Cellular Automata
and Universality

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Cellular Automata:

ECA 1
ECA 1
Cellular Automata:

ECA 28

[Diagram showing cellular automaton evolution]
ECA 28
ECA 54
Multiple Seeds
Disaster

\[ n = 23, \ t = 24, \ p = 690 \]
ECA 45
Cellular Automata:

ECA 90
Cellular Automata:

ECA 30
ECA 30
Cellular Automata:

782359
Cellular Automata:

782340
Cellular Automata:

782353
Cellular Automata:

722797
Langton’s CA
Cellular Automata
A Hard Problem

How does one prove properties of cellular automata?

The short term behavior (first-order logic) is decidable via automata theory.

Almost any question about the long term behavior is undecidable; there is no standardized argument.
Moews

13 states
CA and Correctness Proofs:

Waksman

9 states
Mazoyer

6 states
The paper in TCS is 54 pages long and uses lots of diagrams.

To be sure, the proof is almost certainly correct, but it contains lots of little, unrelated combinatorial facts that are difficult for a human to check – eyes glaze over very quickly.

The proof should be machine checked.
Universality of ECA 110

ECA 110 is given by the following local rule:

000 → 0  
001 → 1  
010 → 1  
011 → 1  
100 → 0  
101 → 1  
110 → 1  
111 → 0

As a Boolean function this comes down to

\[ \rho(x, y, z) = (\overline{x} \land y) \lor (y \oplus z) \]
Another Look

Here is the table again, but ordered differently:

000 → 0  010 → 1
001 → 1  011 → 1
100 → 0  110 → 1
101 → 1  111 → 0

The left column has “control bit” $y = 0$ and amounts to a left shift of $z$. The right column has “control bit” $y = 1$ and amounts to $\text{NAND}(x, z)$.

Since $\text{NAND}$ is a functionally complete set of Boolean operations . . .
One-Point Seed

We get a half-light-cone: the configuration grows to the left only, at speed 1. Does not look particularly complicated.
Finite Seed
Random Seed
After 1000 Steps
A background pattern naturally evolves; think of this as vacuum.
Particles can move in this vacuum.
More Particles
Interactions
Interactions
Interactions
Interactions

and interact ...
Interactions

and interact ...
Interactions

and interact . . .
Putting it Together

In the mid-90’s, Matthew Cook, working at WRI, developed a fancy simulation system that allowed him to design and experiment with large configurations on ECA 110.

Based on his observations, he was able to show universality assuming almost periodic configurations by simulating cyclic tag systems, a variant of Post tag systems.

This is arguably the most interesting result in the study of computational universality in the last two or three decades. It’s also the only result that lead to a law suit.
A number of people have checked the proof in great detail and there are even attempts at producing a more formalized versions of it.

Again, the proof is probably correct but it is really impossible to check all the details by hand – we need a computer-checkable version.