Homework 3

Due in class on Thursday, September 27

Collaboration is allowed on this homework. You must hand in homework assignments individually. You may not use an alignment program to calculate the alignments on this problem set.

Your name:

List the names of the people you worked with.

*Homework must be submitted in class on Thursday, Sept. 27.*
1. The robotics department has created a robot that plays Rock-Paper-Scissors. In each round of this game, two players simultaneously present a hand signal representing a rock (closed fist), paper (open palm) or scissors (ring and index fingers extended). The winner of the round depends on which of the two signals is dominant:

<table>
<thead>
<tr>
<th>Hand Signal</th>
<th>Hand Signal</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper</td>
<td>Scissors</td>
<td>Scissors cut Paper: Scissors wins</td>
</tr>
<tr>
<td>Scissors</td>
<td>Rock</td>
<td>Rock blunts Scissors: Rock wins</td>
</tr>
</tbody>
</table>

If both players present the same signal, then the round is a tie.

The people programming the robot have installed a basic move-set, with the following behavior:

- The robot will always play Scissors on the first round.
- The robot never presents the same signal twice in a row.
- After playing Scissors, the robot will play Rock 25% of the time and Paper 75% of the time.
- After playing Paper, the robot will play Rock 25% of the time and Scissors 75% of the time.
- After playing Rock, the robot plays the other two moves with equal probability.

Your goal is to construct a Markov model of this robot’s sequence of moves.

(a) Draw the topology of the Markov chain. Use as few states as possible. Clearly label your nodes and transition probabilities.
(b) What is the initial state probability distribution?

(c) What is the transition matrix?

(d) Someone left the robot playing over a long weekend. When you return after the holiday, the robot has generated a very long sequence of hand signals. Using this data, you calculate the frequency of each of the hand signals. These frequencies correspond to the stationary distribution of the model. Write down the system of equations that specify the stationary distribution for this model.
(e) What is the stationary distribution?

(f) You plan a marathon match with the robot that will last 5000 rounds. On each round, you will be awarded 1 point if you win, -1 point if the robot wins, and 0 points if you tie. If you can only display one hand signal, which hand signal will maximize your score? Why? What is the expected score you will obtain with this hand signal? You may ignore transient effects from the beginning of the game.
2. Consider the same robot that you analyzed in Problem 1. You have a time lapse photography set-up that takes a snapshot of the robot once every six minutes. The robot can make a move once every two minutes during that time.

(a) Derive a transition matrix that gives the probability of observing each of the three hand signals in the next snapshot, given the hand signal in the current snapshot.

(b) What is the probability that the robot displays the same hand signal in the Round 1 and the Round 4?
(c) What is the probability that the robot displays the same hand signal in Rounds 5000 and 5003?

(d) What is the conditional probability that the robot displays Scissors, given that the hand signal the robot displayed three rounds earlier was Paper?