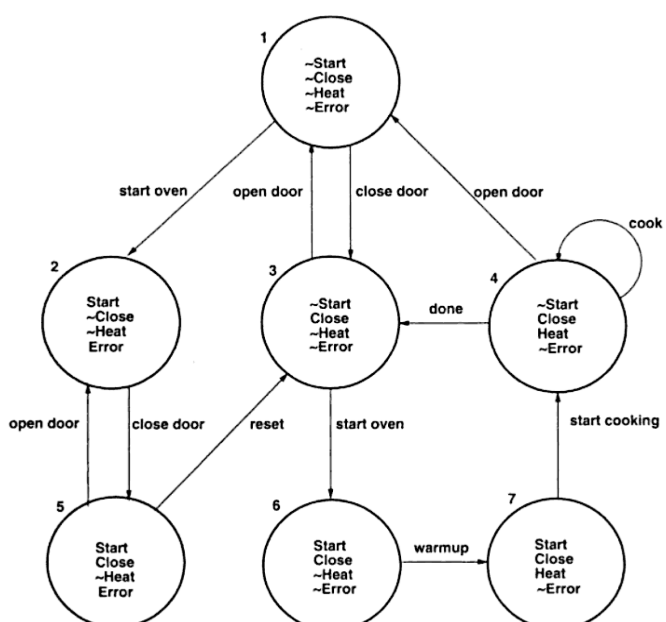
15-817: Assignment 2

Part A. Binary Decision Diagrams. In this assignment, the word "BDD", unless otherwise indicated, is to be understood as referring to a *reduced, ordered* binary decision diagram.

1. Draw a BDD for the formula $\mathbf{XOR}(x_2, x_1)$, where **XOR** is the *exclusive-or* operator.
2. Draw a BDD for the formula $\mathbf{XOR}(x_3, \mathbf{XOR}(x_2, x_1))$. (Note that this formula is true iff an odd number of $\{x_3, x_2, x_1\}$ are true; it is not the same as the exclusive-or of $\{x_3, x_2, x_1\}$, which is true iff exactly one of $\{x_3, x_2, x_1\}$ is true. Henceforth, we will write "$x_3 \oplus x_2 \oplus x_1$" with the same meaning as $\mathbf{XOR}(x_3, \mathbf{XOR}(x_2, x_1))$.) Make sure that you reduce your BDD, combining identical nodes into a single node.
3. How many x_1 nodes are in a (reduced) BDD for $x_4 \oplus x_3 \oplus x_2 \oplus x_1$? (Order the variables so that x_4 is the topmost variable (the first one to be decided) in the BDD.) In general, how many x_i nodes are there in a (reduced) BDD for $x_n \oplus \ldots \oplus x_1$, where x_n is the topmost decision variable? (Consider both $i \neq n$ and $i = n$.)
4. How many total nodes are there in the (reduced) BDD for $x_n \oplus \ldots \oplus x_1$? (Don't count the terminal nodes `true` and `false`.) Show your work.
5. Draw an BDD for $x_6 \oplus x_5 \oplus x_4 \oplus x_3 \oplus x_2 \oplus x_1$. Your drawing should be drawn in an orderly manner and its structure should be visually apparent.
6. Give an example of something that is known to require an exponentially-sized BDD. (Hint: This was mentioned in class. If you don't remember the answer, Google is your friend.)

Part B. Bounded Model Checking. Consider the microwave-oven example given on page 39 of the textbook, shown to the right.



1. What is the recurrence diameter of this transition system?
2. What is the least number of steps needed to reach any state, starting from State 1 (the topmost state)?

Part C. Partial Order Reduction. Suppose that performing action A and then B has the same effect as performing B and then A. It might be tempting to just use the $\langle A, B\rangle$ order and ignore the $\langle B, A\rangle$ order. Why is this wrong? Give a concrete counterexample of where it goes wrong.