

# Software Verification using Predicate Abstraction and Iterative Refinement: Part 1

15-414 Bug Catching: Automated  
Program Verification and Testing

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# Outline

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Overview of Model Checking

Creating Models from C Code: Predicate Abstraction

Eliminating spurious behaviors from the model: Abstraction Refinement

Concluding remarks : research directions, tools etc.



# Model Checking

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**Algorithm** for answering **queries** about behaviors of **state machines**

- Given a state machine  $M$  and a query  $\phi$  does  $M \models \phi$  ?

Standard formulation:

- $M$  is a **Kripke structure**
- $\phi$  is a **temporal logic** formula
  - Computational Tree Logic (CTL)
  - Linear Temporal Logic (LTL)

Discovered independently by **Clarke & Emerson** and **Queille & Sifakis**  
in the early 1980's



# Scalability of Model Checking

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Explicit statespace exploration: early 1980s

- Tens of thousands of states

**Symbolic** statespace exploration: millions of states

- Binary Decision Diagrams (**BDD**) : early 1990's
- Bounded Model Checking: late 1990's
  - Based on propositional satisfiability (**SAT**) technology

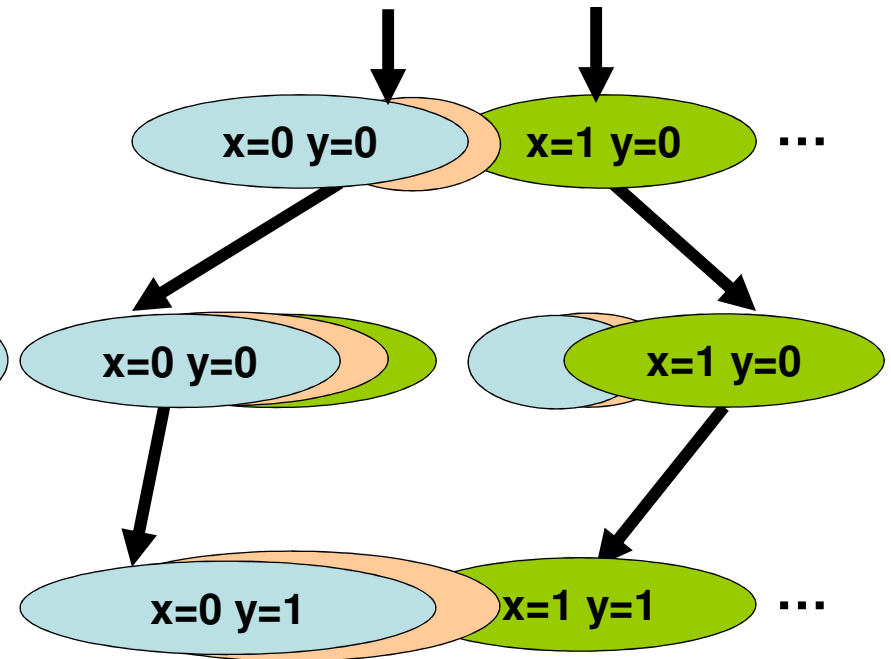
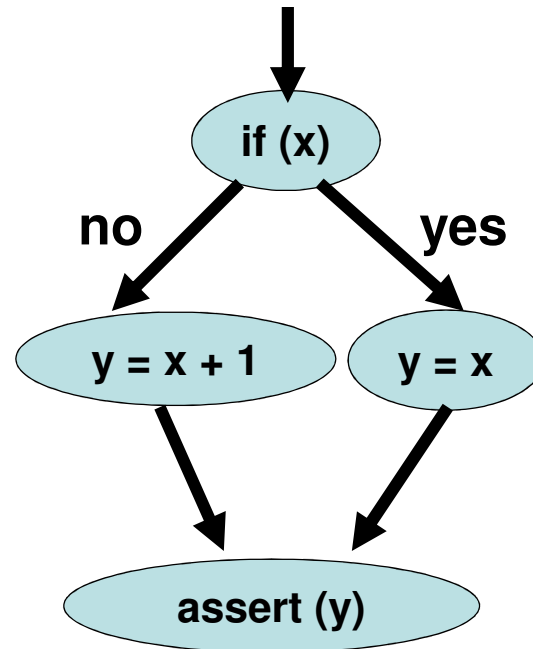
**Abstraction** and **compositional** reasoning

- $10^{120}$  to effectively **infinite** statespaces (particularly for **software**)



# Models of C Code

```
if (x) {  
    y = x;  
} else {  
    y = x + 1;  
}  
assert (y);
```



Program: Syntax    Control Flow Graph

Model: Semantic    **Infinite State**



# Existential Abstraction

**Partition** concrete statespace into abstract states

- Each abstract state  $S$  corresponds to a set of concrete states  $s$
- We write  $\alpha(s)$  to mean the abstract state corresponding to  $s$
- We define  $\gamma(S) = \{ s \mid S = \alpha(s) \}$

Fix the transitions **existentially**

- $S \rightarrow S' \iff \exists s \in \gamma(S) . \exists s' \in \gamma(S') . s \rightarrow s'$
- $S \rightarrow S' \Leftarrow \exists s \in \gamma(S) . \exists s' \in \gamma(S') . s \rightarrow s'$

**Strong & sometimes  
not computable**

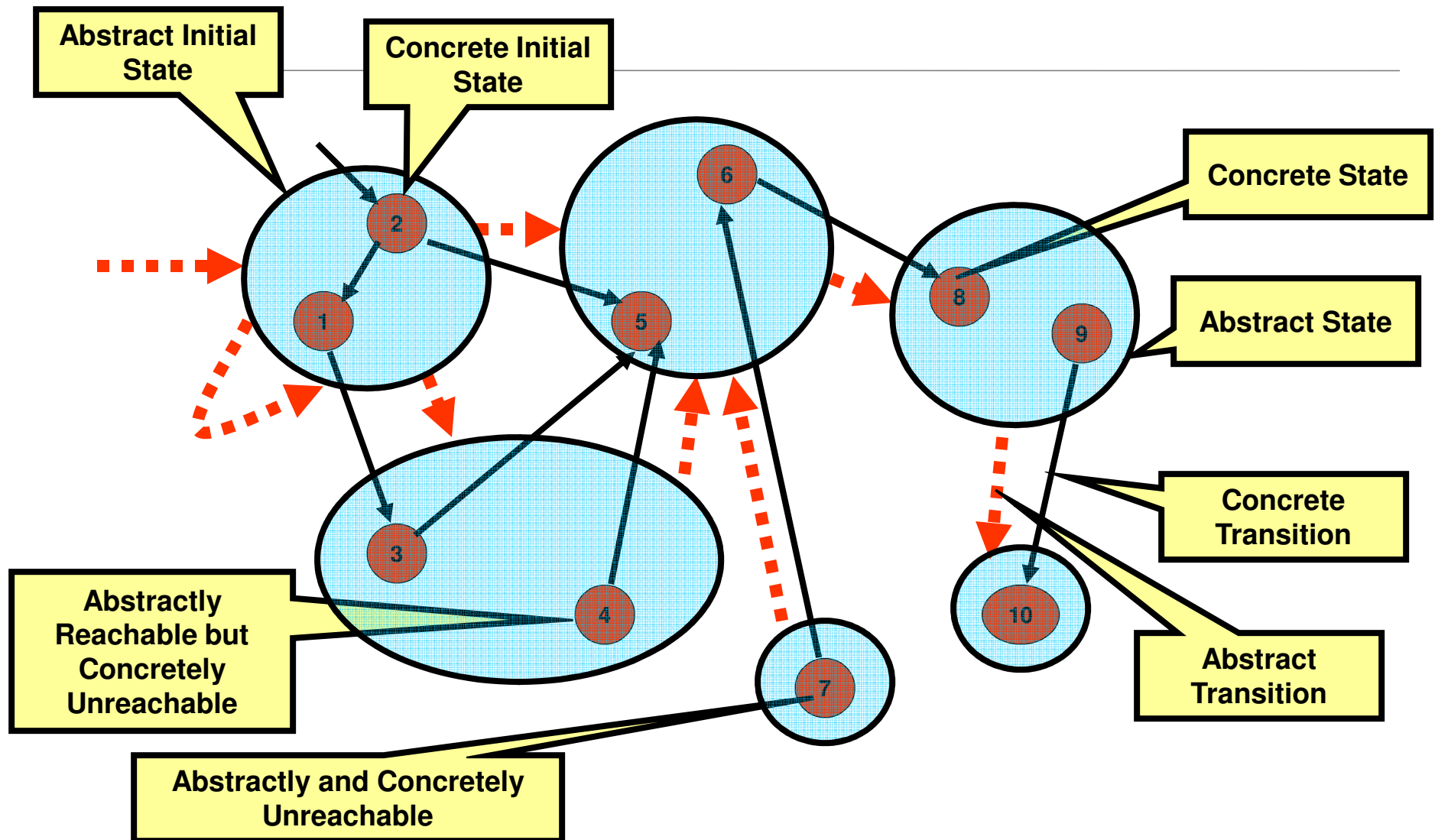
**Weak: computable**

Existential Abstraction is **conservative** [ClarkeGrumbergLong94]

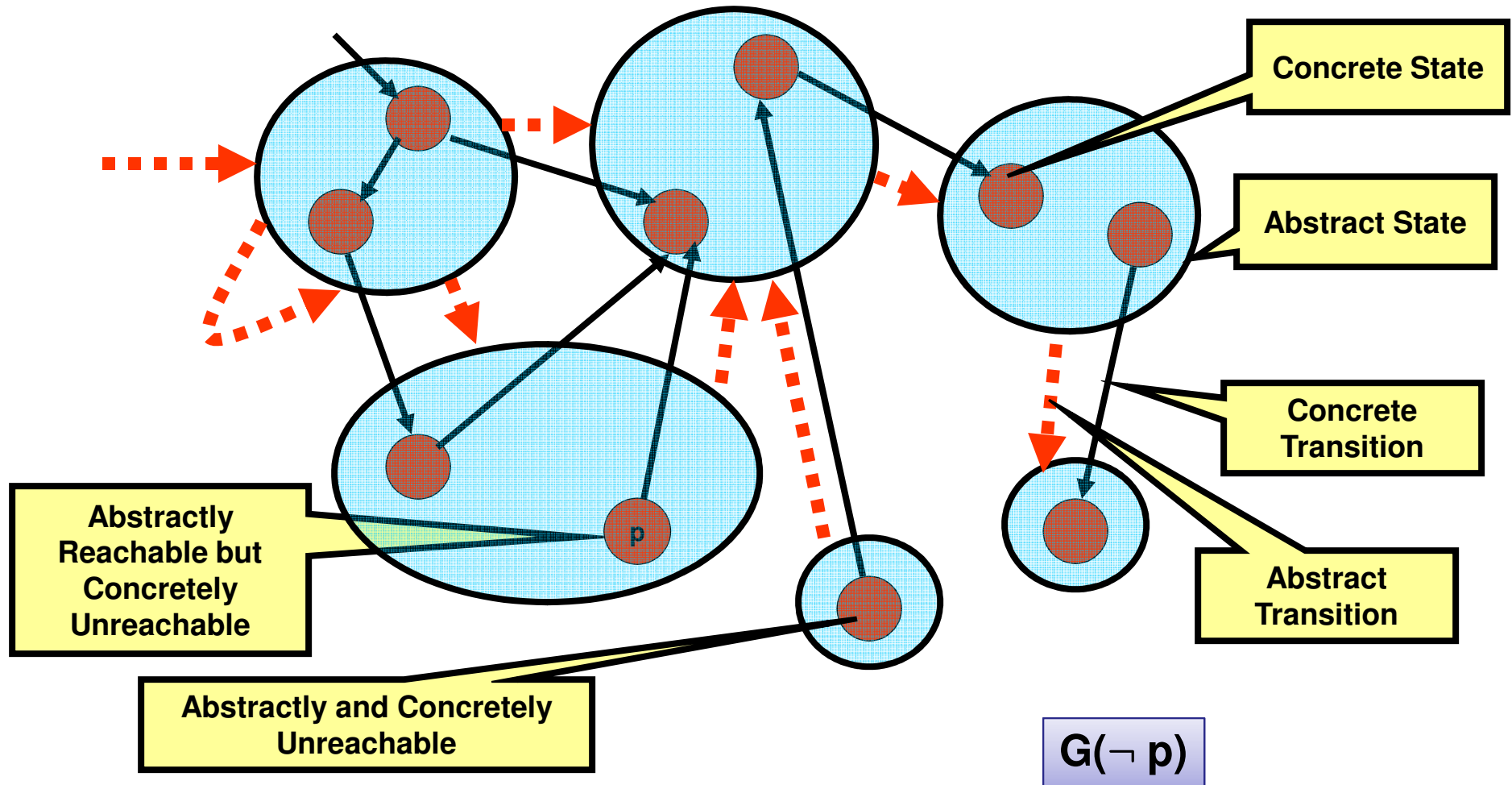
- If a ACTL\* property holds on the abstraction, it also holds on the program
  - LTL is a subset of ACTL\*
- However, the converse is not true: a property that fails on the abstraction may still hold on the program
- Existential abstraction can be viewed as a form of abstract interpretation



# Example of Existential Abstraction

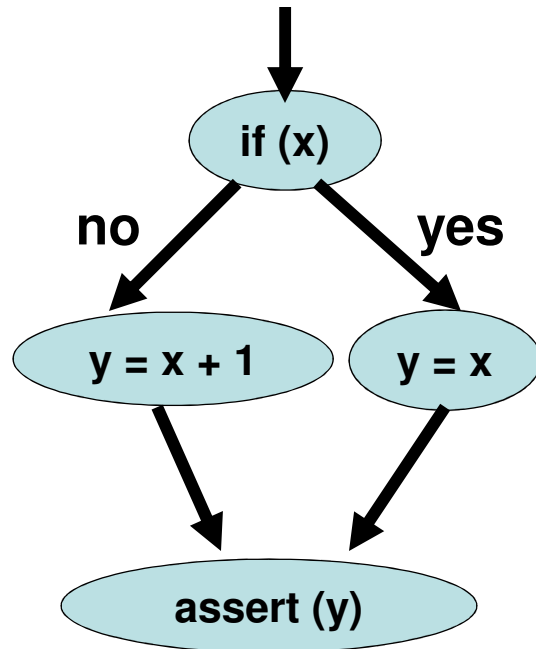


# Example of Existential Abstraction



# Predicate Abstraction

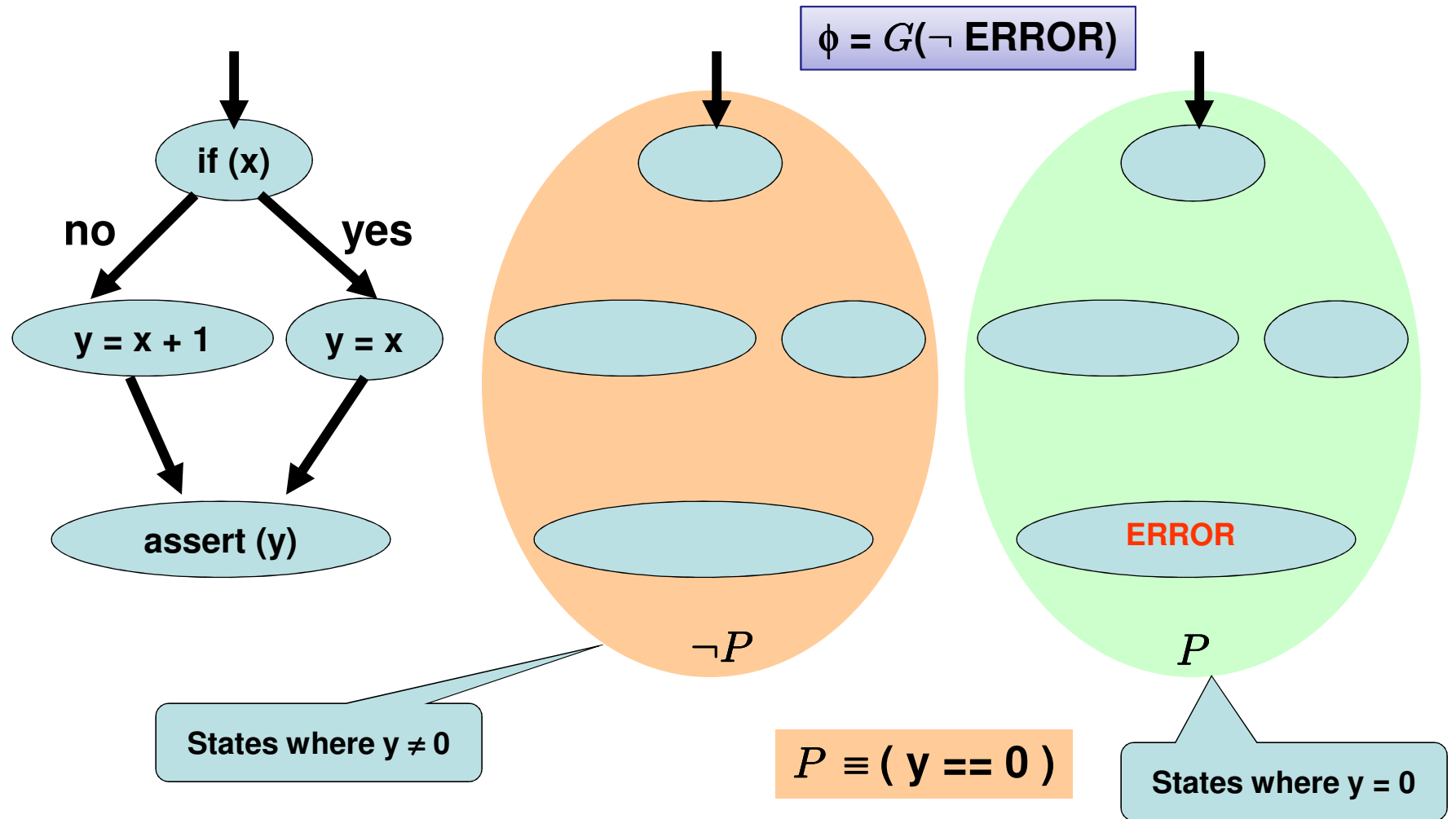
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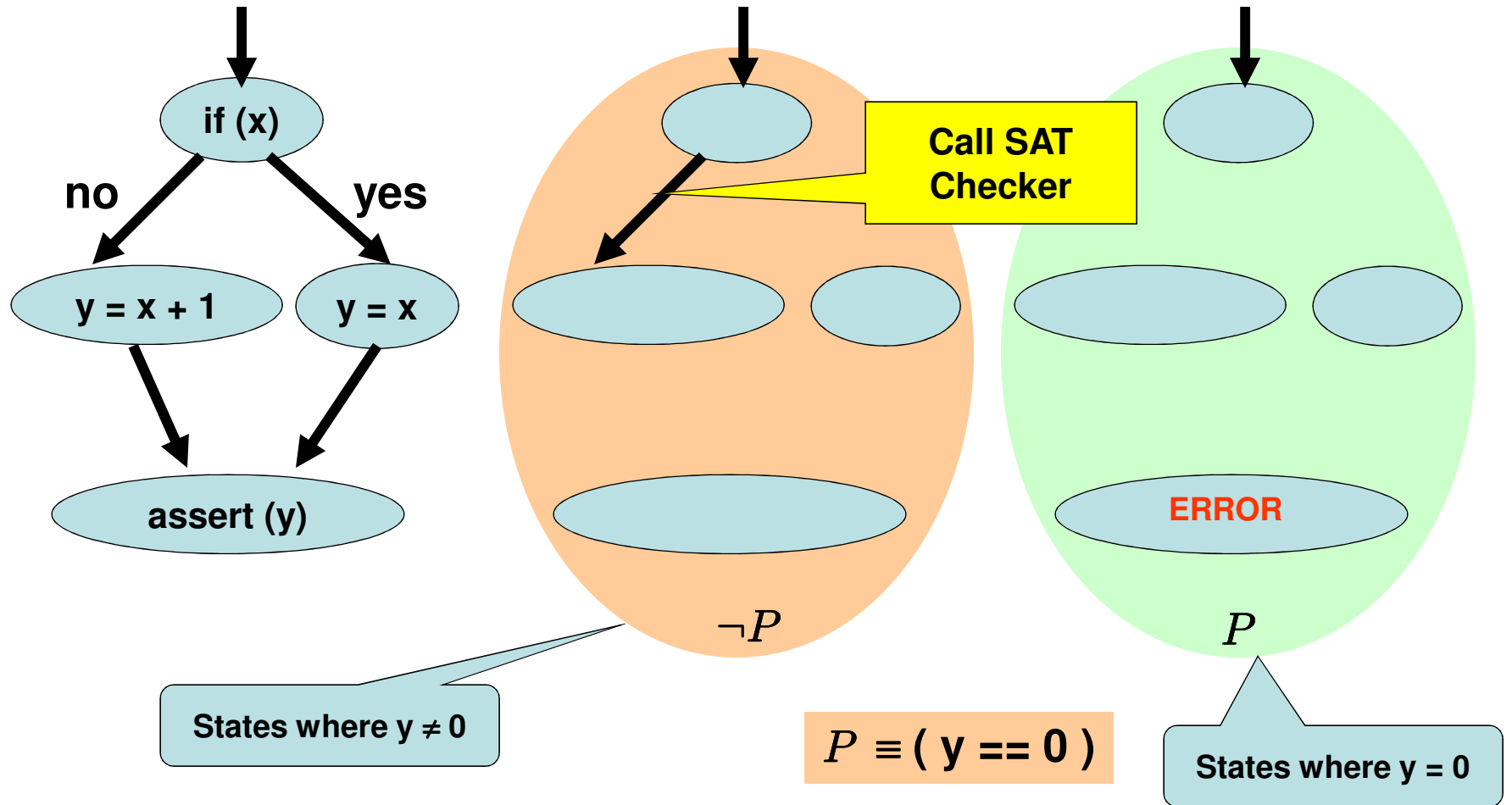
**Partition** the statespace  
based on values of a  
**finite** set of **predicates**  
on program variables



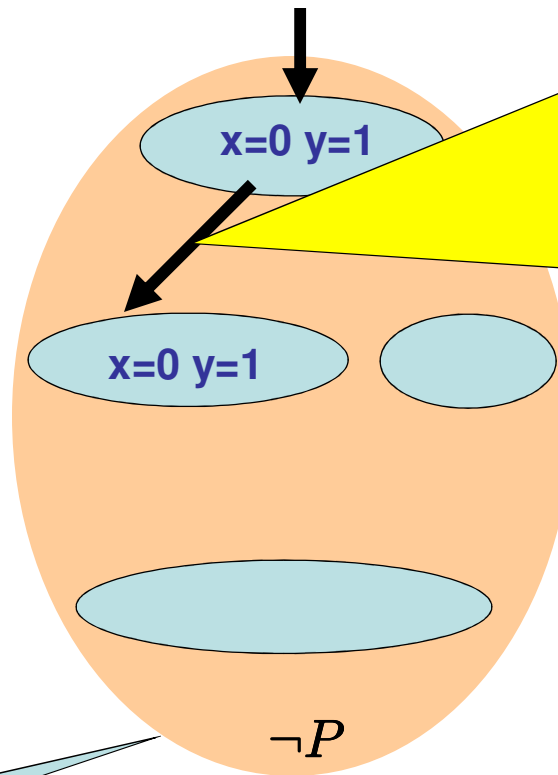
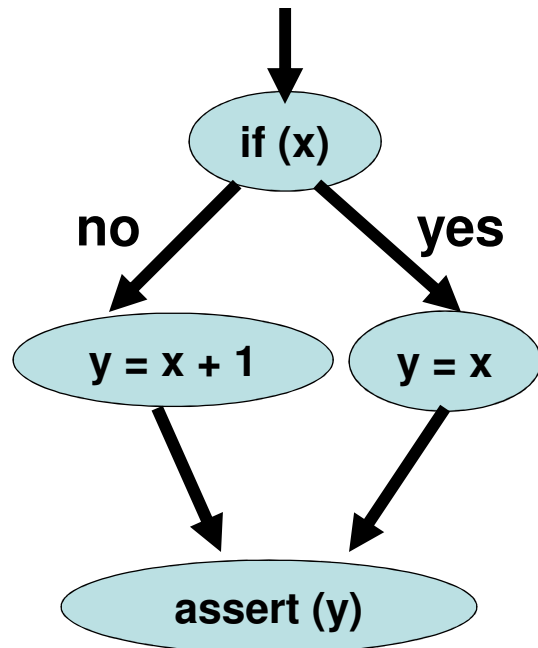
# Predicate Abstraction



# Predicate Abstraction



# Predicate Abstraction



**SAT Checker Query:**

$y \neq 0 \wedge$   
 $x = 0 \wedge$   
 $x' = x \wedge$   
 $y' = y \wedge$   
 $y' \neq 0$

**SAT Checker Answer:**

**SAT and here's a solution**

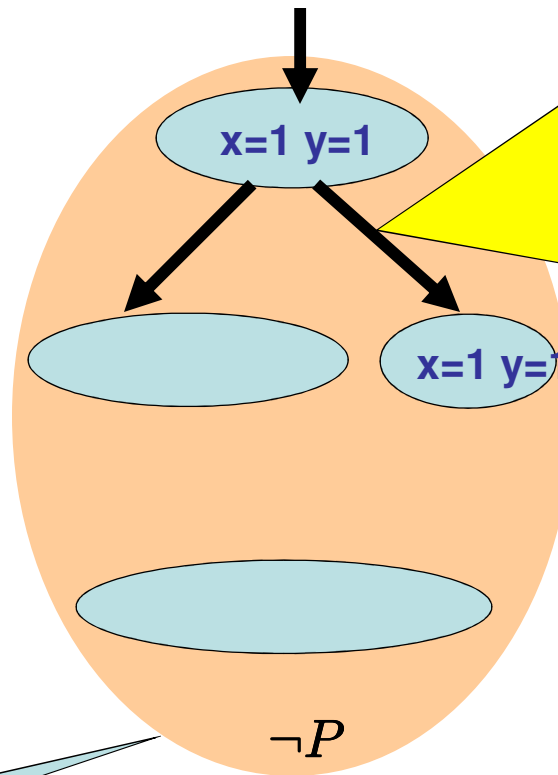
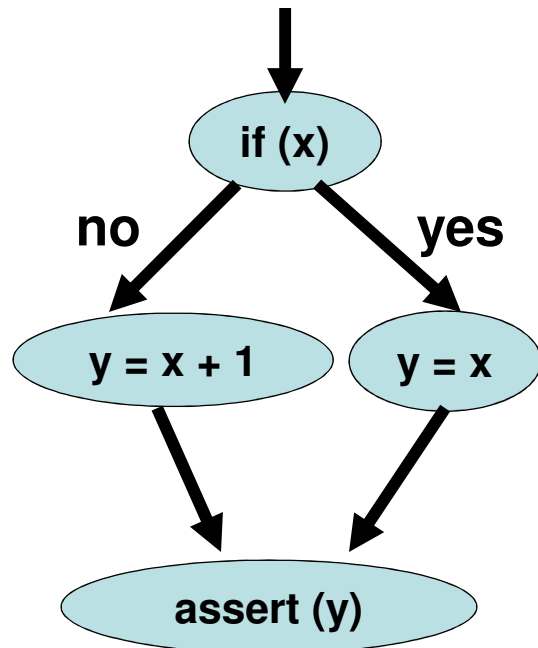
$x=0, y=1, x'=0, y'=1$

States where  $y \neq 0$

$P \equiv (y == 0)$



# Predicate Abstraction



States where  $y \neq 0$

$P \equiv (y == 0)$

**SAT Checker Query:**

$y \neq 0 \wedge$   
 $x \neq 0 \wedge$   
 $x' = x \wedge$   
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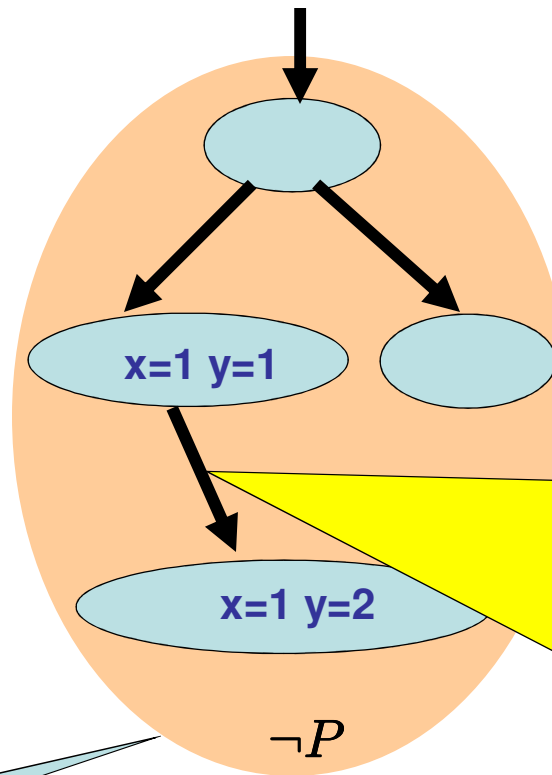
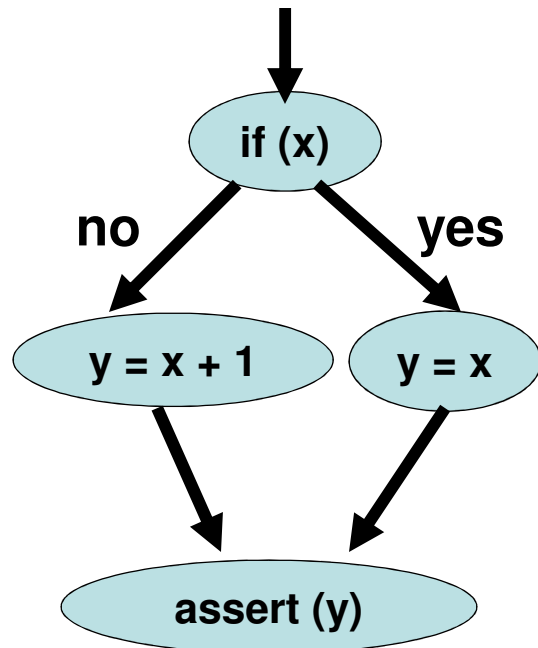
**SAT Checker Answer:**

**SAT and here's a solution**

$x=1, y=1, x'=1, y'=1$



# Predicate Abstraction



States where  $y \neq 0$

$P \equiv (y == 0)$

**SAT Checker Query:**

$y \neq 0 \wedge$   
 $x' = x \wedge$   
 $y' = x+1 \wedge$   
 $y' \neq 0$

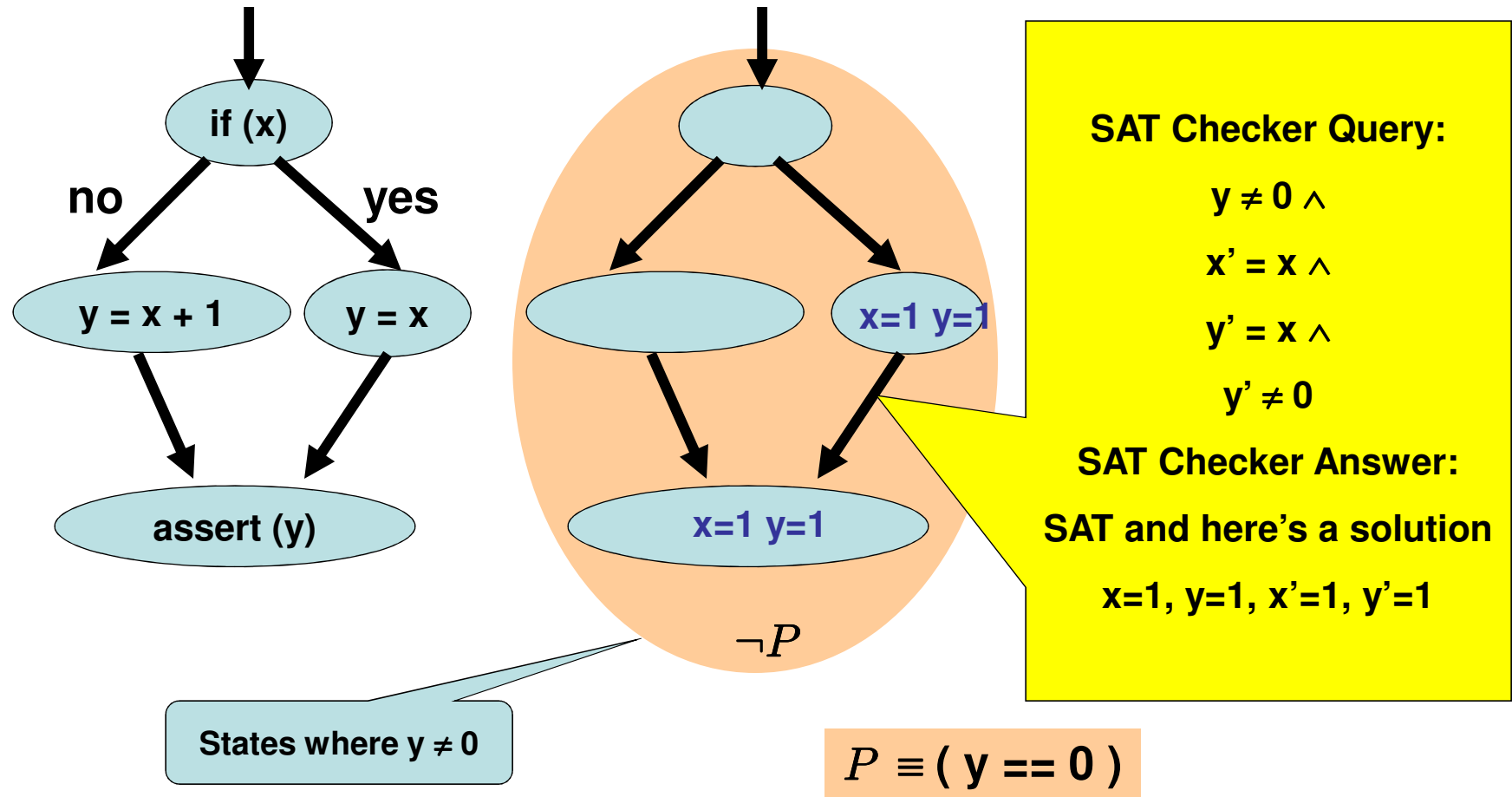
**SAT Checker Answer:**

**SAT and here's a solution**

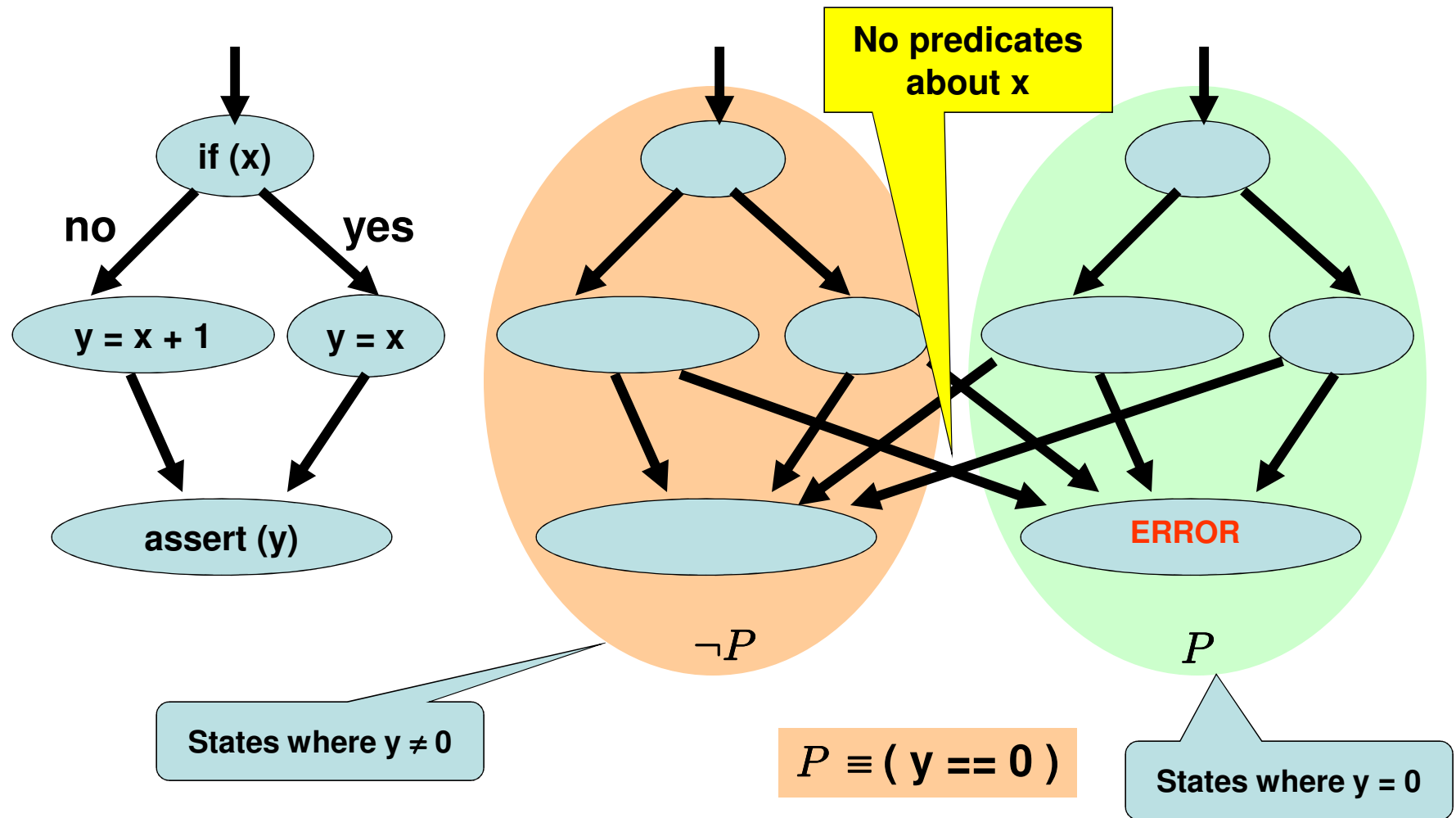
$x=1, y=1, x'=1, y'=2$



# Predicate Abstraction



# Predicate Abstraction



# Imprecision due to Predicate Abstraction

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Counterexamples generated by model checking the abstract model may be **spurious**, i.e., not concretely realizable

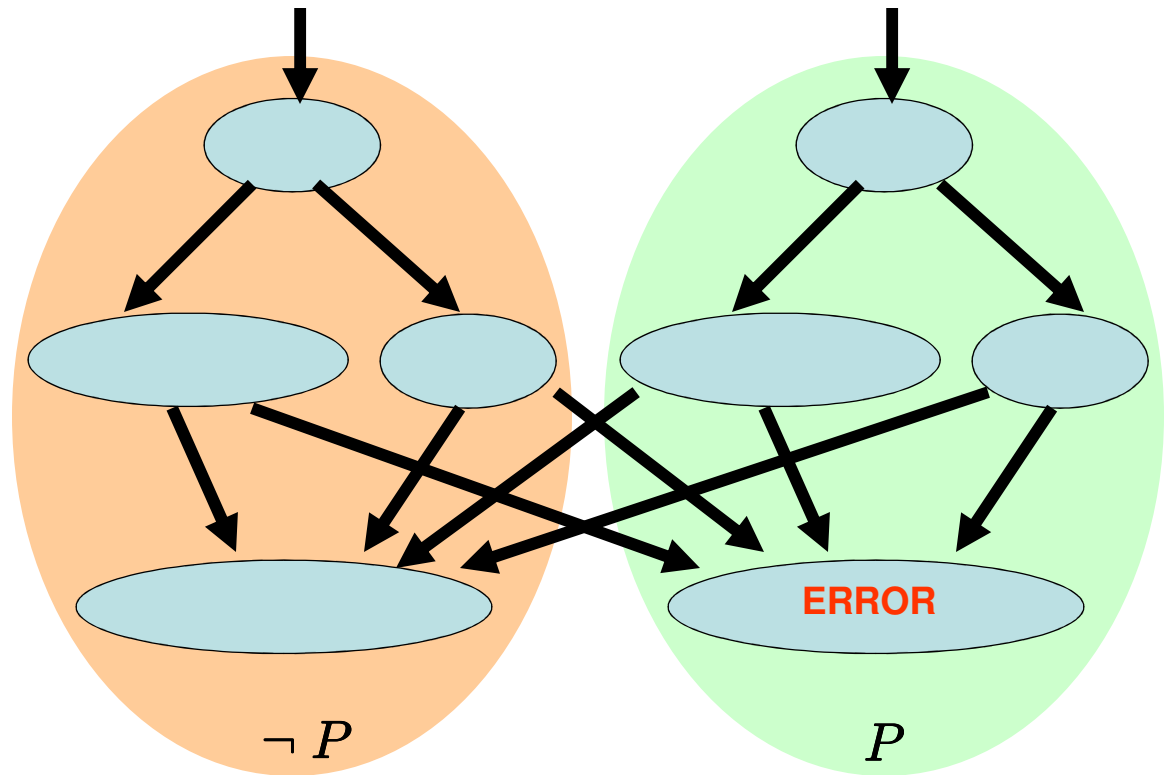
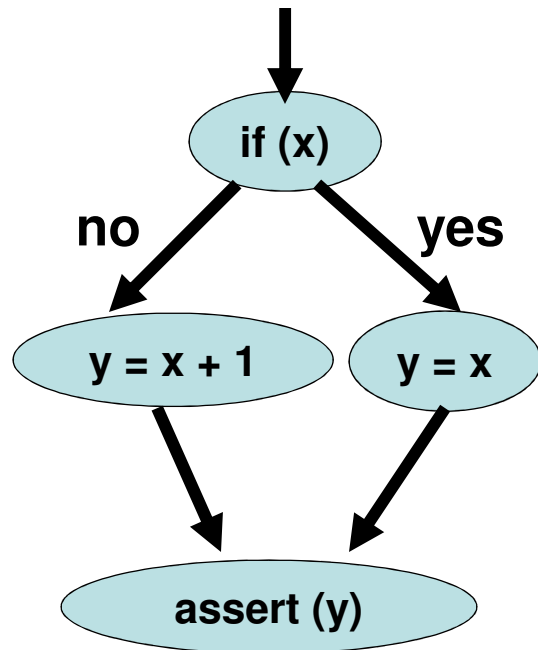
Need to **refine** the **abstraction** iteratively by changing the set of predicates

Can infer new set of predicates by analyzing the spurious counterexample

- Lot of research in doing this effectively
- Counterexample Guided Abstraction Refinement (CEGAR)
- A.K.A. Iterative Abstraction Refinement
- A.K.A. Iterative Refinement



# Model Checking

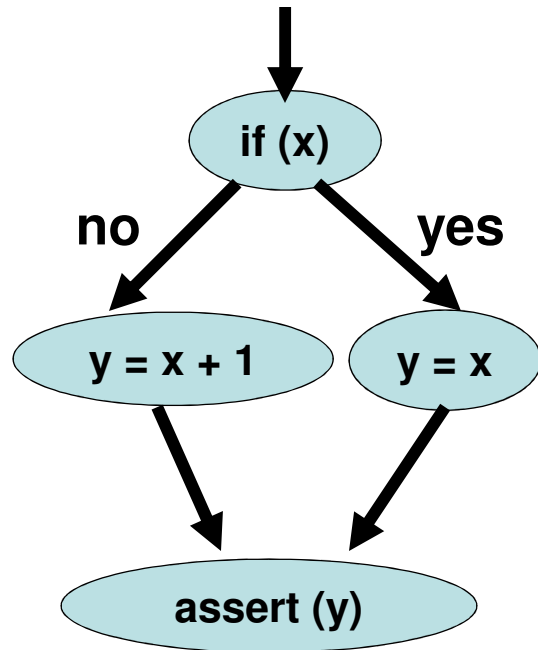


$\phi = G(\neg \text{ERROR})$

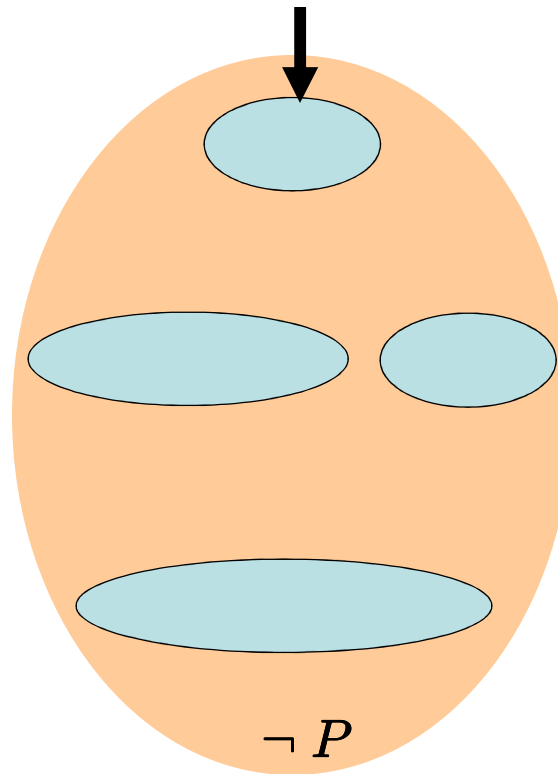
$P \equiv (y == 0)$



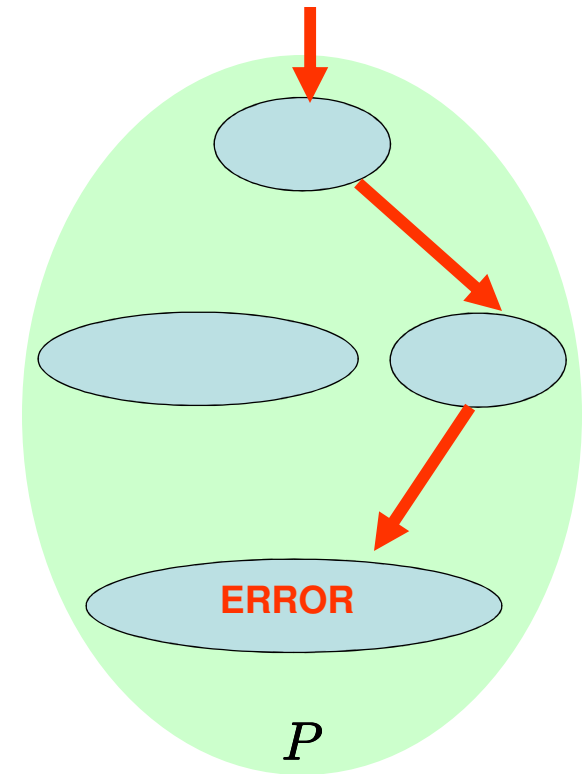
# Model Checking



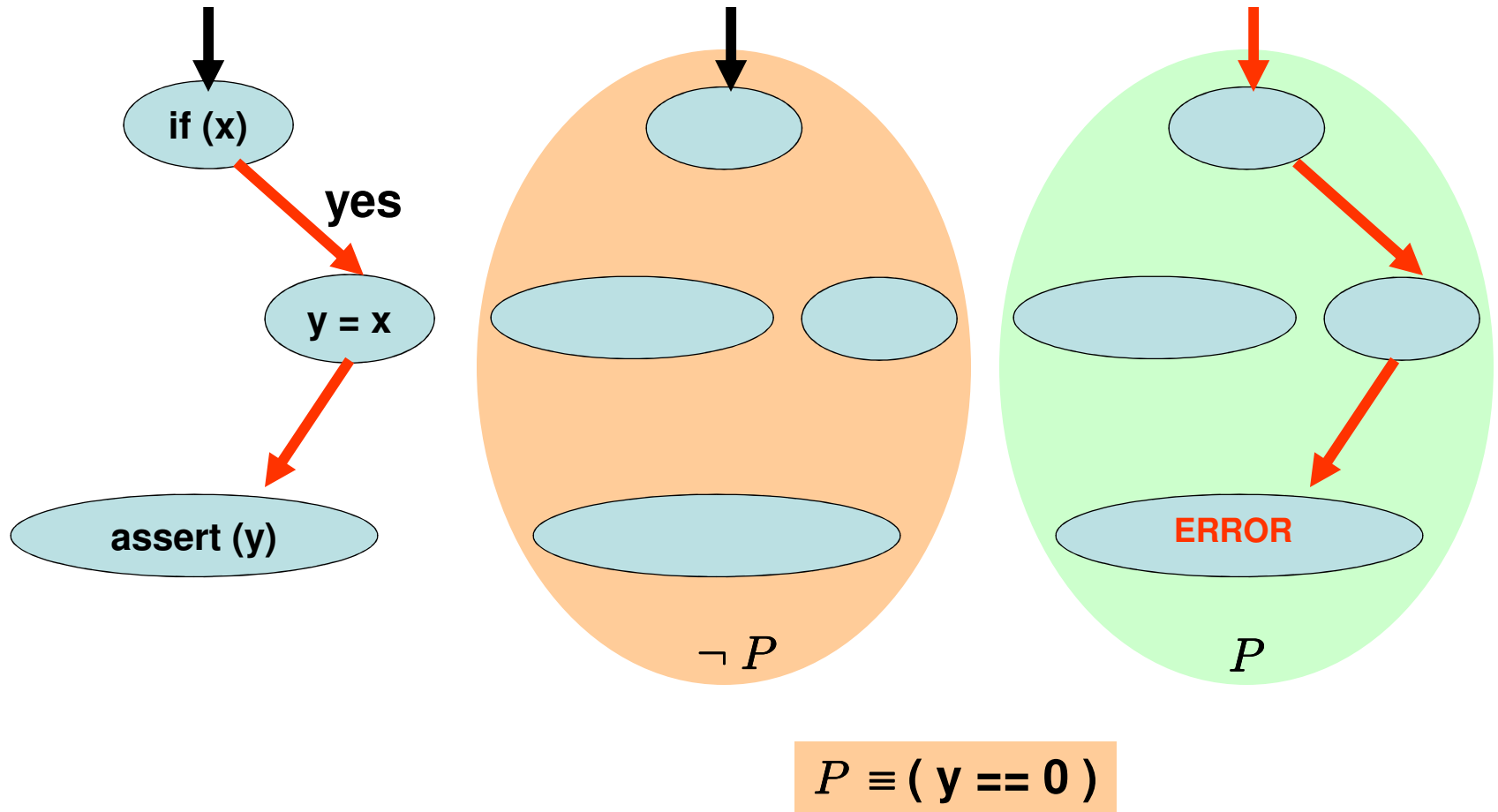
$\phi = G(\neg \text{ERROR})$



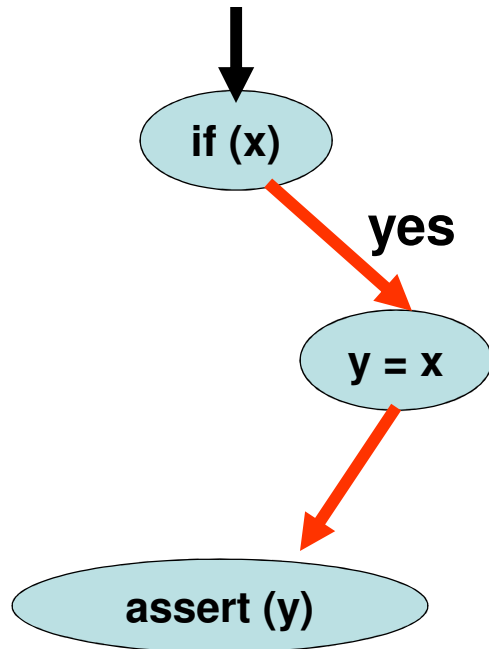
$P \equiv (y == 0)$



# Model Checking



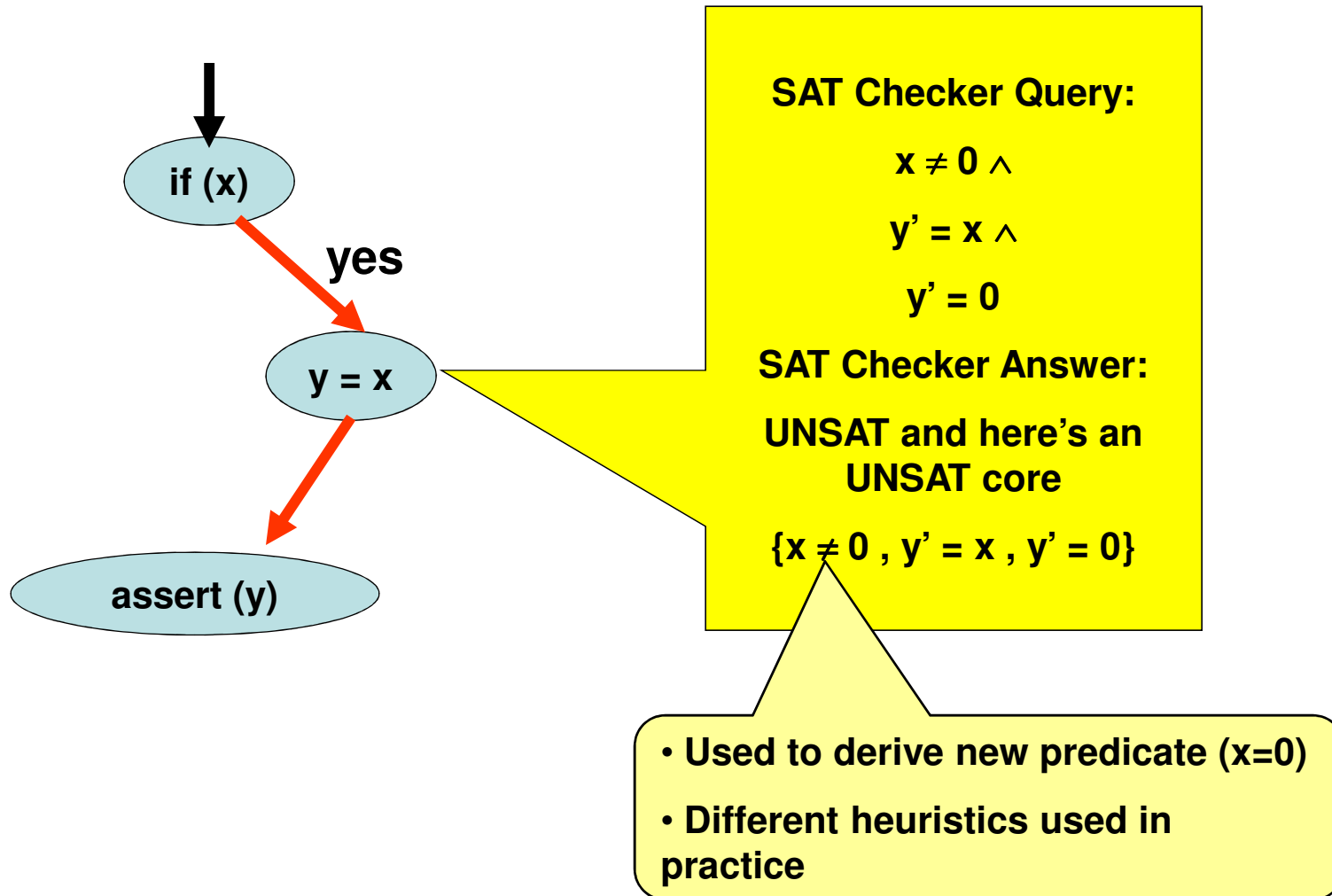
# Counterexample Validation



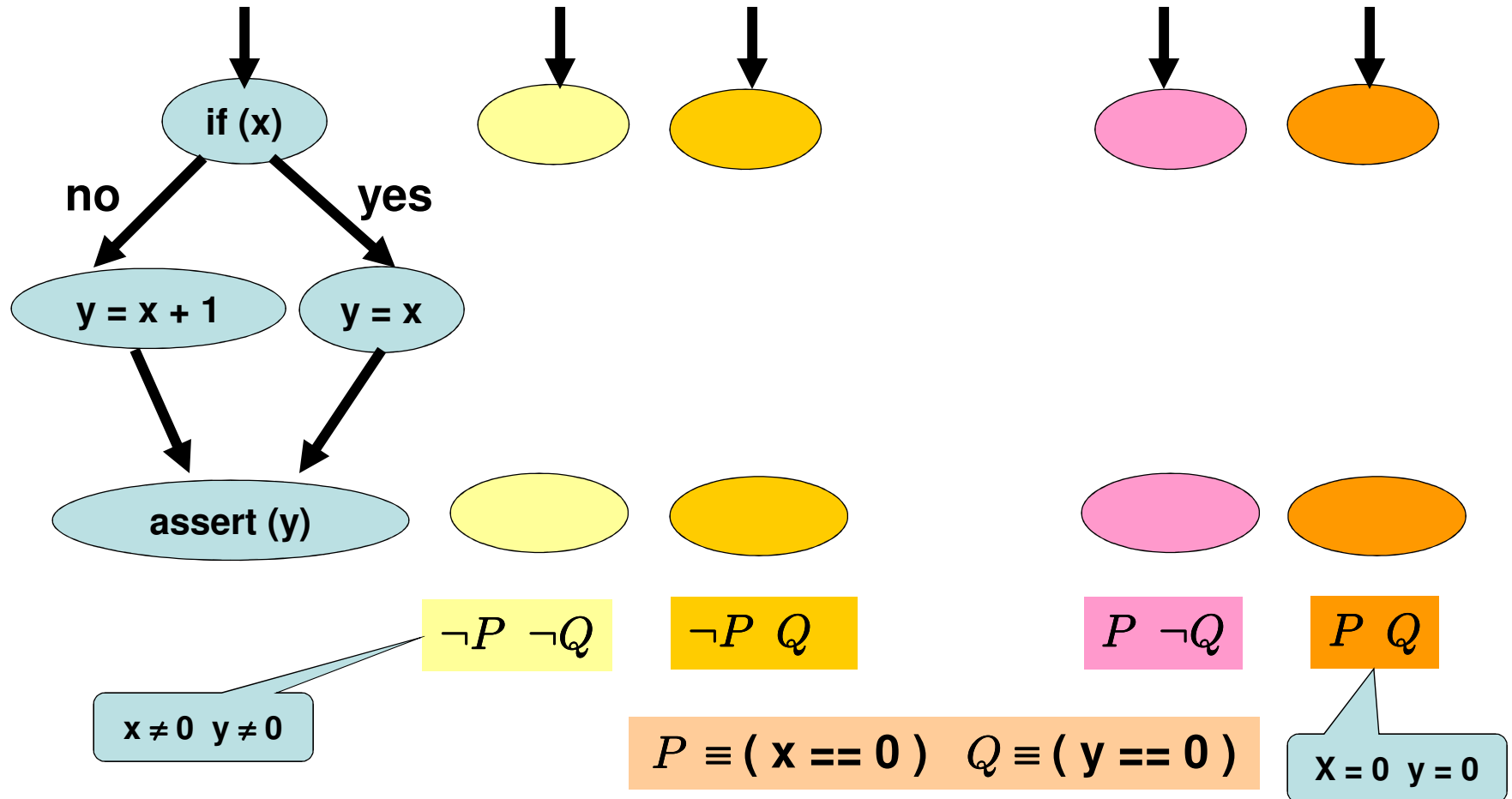
- Simulate counterexample symbolically
- Call SAT Checker to determine if the post-condition is satisfiable
- In our case, Counterexample is spurious
- New set of predicates  $\{x==0, y==0\}$



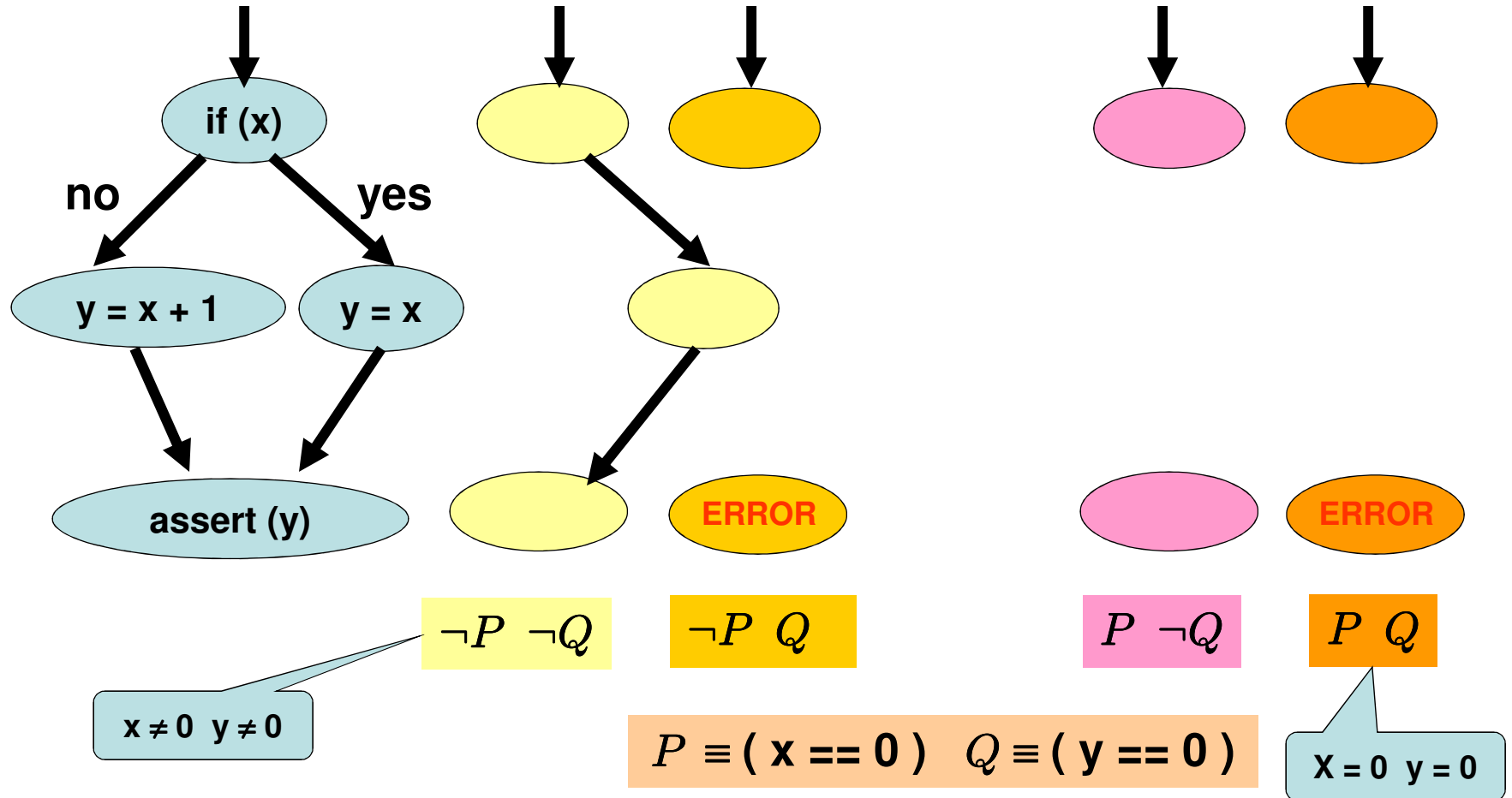
# Counterexample Validation



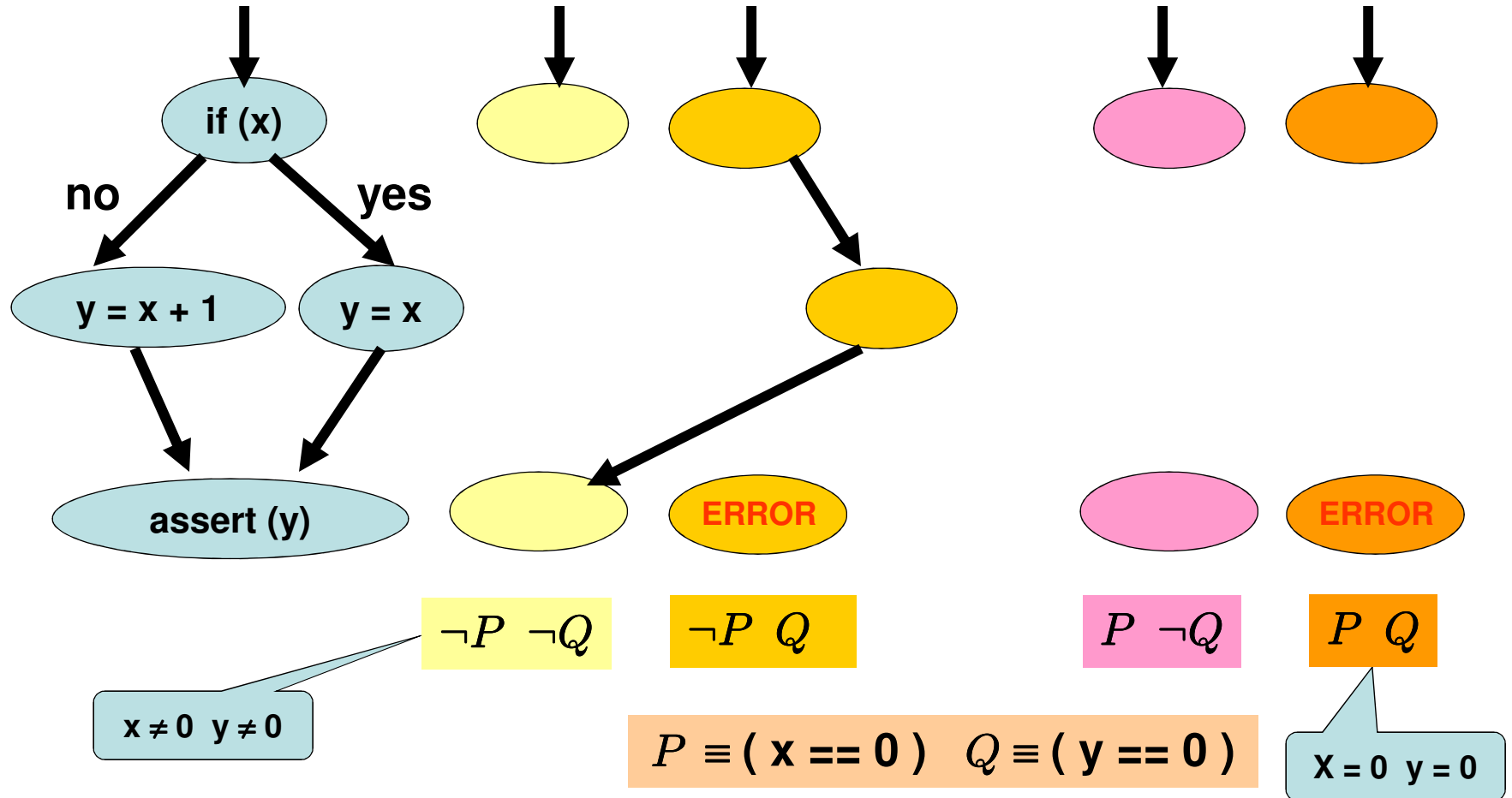
# Predicate Abstraction: 2<sup>nd</sup> Iteration



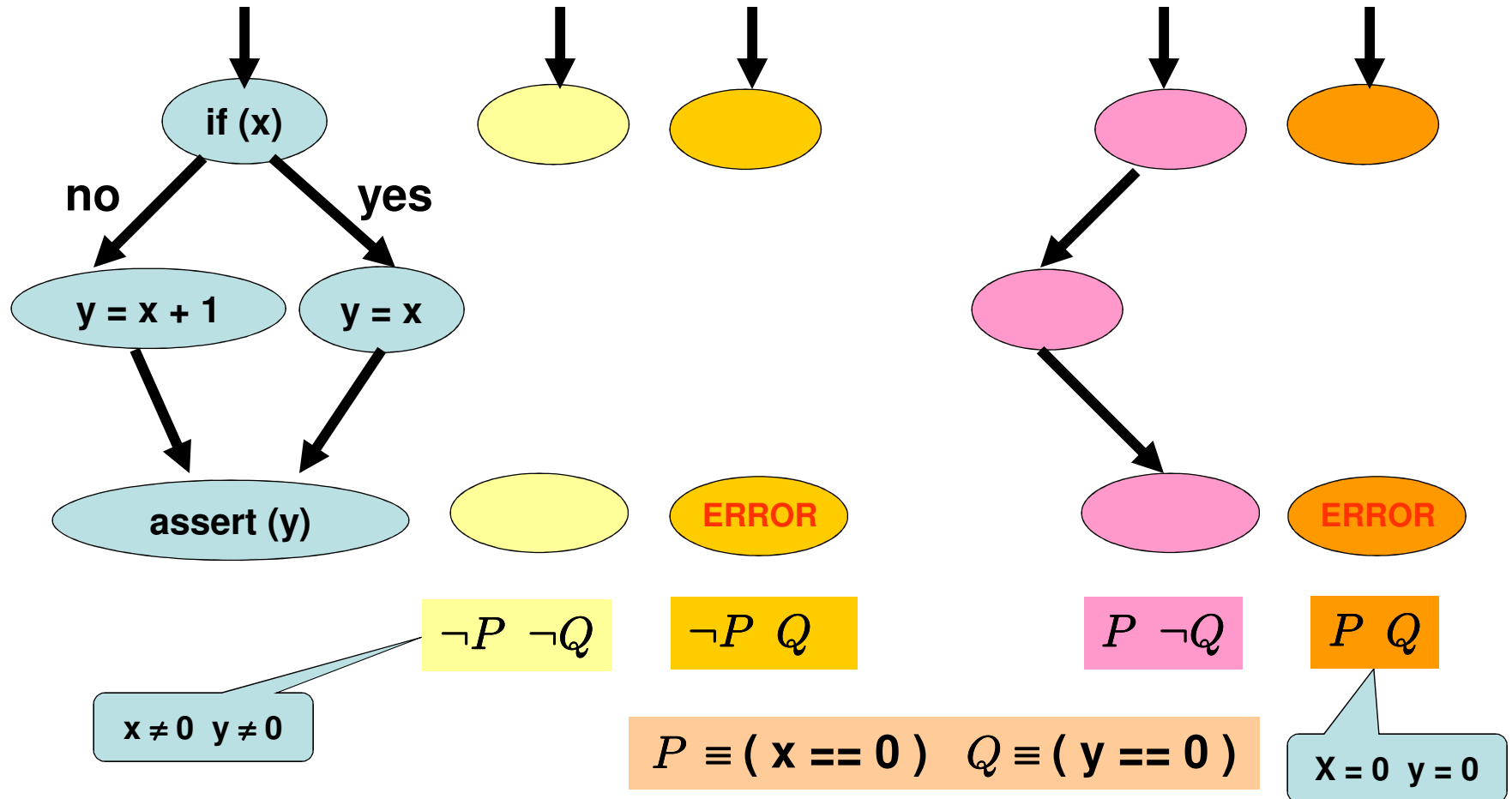
## Predicate Abstraction: 2<sup>nd</sup> Iteration



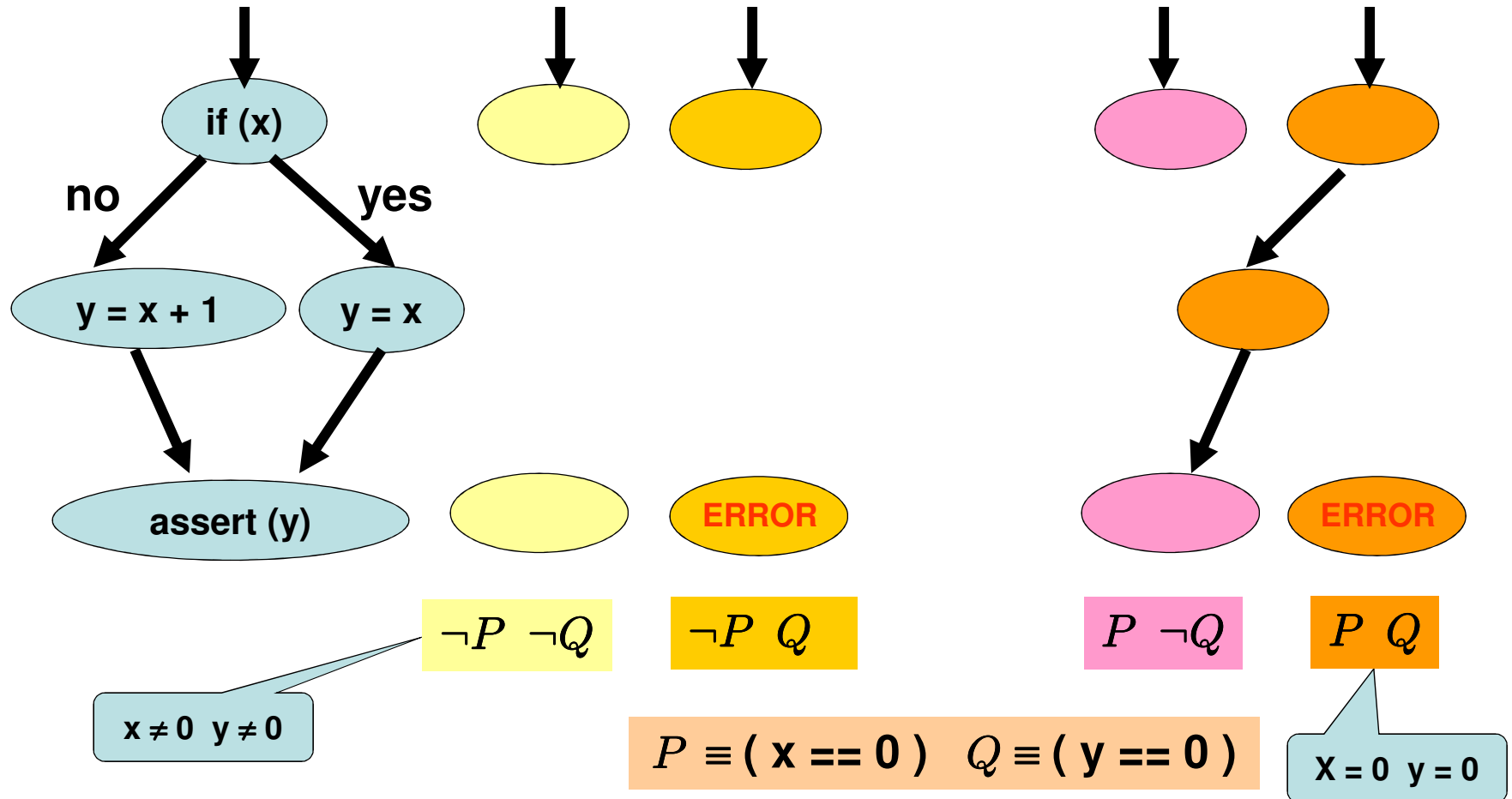
## Predicate Abstraction: 2<sup>nd</sup> Iteration



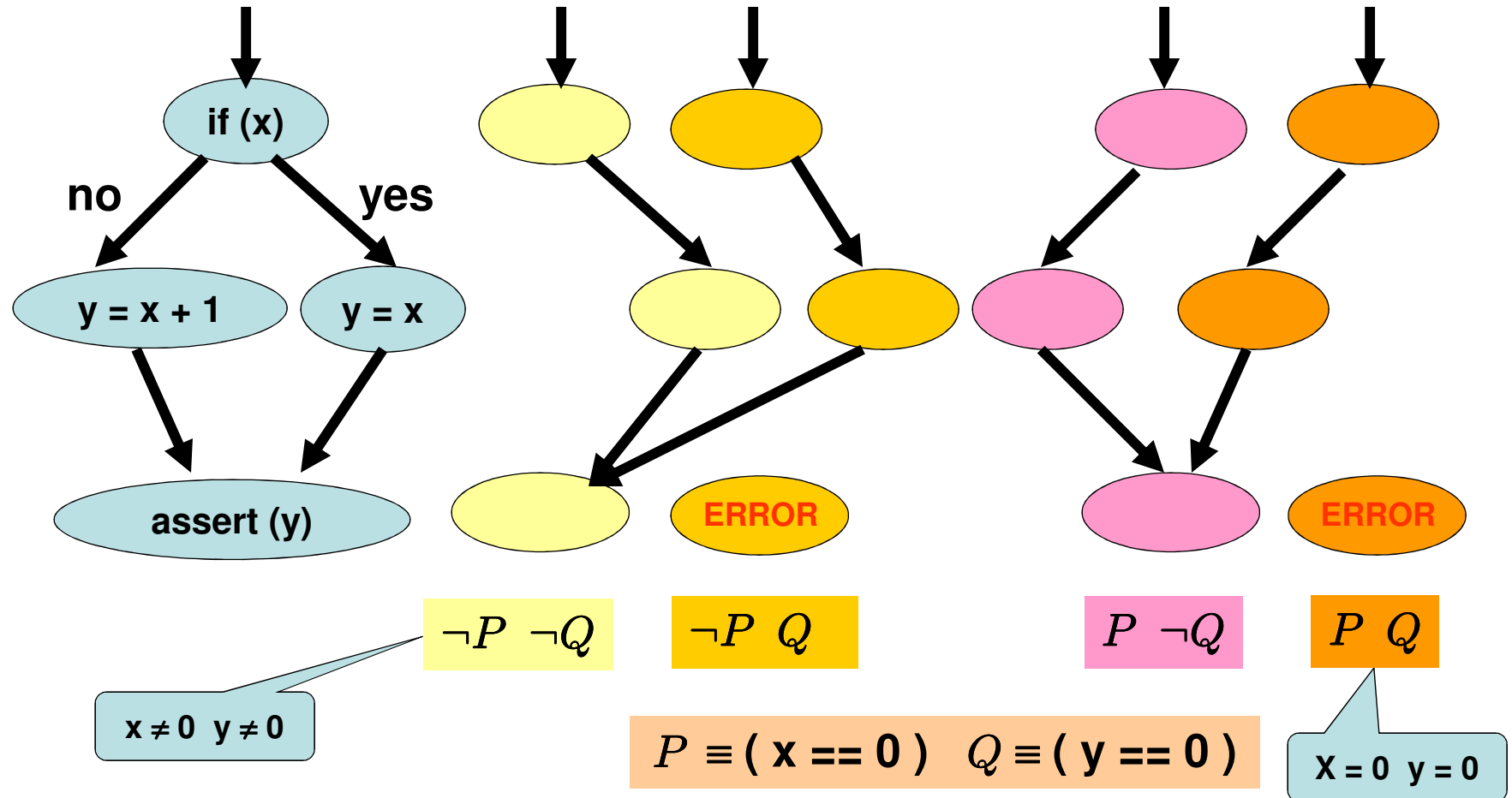
# Predicate Abstraction: 2<sup>nd</sup> Iteration



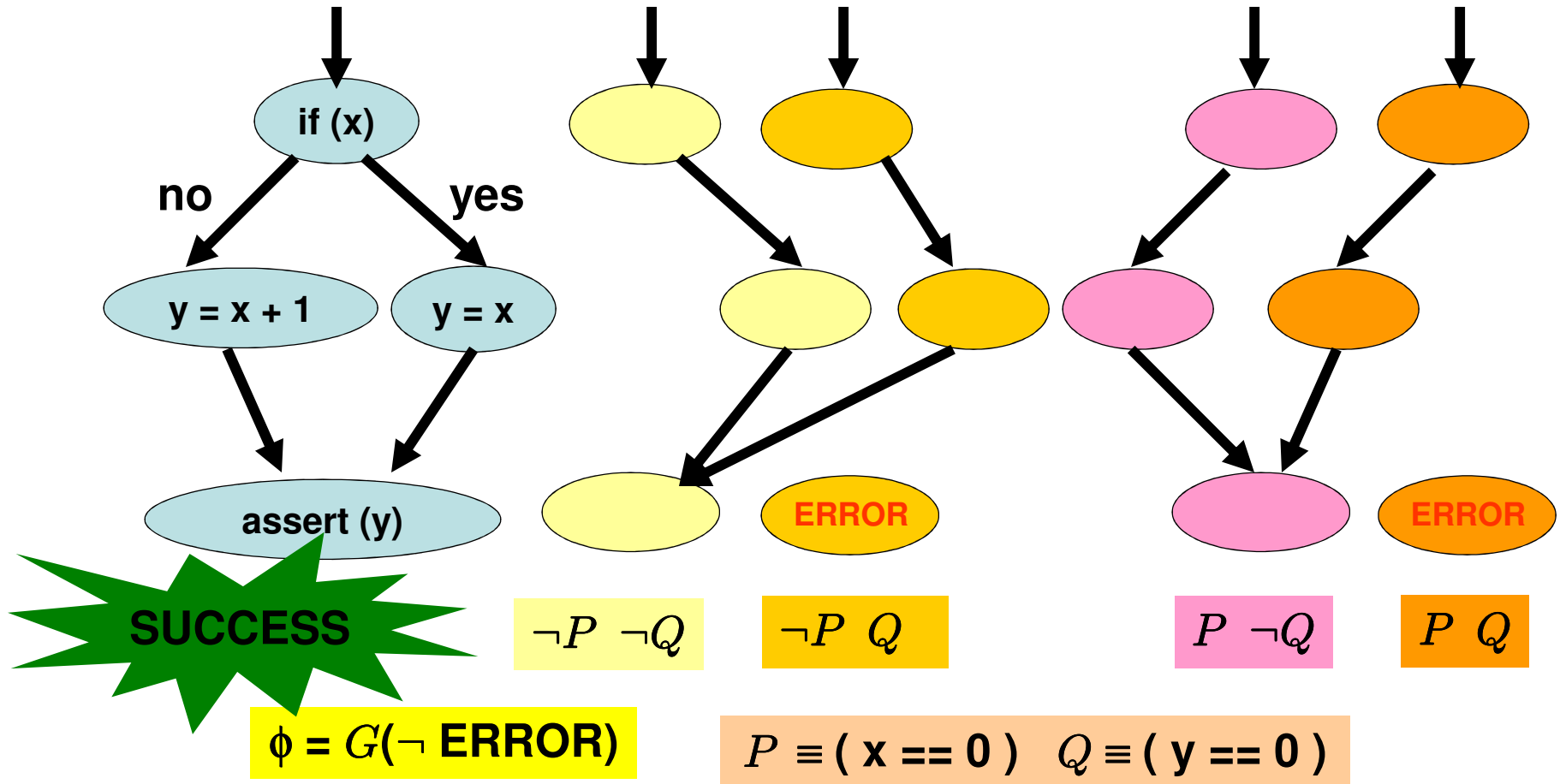
# Predicate Abstraction: 2<sup>nd</sup> Iteration



# Predicate Abstraction: 2<sup>nd</sup> Iteration



# Model Checking: 2<sup>nd</sup> Iteration



# Iterative Refinement: Summary

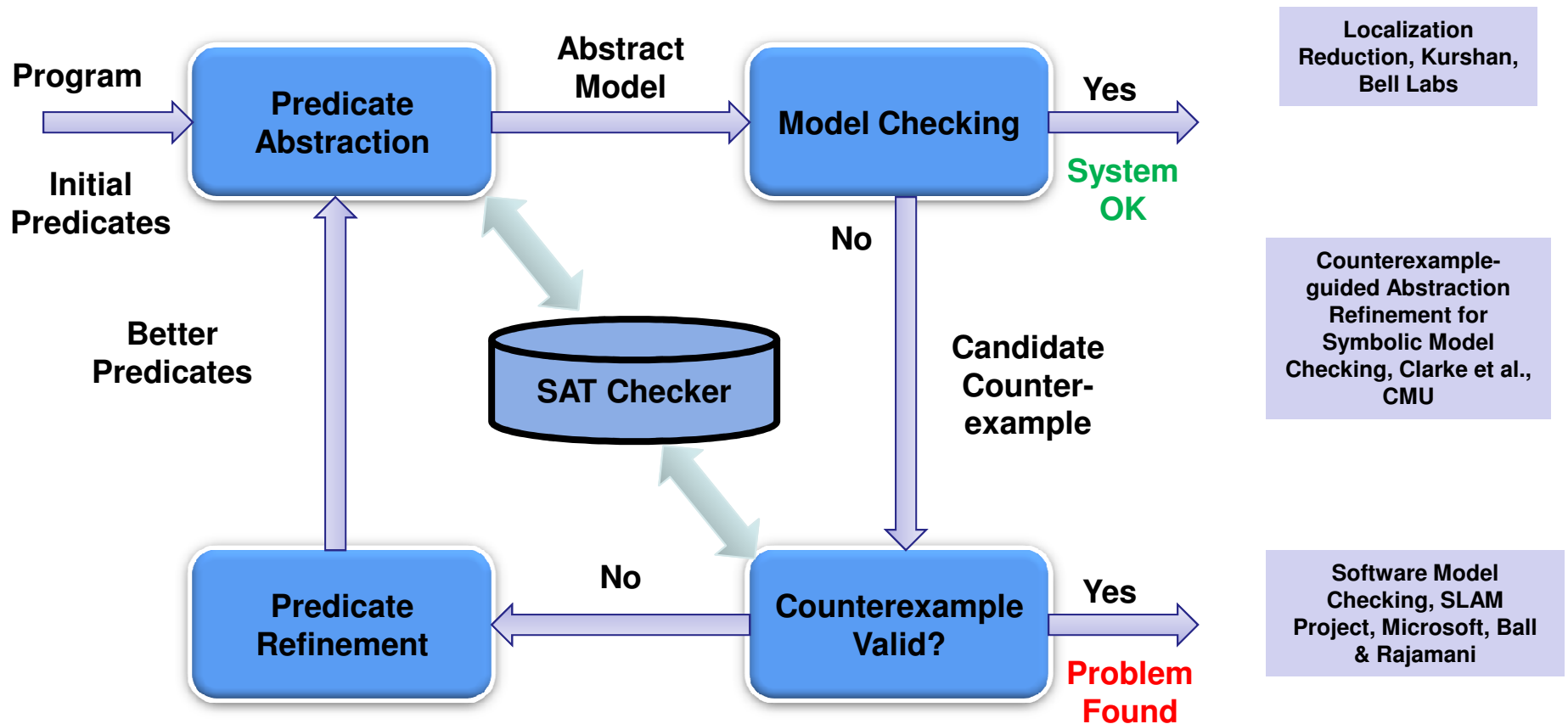
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Choose an initial set of predicate, and proceed iteratively as follows:

1. **Abstraction:** Construct an abstract model  $M$  of the program using the predicate abstraction
2. **Verification:** Model check  $M$ . If model checking succeeds, exit with **success**. Otherwise, get counterexample  $CE$ .
3. **Validation:** Check  $CE$  for validity. If  $CE$  is valid, exit with **failure**.
4. **Refinement:** Otherwise, update the set of predicates and repeat from Step 1.

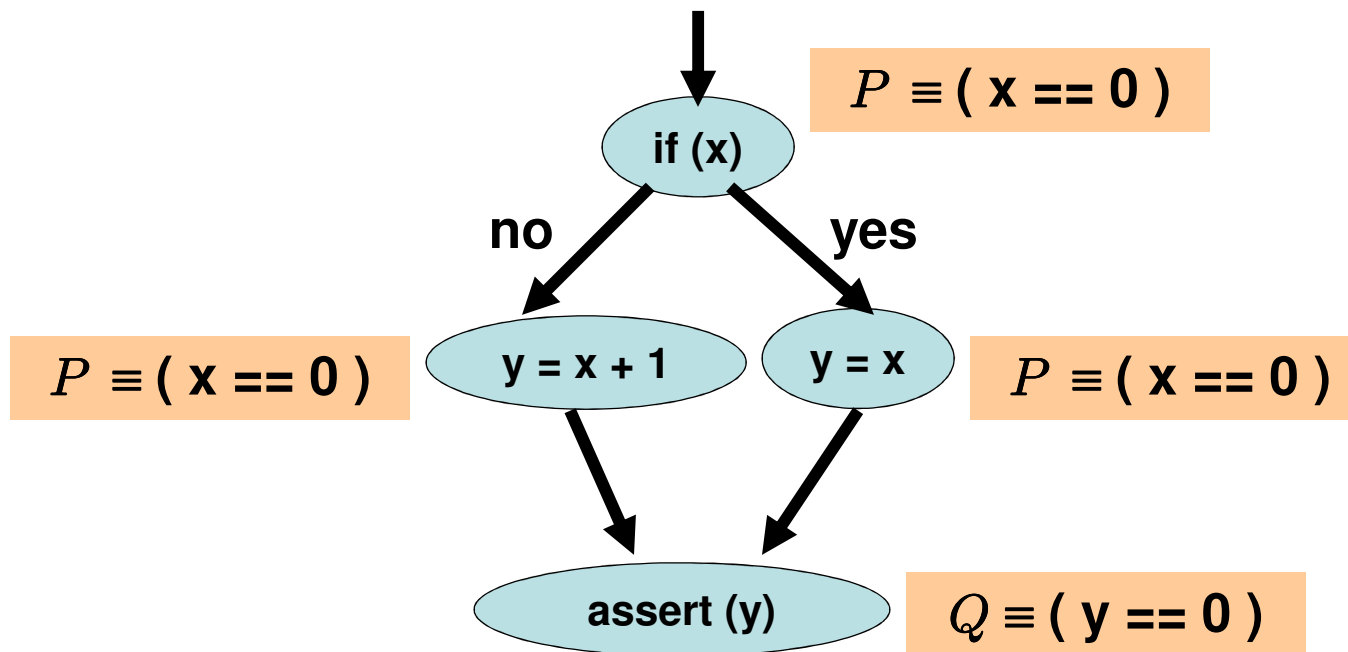


# Iterative Refinement



# Predicate Abstraction: Optimizations

1. Construct transitions on-the-fly
2. Different set of predicates at different control locations



3. Avoid exponential number of theorem-prover calls



# Research Areas

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## Finding “good” predicates

- Technically as hard as finding “good” loop invariants
- Complexity is linear in LOC but exponential in number of predicates

## Combining with static analysis

- Alias analysis, invariant detection, constant propagation
- Inexpensive, and may make subsequent model checking more efficient

## Bounded model checking



# Software Model Checking Tools

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## Iterative Refinement

- SLAM, BLAST, MAGIC, Copper, SATABS, ...

## Bounded Model Checking

- CBMC, ...

## Others

- Engines: MOPED, BEBOP, BOPPO, ...
- Java: Java PathFinder, Bandera, BOGOR, ...
- C: CMC, CPAChecker, ...



**Next lecture**

**Following  
lecture**



# Bibliography

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