Solution to Bonus Problem

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Proof. Given a CFG $L$ over single alphabet $a$, we will show it is also a regular language. Suppose the pumping length of language $L$ is $l$; then for any string $z \in L$ of length larger than $l$, we know we can write into $uvwxy$ such that $uv^iwx^iy$ is also in the language for every $i$. Notice that $uv^{i+1}wx^{i+1}y$ is just adding $za^{l(|v|+|x|)}$. Also Notice that $|vwx| \leq l$, therefore $(|v| + |x|)$ is divisible by $(l!)$. Therefore, we know the following property of language $L$:

for any $z \in L$, if $|z| > l$, then $z(a^l)^*$ is also in $L$.

Then we can divide the string in $L$ with length larger than $l$ in to at most $l!$ classes.

For $1 \leq i \leq (l!)$, let $z_i$ be the shortest string in $L$ such that

$$|z_i| \geq l, \text{ and } |z_i| - l \equiv i \mod (l!).$$

Notice that for some $i$, there might not exists some $z_i$ that satisfy above equation. We use $S$ to denote the set that contains all the $i$ such that $z_i$ exists.

Now we can write $L$ by the following regular expression,

$$L = \{x \mid |x| \leq l, x \in L\} \cup \bigcup_{i \in S} z_i(a^l)^*.$$