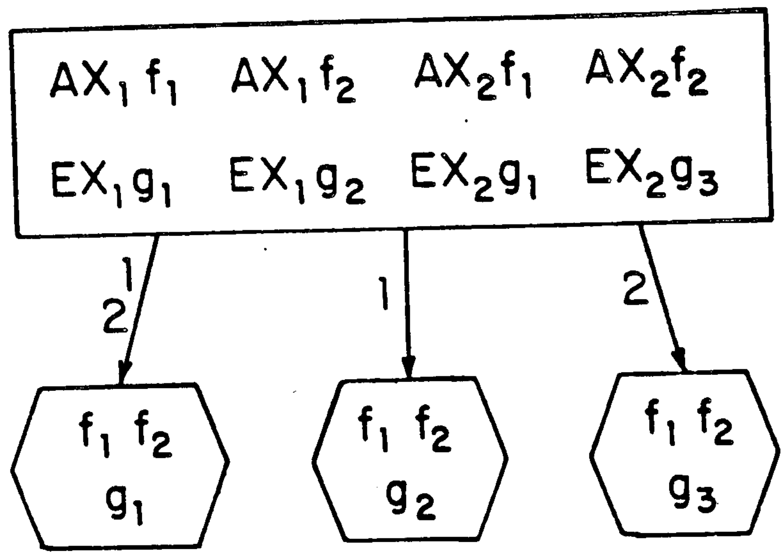
There are two special cases to consider. Let $HA = \cup\{HA_j: j \in [1:k]\}$ and $HE = \cup\{HE_j: j \in [1:k]\}$. If $HA \neq \emptyset$ and $HE = \emptyset$ then split H into $H_1,\ldots,H_k$ where each $H_j = H \cup \{EX_j \text{True}\}$ and proceed as before. If $HA = HE = \emptyset$ the let $\text{Tiles}(H) = \{G\}$ where $G = \{f: f \in H\}$ and let $\text{Blocks}(G) = \{H\}$.

10.4 Deleting Inconsistent Portions of the Tableau

We now apply the rules below to mark as inconsistent certain nodes of the tableau T. First we need the following definition:

A *full subdag* D rooted at node F in T is a finite, directed acyclic subgraph of T satisfying the following 3 conditions:

1. For every OR-node $G \in D$, there exists precisely one AND-node H such that H is a son of G in D and in T.



AX1 f1 AX1 f2 AX2f1 AX2f2
EX1g1 EX1g2 EX2g1 EX2g3
1
2
1
2
f1 f2
g1
f1 f2
g2
f1 f2
g3

Figure 10.2

2. For every AND-node $H \in D$, if H has any sons at all in D, then every son of H in T is a son of H in D.

3. F is the unique node in D from which all other nodes are reachable.

Note that a full subdag D is somewhat like a finite tree. It has a root (either an OR-node or an AND-node) and a frontier consisting of nodes with no successors in D (although they may very well have successor when considered as nodes in T). All nodes of the frontier are AND-nodes.

Here are the marking rules:

markP: Mark as deleted any node F which is immediately inconsistent, i.e, contains a formulae f and its negation $\sim f$.

markOR: Mark as deleted any OR-node G all of whose AND-node sons H_i are already marked deleted.

markAND: Mark as deleted any AND-node H one of whose OR-node sons G_j is already marked deleted.

markEU: Mark as deleted any node F such that $E[f_1 U f_2] \in F$ and there does *not* exist some node F' reachable from F such that $f_2 \in F'$ and for all F'' on some path from F' back to $F, f_1 \in F''$.

markAU: Mark as deleted any node F such that $A[f_1 U f_2] \in F$ and there does *not* exist a full subdag D rooted at F such that for all nodes F' on the frontier of D, $f_2 \in F'$ and for all non-frontier nodes F'' in D, $f_1 \in F''$.

Apply the marking rules as long as possible. Marking must eventually stop because each successful application of a marking rule marks as deleted one node and there are only a finite number of nodes in T.

If the root of T is marked, then f_0 is unsatisfiable. If the root of T is unmarked, then the subgraph of T induced by the remaining unmarked nodes can be unraveled into a finite model of f_0.

10.5 Unravelling the Tableau into a Model

Let T^* be the subgraph of T that remains after all marked nodes and incident arcs have been deleted. We will construct a finite model M of f_0 by "unravelling" T^*: For each AND-node H in T^*, and for each eventuality formula $g \in H$, there is a full subdag rooted at H which certifies that g is fulfilled. (We know this subdag exists because H is not marked by rule markAU or markEU on account of g.) We use these subdags to construct, for each AND-node H, a model fragment MH such that every eventuality in H is fulfilled within MH. We then splice together these fragments to obtain M.

10.6 Selecting Subdags

If H is in T^* and $g \in H$ is an eventuality formula, then there is a full subdag rooted at H whose frontier nodes immediately fulfill g. There may be more than one such subdag. We wish to choose one of minimal size where the size of a subdag is the length of the longest path it contains. Our approach is to tag each node in T^* with the size of the smallest subdag for g rooted at the node.

Suppose, for example, that $g = A[fUh]$. Initially, we set $tag(F) = 0$ for all nodes F such that $h \in F$ and we set $tag(F) = \infty$ for all other nodes F. Then we let the size of full subdags radiate outward by making card(T^*) passes over the tableau. During each pass we perform the following step for each node F:

> *if* F is an AND-node H such that $A[f\cup h] \in H$ and $tag(H) = \infty$ and
> $tag(G) < \infty$ for all $G \in Tiles(H)$ and $f \in H$
> *then* let $tag(H) := 1 + \max\{tag(G): G \in Tiles(H)\}$;
> *if* F is an OR-node G such that $A[f\cup h] \in G$ and $tag(G) = \infty$ and
> $tag(H) < \infty$ for some $H \in Blocks(G)$
> *then* let $tag(G) := \min\{tag(H): H \in Blocks(G)\}$;

After executing all card(T^*) passes, if $tag(F) = k < \infty$ then there is a full subdag for g rooted at F of minimal size = k. To select a specific full subdag D we perform a construction in stages.

Initially let D_0 consist of the single node F.

In general, obtain D_{i+1} from D_i as follows:

> *for* all nodes $F \in frontier(D_{i+1})$ *do*
> *if* F = some OR-node G
> *then* choose an AND-node $H \in Blocks(G)$ with a minimal tag value
> (if there is more than one H eligible,
> choose one with a maximal card(Tiles(H)) value;
> if there is still more than one H eligible, choose the
> one of lowest index.)
> attach H as the successor of G;
> *if* F = some AND-node H
> *then* add each member of Tiles(H) as a successor of F

Halt with $D = D_i$ when all frontier nodes of D_i are AND-nodes H with $tag(H) = 0$. Let DAG[H,g] denote the subdag naturally induced by the AND-nodes of D. (Note: In the case where $g = E[f\cup h]$, the construction of DAG[H,g] is similar.)

10.7 Construction of Fragments from Dags

For each AND-node H in T^*, we construct the fragment MH to have these properties:

(1) MH is a dag consisting of (copies of) AND-nodes with root H.

(2) MH is *generated* by T^* in this sense: for all nodes H_0 in MH, if $\{H_1,\ldots,H_k\}$ is the set of successors of H_0 in MH, then there exist OR-nodes $G_1,\ldots,G_k$ in T^* such that $\mathrm{Tiles}(H_0) = \{G_1,\ldots,G_k\}$ and $H_i \in \mathrm{Blocks}(G_i)$ for all $i \in [1:k]$. If the arc (H_0,H_i) in MH has labels $j_1,\ldots,j_n$ then the arc (H_0,G_i) has labels $j_1,\ldots,j_n$ in T^*.

(3) All eventuality formulae in H are fulfilled in MH.

We construct MH in stages. Let $g_1,g_2,\ldots,g_m$ be a list of all eventuality formulae occurring in H. We build a sequence of dags $MH^1,\ldots,MH^m = MH$ so that, for each $j \in [1:m]$, MH^j is a subgraph of MH^{j+1} and $g_1,\ldots,g_j$ are fulfilled in MH^j.

Let $MH^1 = \mathrm{DAG}[H,g_1]$. To obtain MH^{i+1} from MH^i do the following:

```
Identify any two nodes on frontier(MH^i) with the same label;
for all H' ∈ frontier(MH^i) do
  if g_{i+1} ∈ H'
  then attach (a copy of) DAG[H',g_{i+1}] to MH^i at H'
end
```

Finally, let $MH = MH^m$.

10.8 Constructing the Model from Fragments

We construct M by splicing together fragments. Again, the construction is done in stages:

Let $M^1 = MH_0$ where $H_0 \in \text{Blocks}(\{f_0\})$ is chosen arbitrarily.
To construct M^{i+1} from M^i do the following:

```
for each H ∈ frontier(M^i) do
  if there is a non-frontier node H' that is the root of fragment
     MH' in M^i and has the same label as H
  then merge H and H'
  else attach MH to M^i at H
end
```

The construction halts with $i = N$ when frontier(M^N) is empty.
Let $M = M^N$.

THEOREM. *The root of the fully marked tableau* T^* *for* CTL *formula* f_0 *is unmarked iff* f_0 *is satisfiable. If* f_0 *is satisfiable, it is satisfiable in a finite model of size* $O\left(c^{\text{length}(f_0)}\right)$ *for some* $c > 1$. □

The proof of this theorem will not be given; however, the proof of the corresponding theorem for UB is presented in [EH81].