

FLAC Assignment 6

Exercise 1. Give context-free grammars that generate the following languages. In all parts the alphabet is Σ is $\{0, 1\}$.

- a. $\{w \mid w \text{ contains at least three 1s}\}$
- b. $\{w \mid w \text{ starts and ends with the same symbol}\}$
- c. $\{w \mid \text{the length of } w \text{ is odd}\}$
- d. $\{w \mid \text{the length of } w \text{ is odd and its middle symbol is a 0}\}$
- e. $\{w \mid w = w^R, \text{ i.e., } w \text{ is a palindrome (of either odd or even length)}\}$
- f. The empty set

(Note: You may check your answers to parts (a) and (d) in the book; see Exercise 2.4 on page 128 and 132. But don't peek without first trying it yourself!)

Exercise 2. Give the state diagrams of pushdata automata for the following languages.

- d. The language of Exercise 1(d).
- e. The language of Exercise 1(e).
- f. $\{w\#v \mid w \text{ has more occurrences of 1 than does } v\}$. You may assume that the input string has no more than one occurrence of “#”; strings with more than one “#” are don't-care inputs that can be ignored to simplify the design of your PDA.

Exercise 3. Show the intersection of a context-free language C with a regular language R is always context-free.

Exercise 4. Show that the language $\{0^n 1^m 0^n 1^m \mid n \geq 0\}$ is not context-free.

Exercise 5. Show that the language $\{ww \mid w \in (0+1)^*\}$ is not context-free. Hint: Intersect with $0^*1^*0^*1^*$ and use the results from Exercises 3 and 4.

Exercise 6 (bonus). Is the following language context-free? Prove your answer.

$$\{ww' \mid w \in (a+b)^*, w' \in (a+b)^*, w \neq w', \text{ and } |w| = |w'|\}$$