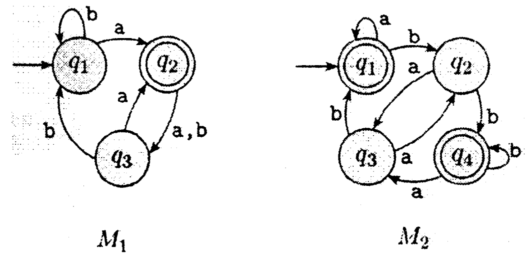
Name: ____

FLAC Assignment 1

Please read all questions ASAP. If a question is unclear, please email the TAs. Clarifications may also be posted on the Assignments page of the course website. Remember to staple together all pages of your completed assignment.



Exercise 1.1. To the right are the state diagrams of two DFAs, M_1 and M_2. Answer the following questions about these machines.

a. What is the start state of M_1?

b. What is the set of accepting states of M_1?

c. What is the start state of M_2?

d. What is the set of accepting states of M_2?

e. What sequence of states does M_1 go through on input `aabb`?

f. Does M_1 accept the string `aabb`?

g. Does M_2 accept the empty string Λ?

Exercise 1.4. Give state diagrams of DFAs recognizing the following languages. In all cases, the alphabet is $\{0, 1\}$.

a. $\{w \mid w$ begins with a 1 and ends with a 0$\}$.

b. $\{w \mid w$ contains at least three 1s$\}$.

c. $\{w \mid w$ contains the substring 0101, i.e., $w = x0101y$ for some (possibly empty) x and $y\}$.

d. $\{w \mid w$ has length at least 3 and its third symbol is a 0$\}$.

e. $\{w \mid w$ starts with 0 and has odd length, or starts with 1 and has even length$\}$.

f. $\{w \mid w$ doesn't contain the substring 110$\}$.

g. $\{w \mid$ the length of w is at most 5$\}$.

h. $\{w \mid w$ is any string except 11 and 111$\}$.

i. $\{w \mid$ every odd position of w is a 1$\}$.

j. $\{w \mid w$ contains at least two 0s and at most one 1$\}$.

k. $\{\Lambda, 0\}$. (Here, "Λ" denotes the empty string.)

l. $\{w \mid w$ contains an even number of 0s, or exactly two 1s$\}$.

m. The empty set.

n. All strings except the empty string.

Exercise 1.5. Give NFAs with the specified number of states recognizing each of the following languages.

a. The language $\{w \mid w$ ends with $00\}$ with three states.

b. The language of Exercise 1.4c with five states.

c. The language of Exercise 1.4l with six states.

d. The language $\{0\}$ with two states.

e. The language $0^*1^*0^*0$ with three states.

f. The $\{\Lambda\}$ with one state.

g. The language 0^* with one state.

Exercise 1.10. **(a)** Show by giving an example that, if M is an NFA that recognizes language C, then swapping the accepting and non-accepting states in M doesn't necessarily yield a new NFA that recognizes the complement of C. **(b)** Is the class of languages recognized by NFAs closed under complementation? Explain your answer.

Exercise 1.25. If R_1 and R_2 are regular languages, is

$$\{w \mid w \in R_1 \text{ and } w^R \in R_2\}$$

a regular language? If so, prove it; if not, give a counterexample. (Recall that "w^R" denotes the reversal of the string w.)

Exercise 1.31. Consider a new kind of finite automaton called an all-paths-NFA. An all-paths-NFA M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that recognizes $x \in \Sigma^*$ if every possible computation of M on x ends in a state from F. Note in contrast, that an ordinary NFA accepts a string if *some* computation ends in an accept state. Prove that all-paths-NFAs recognize the class of regular languages.

Exercise 1.42 (extra credit). If A is any language, let $A_{\frac{1}{2}-}$ be the set of all first halves of strings in A so that

$$A_{\frac{1}{2}-} = \{x \mid \text{ for some } y,\ |x| = |y| \text{ and } xy \in A\}$$

Show that if A is regular, then so is $A_{\frac{1}{2}-}$.