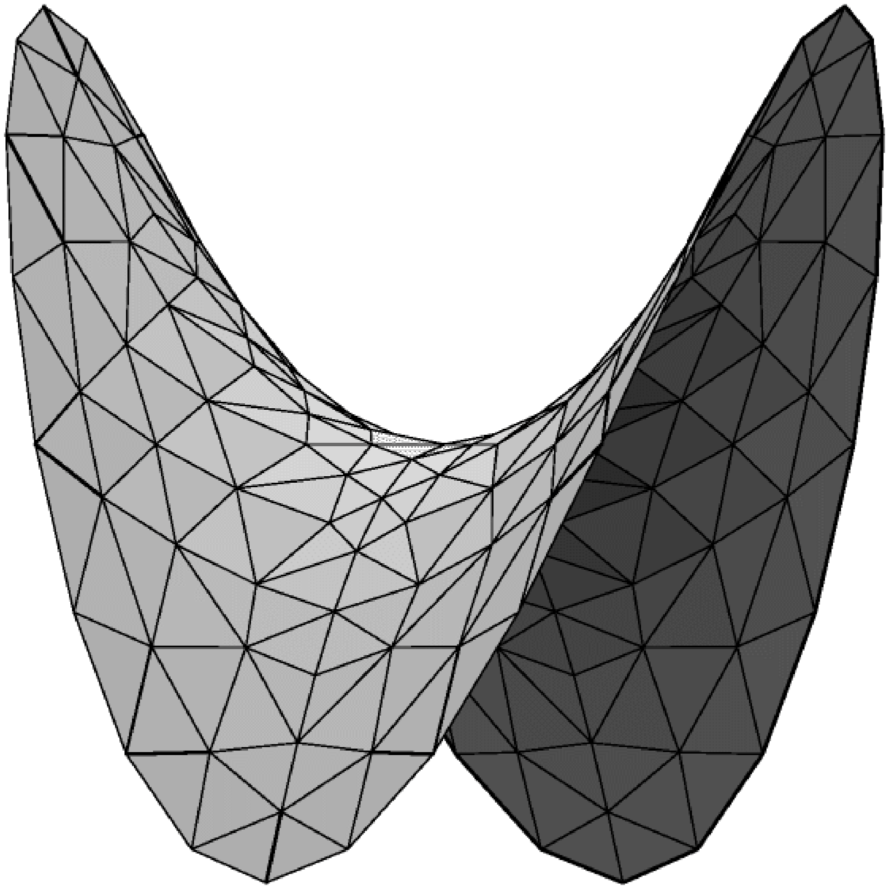
manifold learning
with applications to object recognition



Advanced Perception
David R. Thompson

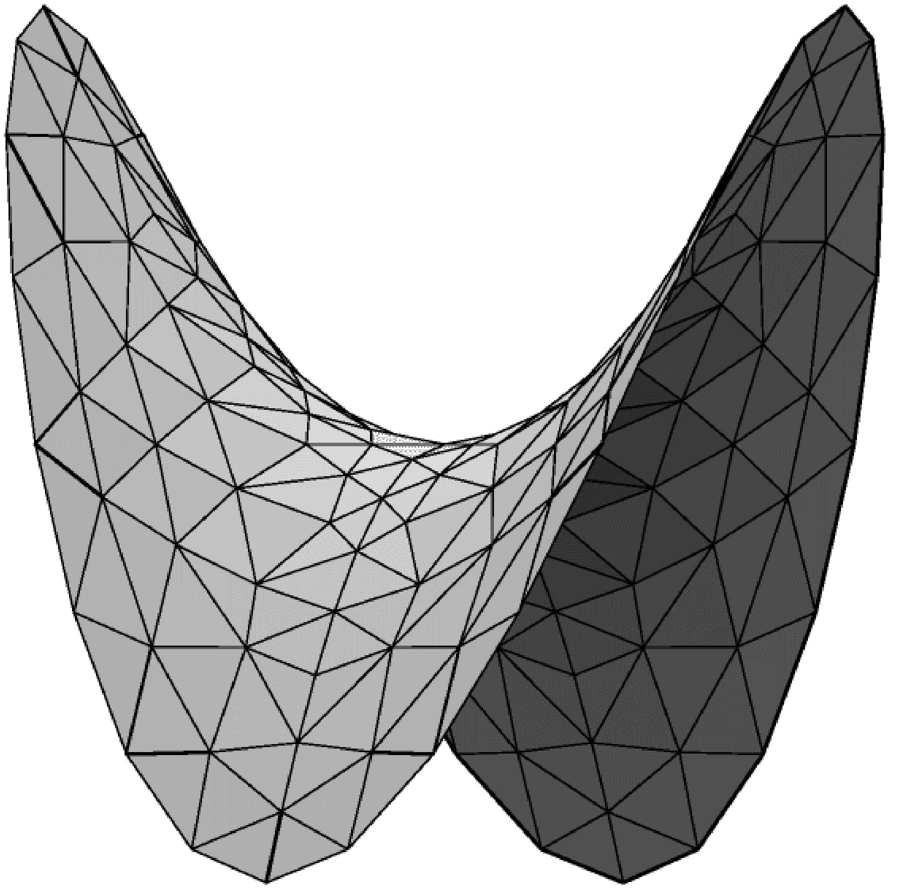
agenda

1. why learn manifolds?

2. Isomap

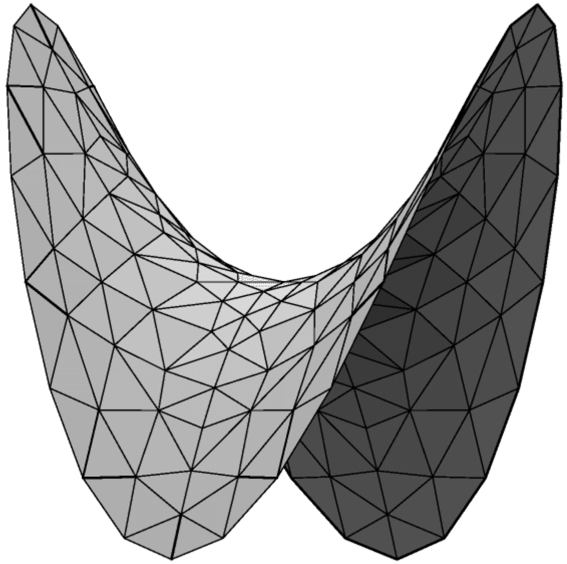
3. LLE

4. applications

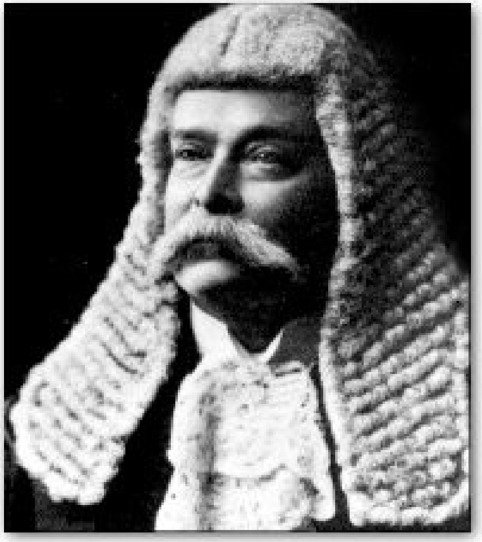


types of manifolds

exhaust
manifold



low-D surface
embedded in
high-D space



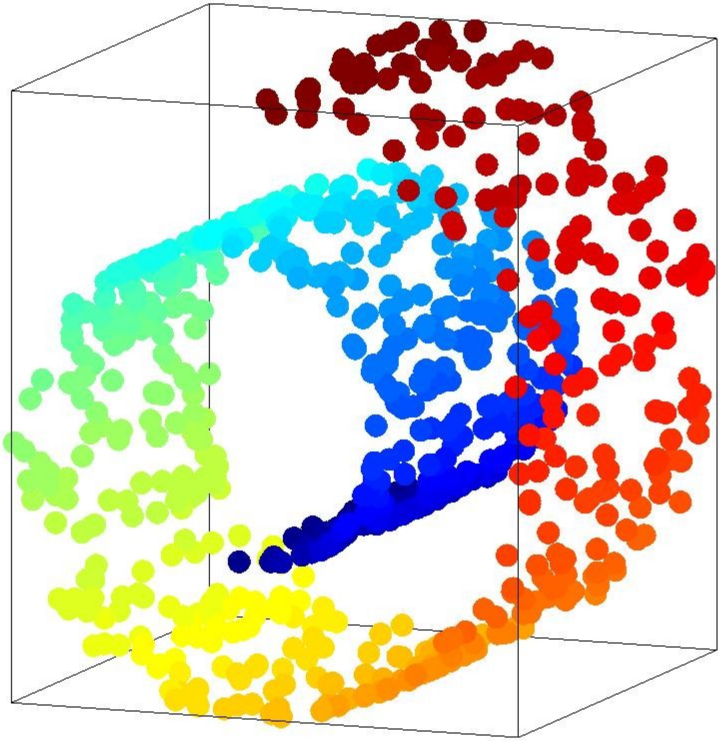
Sir Walter
Synnot Manifold
1849-1928

manifold learning

Find a low-D basis for describing high-D data.

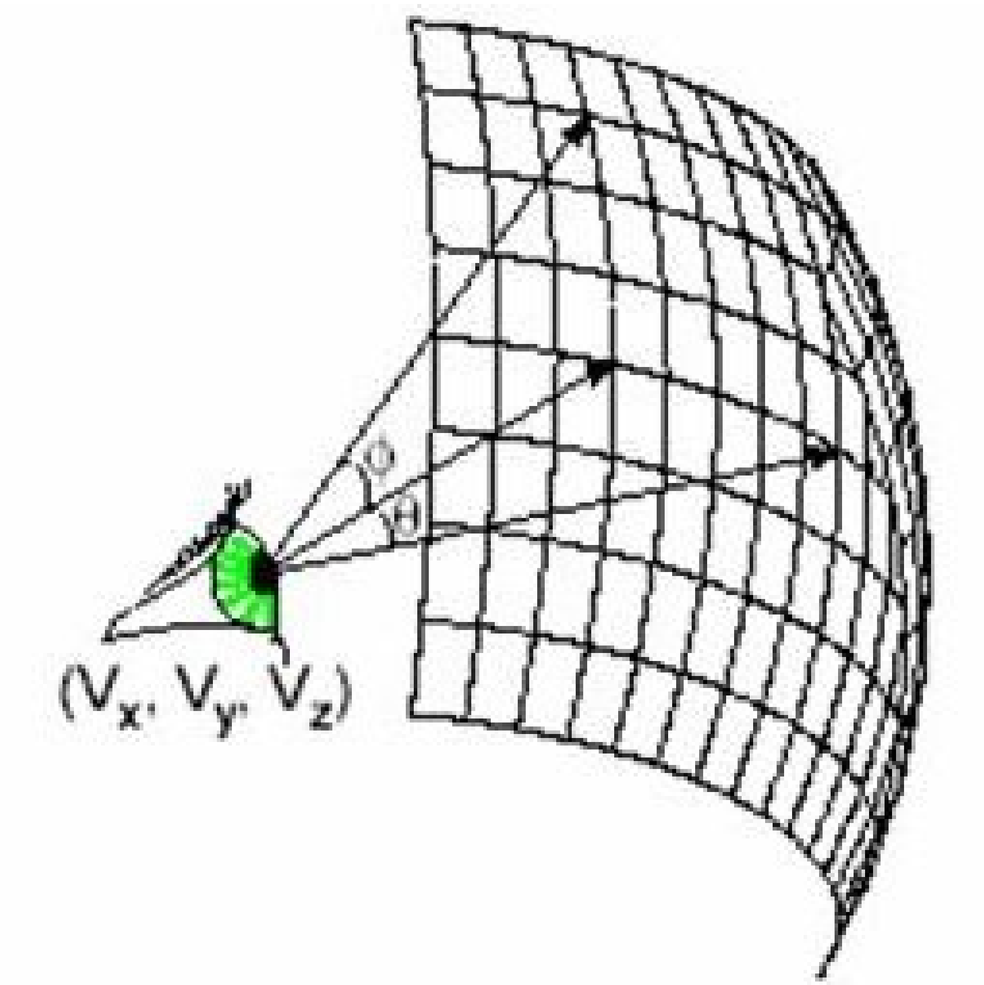
X → X' S.T.
dim(X') << dim(X)

uncovers the intrinsic dimensionality (invertible)



manifolds in vision

plenoptic function / motion / occlusion



manifolds in vision

appearance variation

images from hormel corp.

manifolds in vision

deformation

images from www.golfswingphotos.com

why do manifold learning?

1. data compression

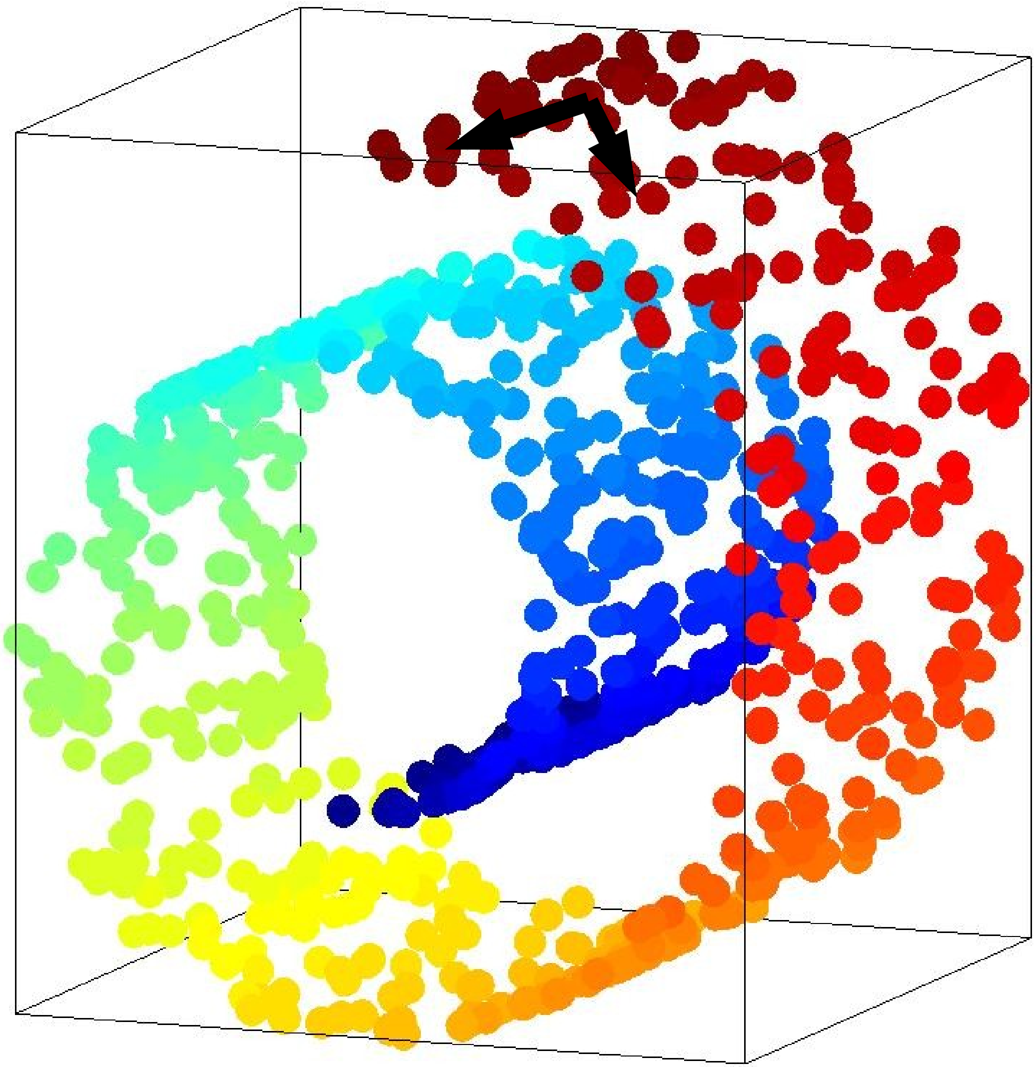
2. “curse of dimensionality”

3. de-noising

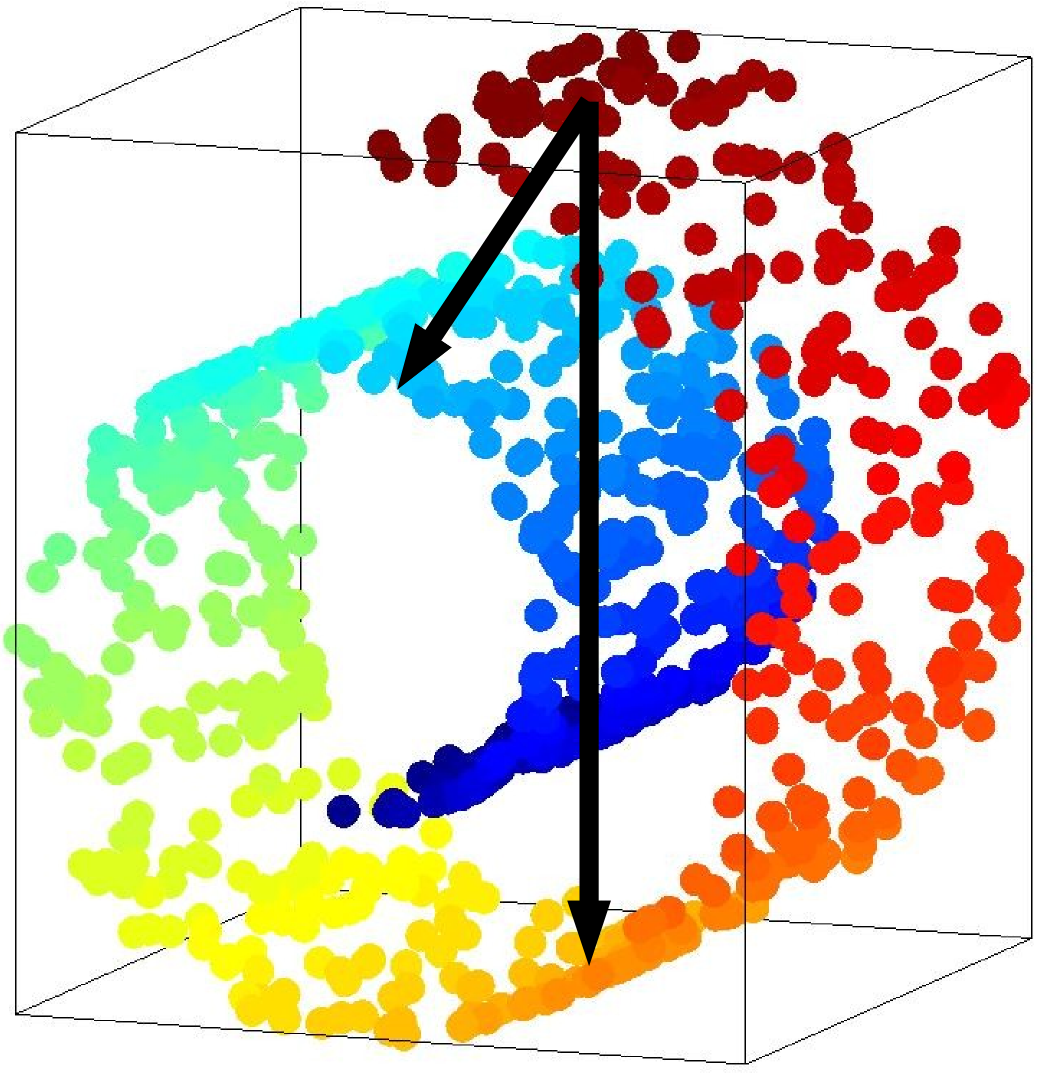
4. visualization

5. reasonable distance metrics *

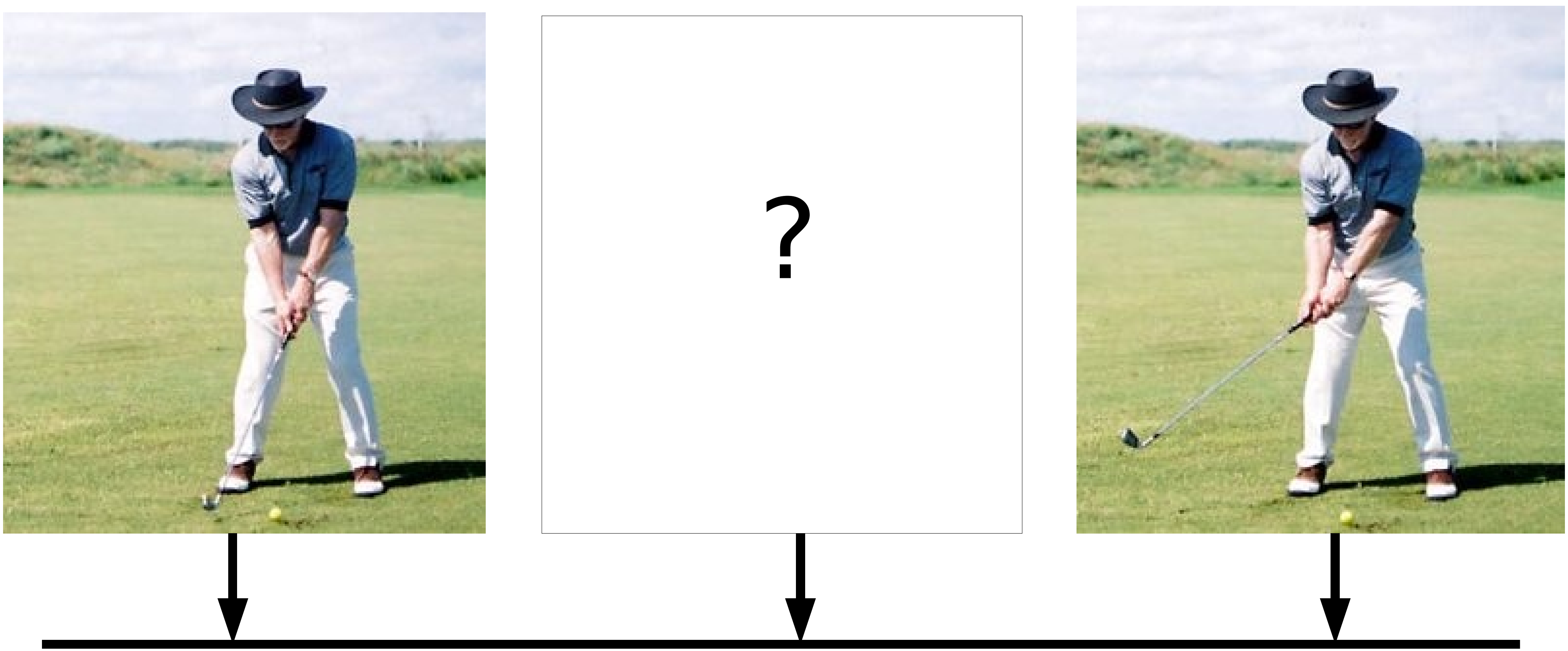
reasonable distance metrics



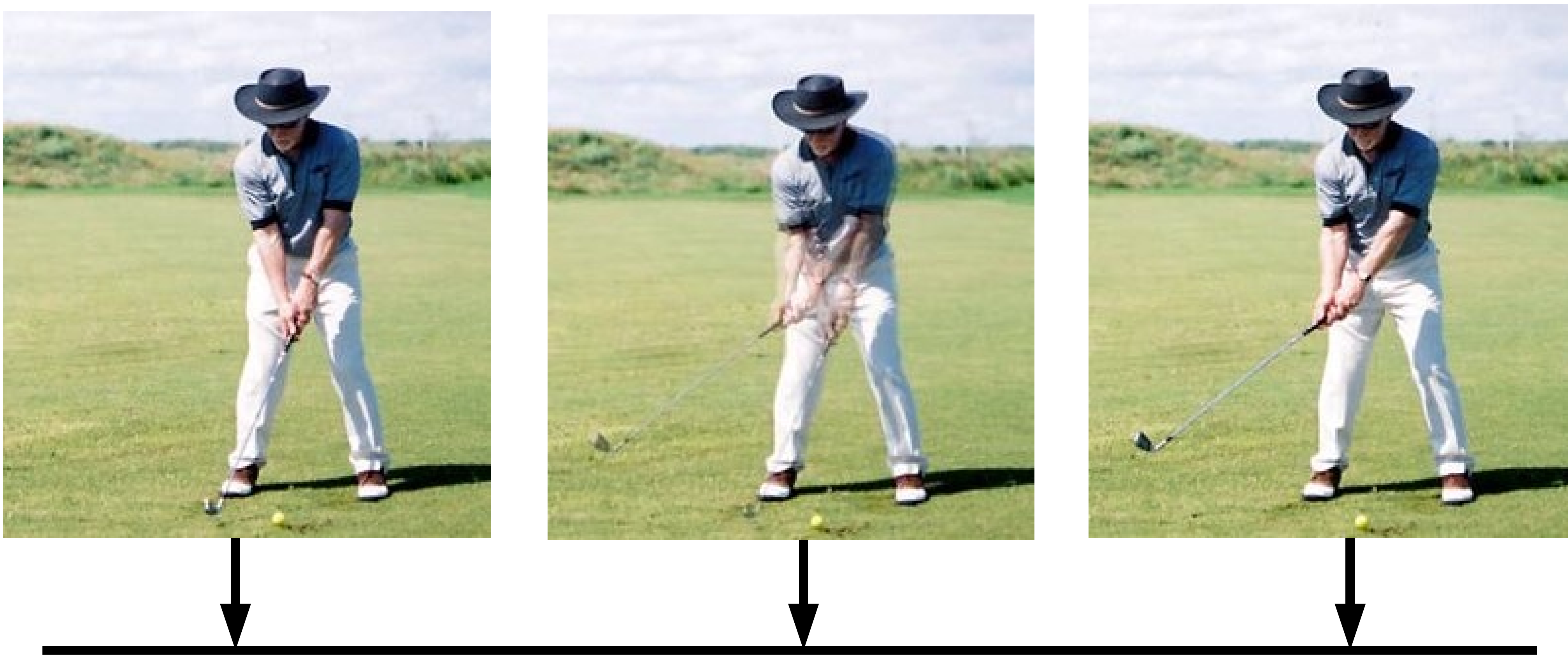
reasonable distance metrics



reasonable distance metrics

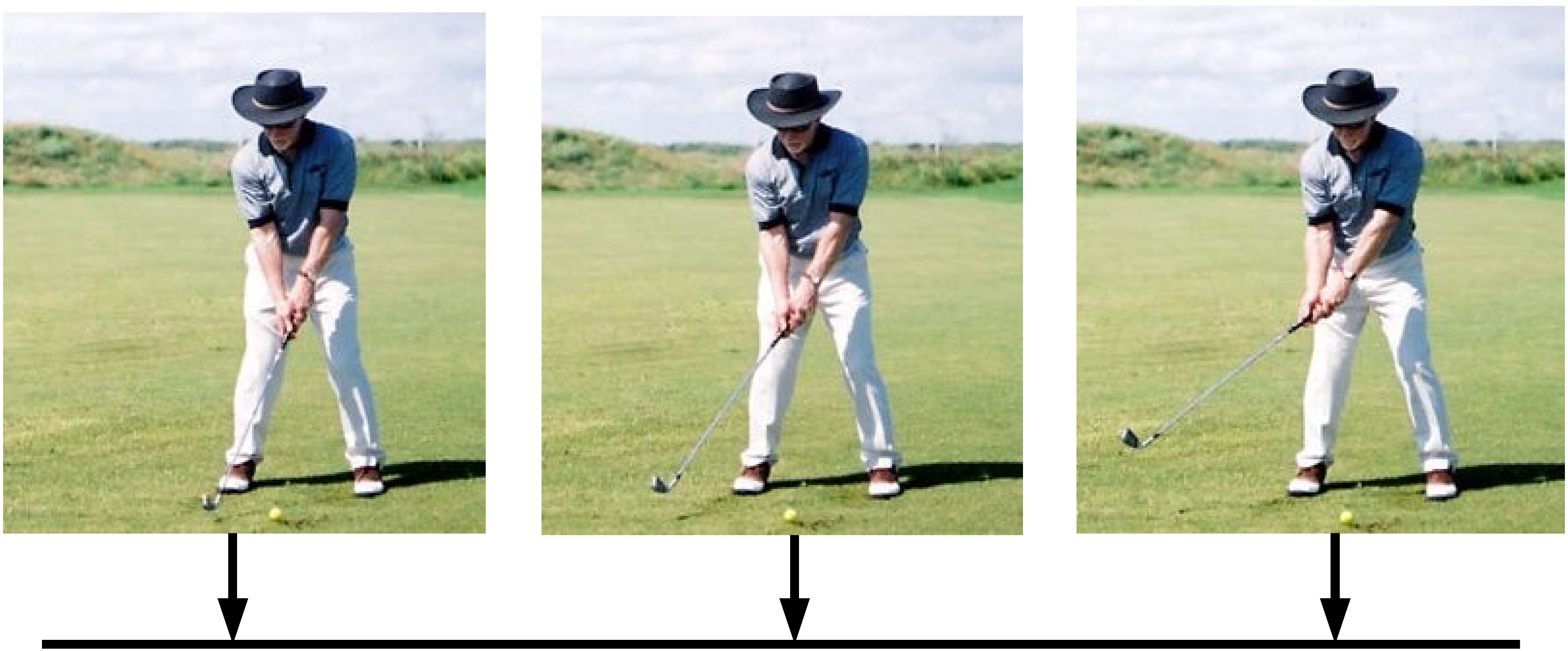


reasonable distance metrics



linear interpolation

reasonable distance metrics



manifold interpolation

agenda

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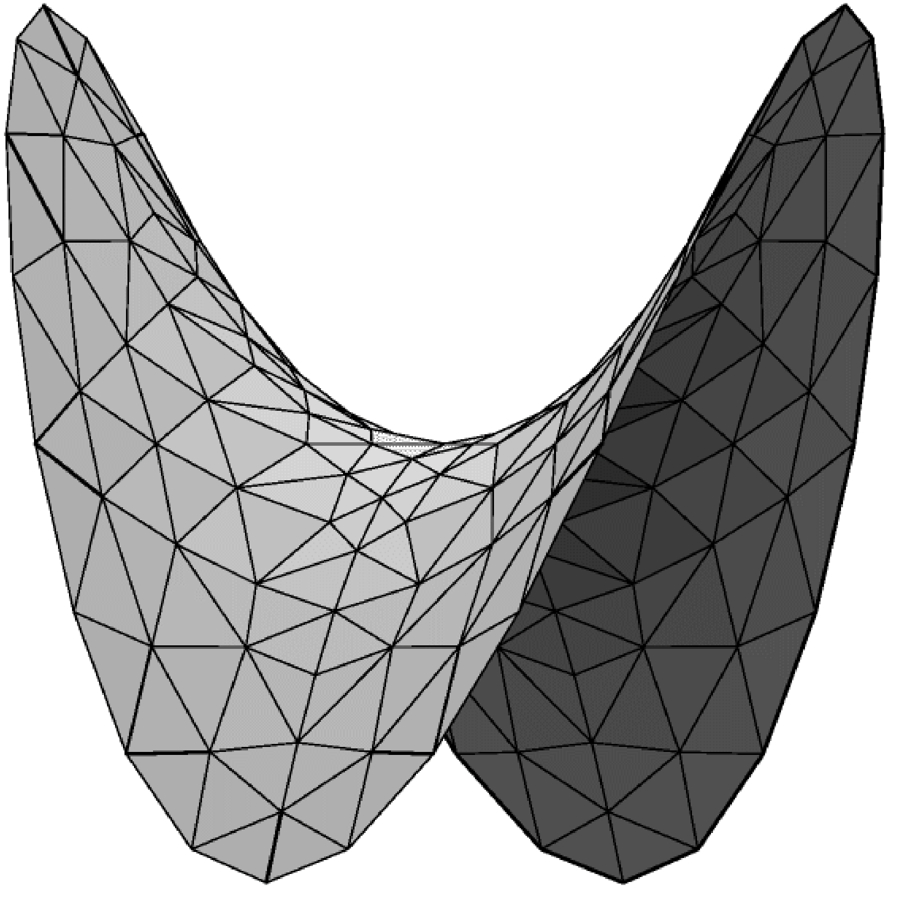
2. Isomap

3. LLE

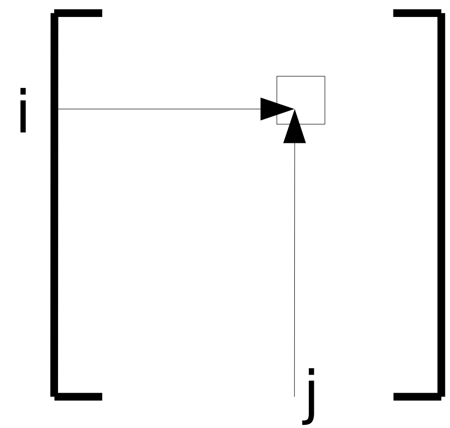
4. applications



Isomap

For *n* data points, and a distance matrix D,

$D_{ij} =$



…we can construct a *m*-dimensional space to preserve inter-point distances by using the top eigenvectors of D scaled by their eigenvalues.

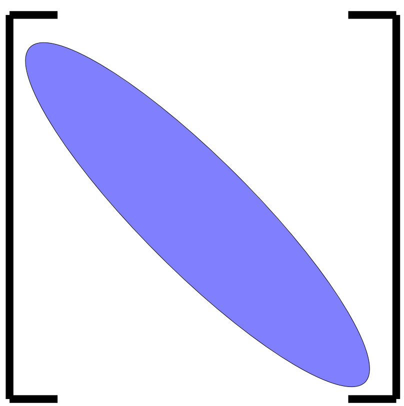
$$y_i = [\ \sqrt{\lambda_1} v_1^i\ ,\ \sqrt{\lambda_2} v_2^i\ ,\ \ldots\ ,\ \sqrt{\lambda_m} v_m^i\ ]$$

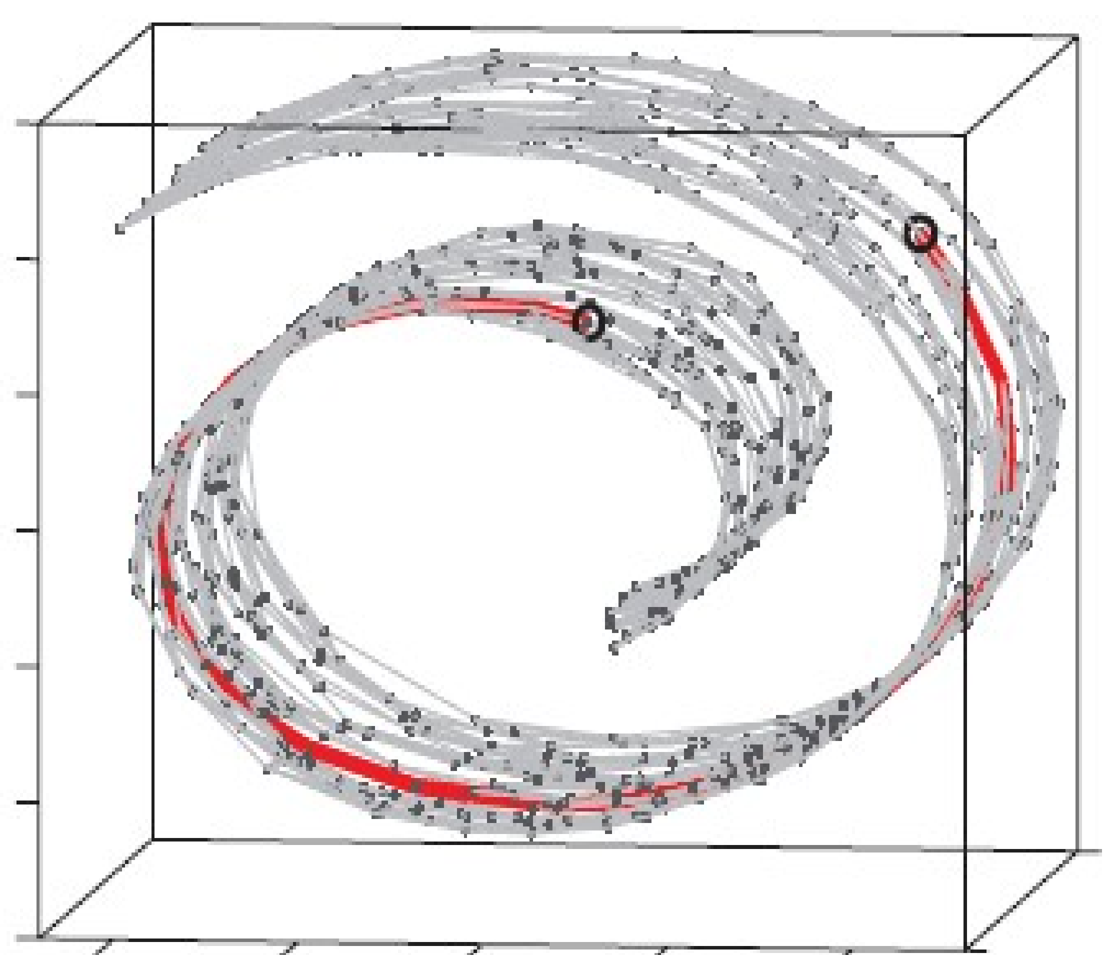
Isomap

Infer a distance matrix using distances along the manifold.

Isomap

1. Build a sparse graph with K-nearest neighbors

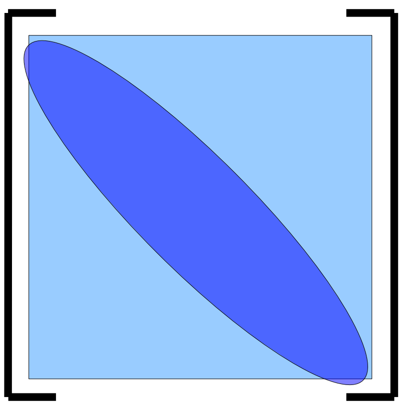
$D_g =$ 

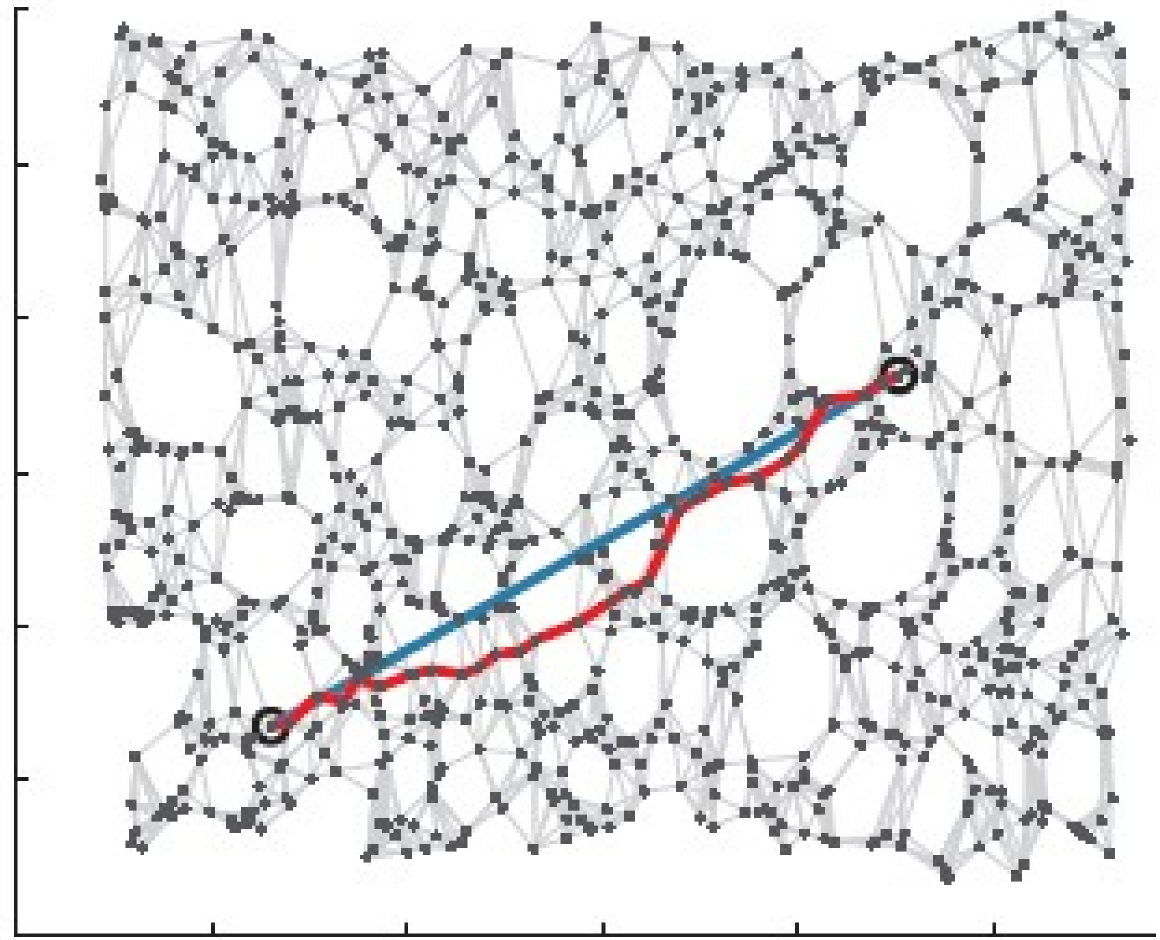


(distance matrix is sparse)

Isomap

2. Infer other interpoint distances by finding shortest paths on the graph (Dijkstra's algorithm).

$D_g =$ 



Isomap

3. Build a low-D embedded space to best preserve the complete distance matrix.

Error function:

inner product distances in graph

inner product distances in new coordinate system

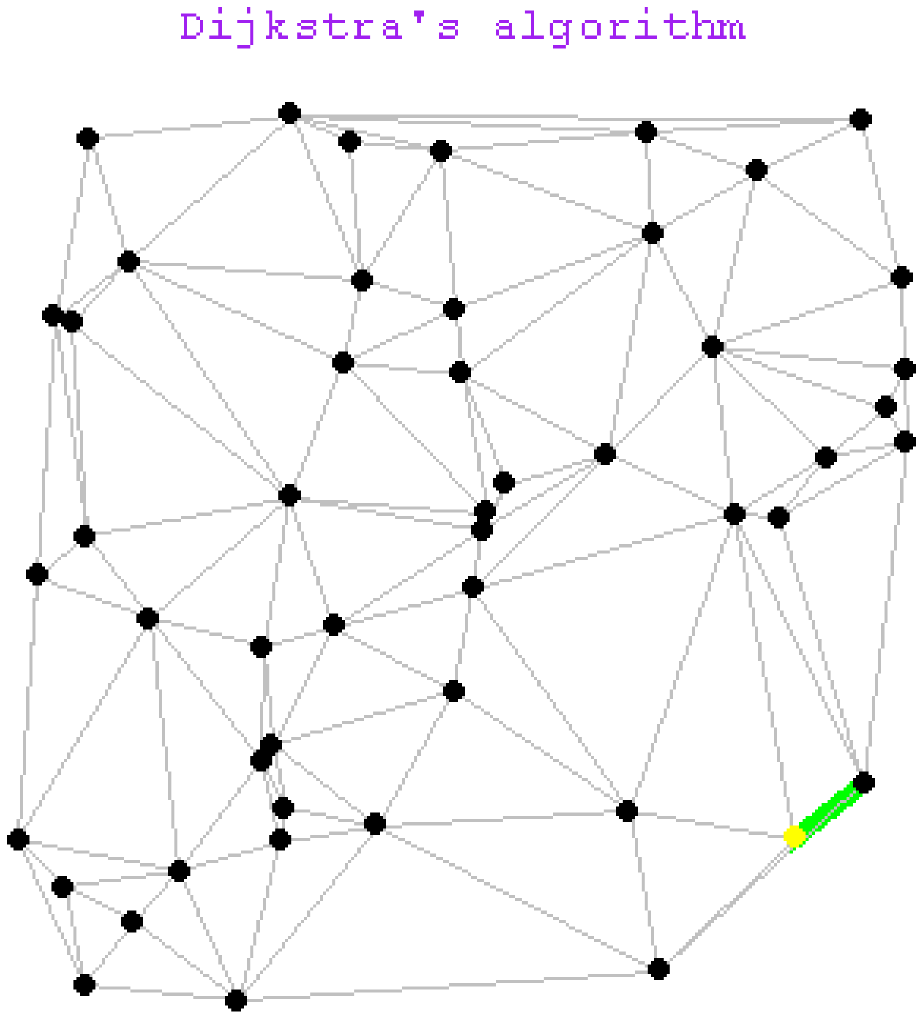
L2 norm

$$E = \|\tau(D_G) - \tau(D_Y)\|_{L^2}$$

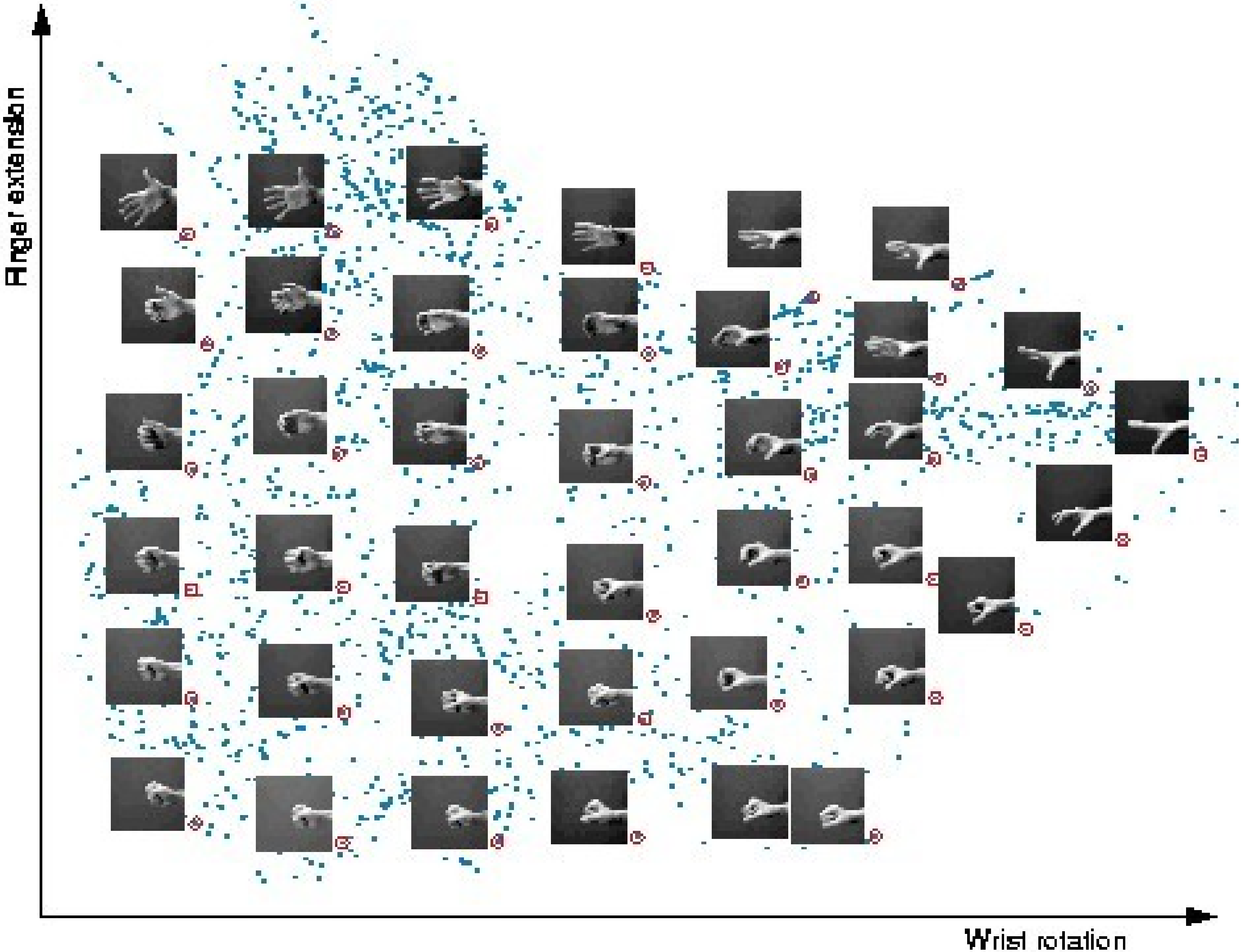
Solution – set points Y to top eigenvectors of D_g

Isomap

- shortest-distance on a graph is easy to compute



Isomap results: hands



Isomap: pro and con

- preserves global structure

- few free parameters

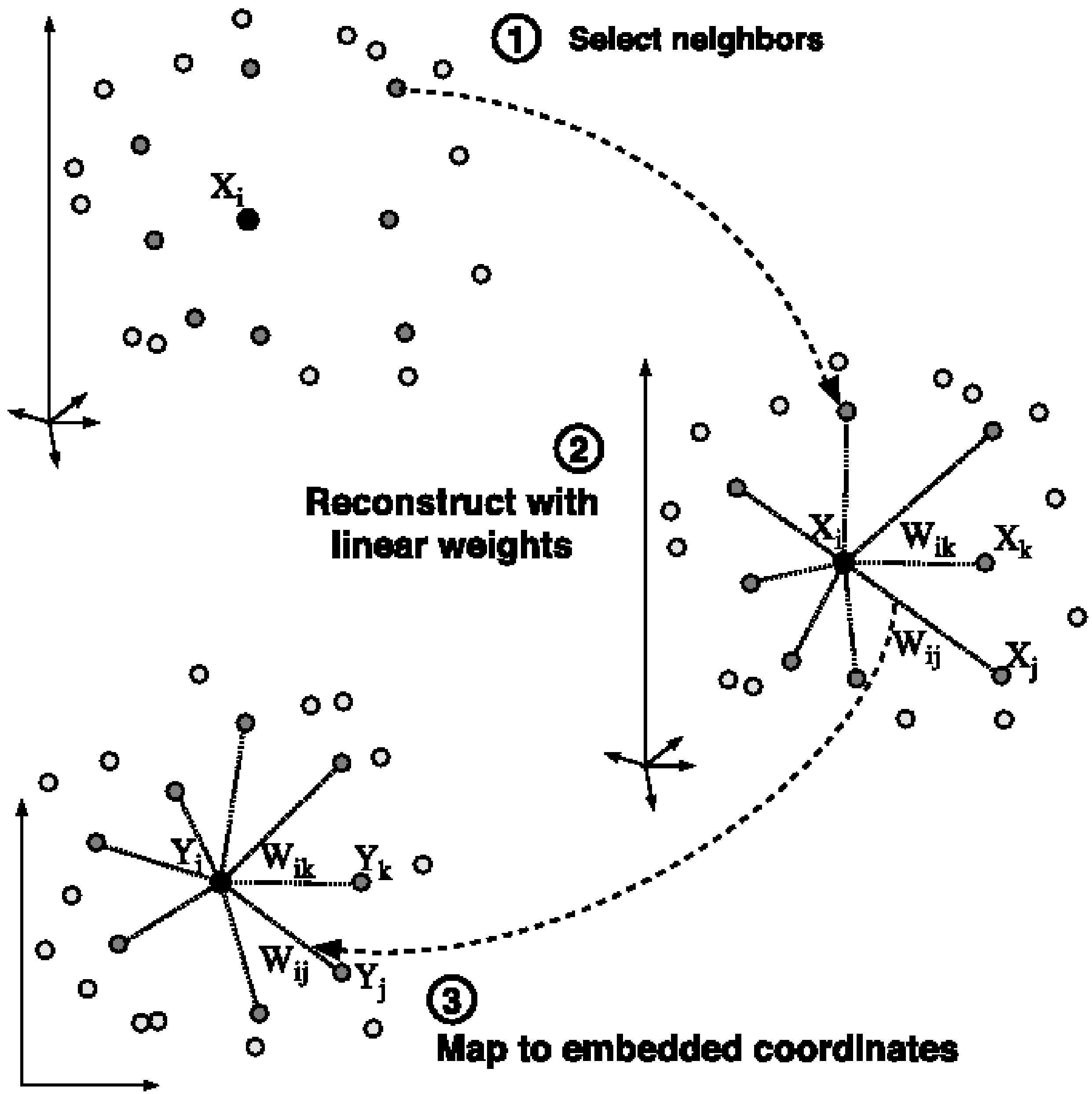
- sensitive to noise, noise edges

- computationally expensive (dense matrix eigen-reduction)

Locally Linear Embedding

Find a mapping to preserve local linear relationships between neighbors

Locally Linear Embedding



LLE: Two key steps

1. Find weight matrix W of linear coefficients:

$$\mathcal{E}(W) = \sum_i \left| \vec{X}_i - \sum_j W_{ij} \vec{X}_j \right|^2$$

Enforce sum-to-one constraint with the Lagrange Multiplier:

$$\frac{1 - \sum_{jk} C_{jk}^{-1} (\vec{x} \cdot \vec{\eta}_k)}{\sum_{jk} C_{jk}^{-1}}$$

$$w_j = \sum_k C_{jk}^{-1} (\vec{x} \cdot \vec{\eta}_k + \lambda)$$

LLE: Two key steps

2. Find projected vectors Y to minimize reconstruction error

$$\Phi(Y) = \sum_i \left| \vec{Y}_i - \sum_j W_{ij} \vec{Y}_j \right|^2$$

must solve for whole dataset simultaneously

LLE: Two key steps

$$\Phi(Y) = \sum_i \left| \vec{Y}_i - \sum\nolimits_j W_{ij} \vec{Y}_j \right|^2$$

We add constraints to prevent multiple / degenerate solutions:

$$\sum\nolimits_i \vec{Y}_i = \vec{0}$$

$$\frac{1}{N} \sum\nolimits_i \vec{Y}_i \otimes \vec{Y}_i = I$$

LLE: Two key steps

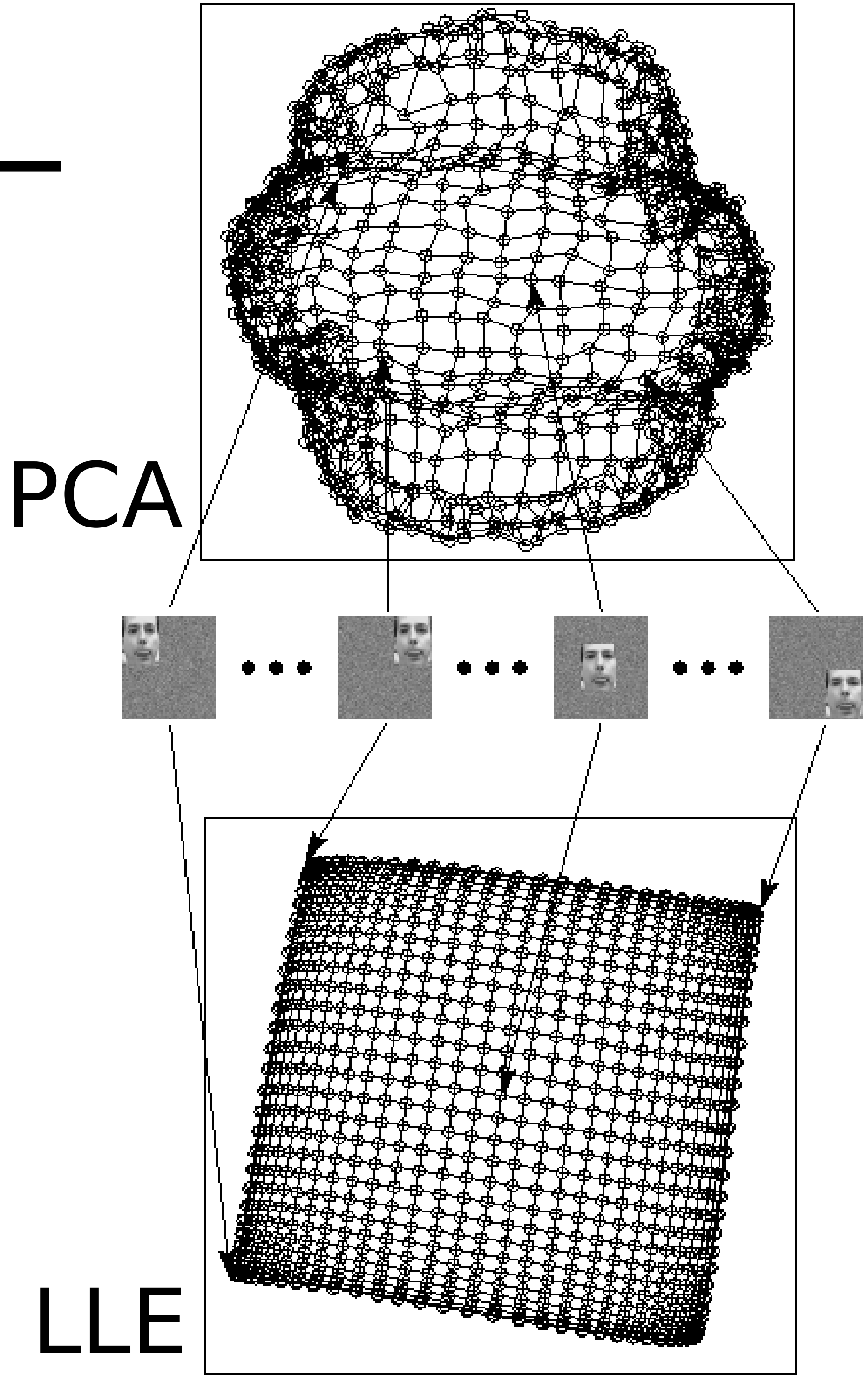
cost function becomes:

$$M_{ij} = \delta_{ij} - W_{ij} - W_{ji} + \sum_k W_{ki} W_{kj}$$

the optimal embedded coordinates are given by bottom *m*+1 eigenvectors of the matrix M

LLE: Result

preserves local topology



LLE: pro and con

- no local minima, one free parameter

- incremental & fast

- simple linear algebra operations

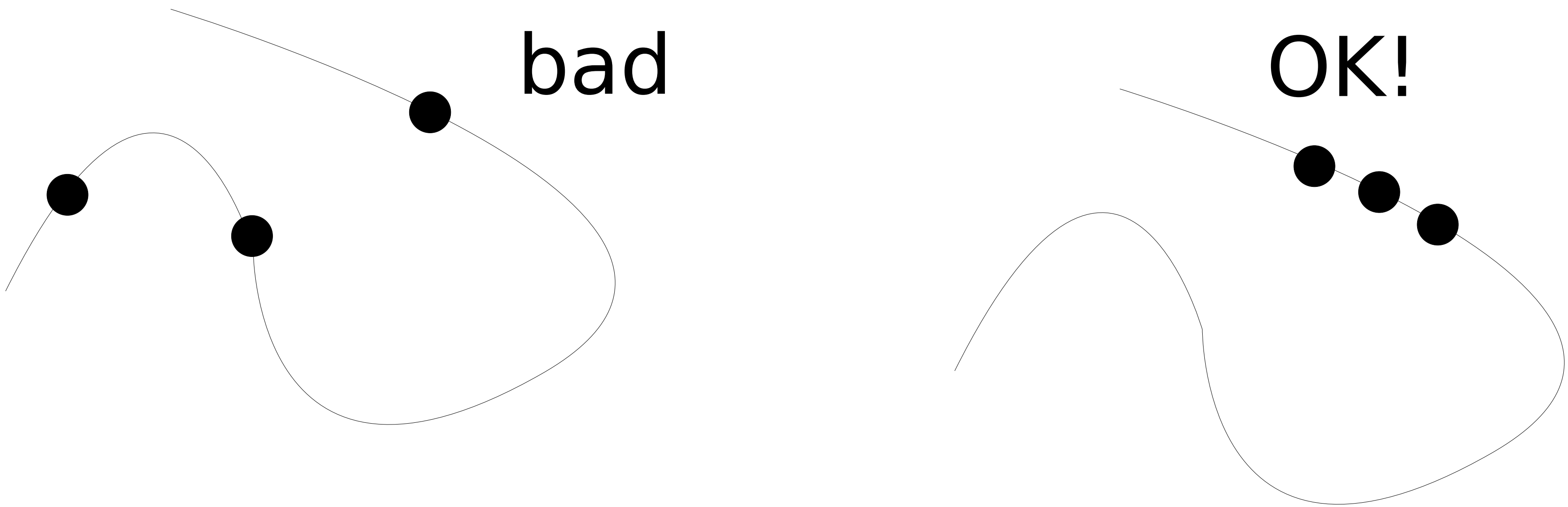
- can distort global structure

Others you may encounter

- Laplacian Eigenmaps (Belkin 2001)
  - spectral method similar to LLE
  - better preserves clusters in data
- Kernel PCA
- Kohonen Self-Organizing Map (Kohonen, 1990)
  - iterative algorithm fits a network of pre-defined connectivity
  - simple, fast for on-line learning
  - local minima
  - lacking theoretical justification

No Free Lunch

the “curvier” your manifold, the denser your data must be



conclusions

Manifold learning is a key tool in your object recognition toolbox

A formal framework for many different ad-hoc object recognition techniques