Model Checking Assignment 1

Exercise 1. The "weak until" operator $\mathbf{W}$ has similar semantics to the "strong until" operator $\mathbf{U}$ except that $g_1 \, \mathbf{W} \, g_2$ doesn't require g_2 to ever hold true if g_1 is globally true:

$$\pi \models g_1 \, \mathbf{W} \, g_2 \quad \Leftrightarrow \quad \text{for all } j \geq 0, \text{ if } \pi^j \models \neg g_1 \text{ then there exists a } k \leq j \text{ such } \pi^k \models g_2$$

Show that $\mathbf{W}$ can be expressed in terms of the existing temporal operators:

a. Express $\pi \models g_1 \, \mathbf{W} \, g_2$ in terms of the existing temporal operators ($\mathbf{F}$, $\mathbf{G}$, $\mathbf{U}$). You may check your answer by consulting the Wikipedia page on Linear Temporal Logic.

b. Prove your expression in part (a) means the same as $\pi \models g_1 \, \mathbf{W} \, g_2$ provided $\pi \models \mathbf{F} \, g_2$.

c. Prove your expression in part (a) means the same as $\pi \models g_1 \, \mathbf{W} \, g_2$ provided $\pi \models \neg \, \mathbf{F} \, g_2$.

Exercise 2. Write an expression equivalent to $\mathbf{AG} \, \mathbf{EF} \, p$ without using any of the operators $\{\mathbf{A}, \mathbf{F}, \mathbf{G}\}$. (Hint: use the identities on Slide 13 of lecture2.pdf.)

Exercise 3. Prove that $\mathbf{A} \, \mathbf{G} \, \mathbf{F} \, p$ is equivalent to $\mathbf{AG} \, \mathbf{AF} \, p$.

Exercise 4. Show that $\mathbf{A}[f \ \mathbf{U} \ g] \ \equiv \ \neg\mathbf{E}[\neg g \ \ \mathbf{U} \ \ \neg f \wedge \neg g] \ \wedge \ \neg\mathbf{EG} \, \neg g$

Hints:

a. Note that $\neg \, \mathbf{E} \, x = \mathbf{A} \, \neg x$ and $\mathbf{A}(x \wedge y) = (\mathbf{A} \, x) \wedge (\mathbf{A} \, y)$.

b. Rewrite $[f \ \ \mathbf{U} \ \ g]$ as $[(f \ \ \mathbf{W} \ \ g) \ \wedge \ \mathbf{F} \ \ g]$.

c. Show that $(\pi \models \neg(f \, \mathbf{W} \, g)) \equiv (\pi \models \neg g \ \ \mathbf{U} \ \ \neg f \wedge \neg g)$.