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Assignment 1

1 Truth Tables

For each of the pairs of formulae below, construct a truth table that shows if the two formulae are equivalent or not.

- (a) $(x \rightarrow y) \rightarrow y$ and $x \rightarrow (y \rightarrow y)$
- (b) $(x \wedge y) \rightarrow z$ and $x \rightarrow (y \rightarrow z)$

2 Validity

Model the following arguments symbolically and determine their validity. Don't forget to let us know what your atomic propositions stand for!

- (a) If an animal is a dog, it chases its tail. If an animal is a cat, it licks its tail. If an animal chases its tail, it will not lick its tail. Therefore, if an animal is a cat, it is a dog.
- (b) If an animal is a pig, it likes slop. If an animal is a grad student, it likes free food. If an animal likes slop, it does not like gruel. If an animal likes free food, it likes swill. If an animal does not like gruel, it does not like swill. Therefore, if an animal is a grad student, it is not a pig.

3 Tseitin transformation and CNF

In class, we saw how to use Tseitin transformation to transform formulae into CNF, avoiding an exponential increase in size by preserving satisfiability rather than equivalence.

Use Tseitin transformation on the following formulae.

- (a) $(a \wedge b) \vee c$

Hint: You will need to introduce a new variable z such that $z \Leftrightarrow a \wedge b$, that is, $z \Rightarrow a \wedge b$ and $a \wedge b \Rightarrow z$. Rewrite the first implication into two new clauses and the second implication into one new clause.

- (b) $(a \wedge b) \vee ((c \vee d) \wedge e)$

Hint: You will need to introduce three new variables, $x \Leftrightarrow a \wedge b$, $y \Leftrightarrow c \vee d$, $z \Leftrightarrow y \wedge e$.

4 Lewis Carroll Argument

The following argument was proposed by Lewis Carroll, an english mathematician who loved creating logic puzzles and was the author of the famous book “Alice’s Adventures in Wonderland”. Prove or refute its validity. You may (should) use the abbreviated form of argumentation the Professor presented in class instead of presenting a full truth table.

1. All the dated letters in this room are written on blue paper.
2. None of them are in black ink, except those that are written in the third person.
3. I have not filed any of those that I can read.
4. None of those that are written on one sheet are undated.
5. All of those that are not crossed out are in black ink.
6. All of those that are written by Brown begin with “Dear Sir”.
7. All of those that are written on blue paper are filed.
8. None of those that are written on more than one sheet are crossed out.
9. None of those that begin with “Dear sir” are written in the third person

Therefore, I cannot read any of Brown’s letters.

5 Sudoku (Extra Credit)

Solve the following Sudoku for extra credit and practice for an upcoming assignment.

					5	6	8	1
					9	4		
5			1	4			2	
	5	3				8		
6	7						3	4
		4				1	7	
	4			6	3			8
		7	8					
9	1	8	4					