Study guide

This study guide is intended to help you to review for Exam II. This is not an exhaustive list of the topics covered in the class and there is no guarantee that these questions are representative of the questions on the exam. You should also review the notes you took in class, the notes and readings on the syllabus, and your homework assignments.

Markov chains

- Definitions and terminology
  - States, the state probability distribution at time $t$, the initial state probability distribution.
  - The transition probability matrix. What requirements must a matrix satisfy to be a valid transition probability matrix?
  - What is the Markov property?
  - Absorbing states, reflecting states, periodic states.

- We discussed finite-state, discrete-time, time-homogeneous Markov chains. You should understand each of these terms.

- Higher-order Markov chains: Given a transition matrix for 1 time step, you should understand how to construct a transition matrix for $n$ time steps.

- Stationary state distributions. What is the formal definition of a stationary distribution? How can you calculate the stationary distribution of a Markov chain? How can you verify that a given distribution is the stationary distribution? What properties are required for a Markov chain to have a (unique) stationary distribution? What properties prevent a Markov chain from having a stationary distribution?

- Simple random walk models. What are they? How are they related to sequence analysis?

Markov models of nucleotide substitution

- The Jukes Cantor (JC) model
  - What is the basic structure of the JC model (i.e., states, transition matrix)? What are the underlying assumptions?
  - What is the stationary distribution of the JC model?
  - How is the rate parameter of the JC model related to the overall substitution rate?

- More complex models of nucleotide substitution with non-uniform transition probabilities
The Kimura 2 parameter (K2P) assumes that transitions and transversions occur at different rates. (What are transitions and transversions?) Like JC, the K2P model has a uniform stationary distribution.

- More complex models with non-uniform stationary distributions
  - Both the JC and the K2P models have uniform stationary distributions. This distribution is an implicit consequence of the symmetric structure of the transition matrices of these models.
  - In contrast, the Felsenstein model assumes all substitutions proceed at the same rate, but allows for different underlying base frequencies. How is the transition matrix in the Felsenstein model modified to achieve this?

- The Hasegawa, Kishino, Yano (HKY) model combines the innovations of the K2P and Felsenstein models to give a matrix that has separate rates for transitions and transversions and allows for non-uniform base frequencies.

- More complex models allow three or more rates. The most complex of the models within this framework is the General Time Reversible (GTR) model. The GTR allows for a different rate for each of the six possible substitutions and an arbitrary stationary distribution.

- Given a set of non-uniform base frequencies and a transition matrix that implies uniform base frequencies, can you construct a new model that has the same rate structure as the original transition matrix, but with the specified set of non-uniform base frequencies? For example, given an instance of the JC model, could you turn it into an instance of the Felsenstein model?

- Limitations: Properties of sequence evolution that are not captured by the models we learned in class include interactions between different sites in the same sequence, different rates at different sites (site-dependent rate variation) and changes in rate over time (time-dependent rate variation).

Amino acid substitution models and matrices

- Deriving amino acid substitution matrices: overview
  - Substitution models should reflect biophysical properties. Pairs of residues with similar properties represent conservative replacements and should have higher similarity scores than pairs of residues with different properties, which represent non-conservative replacements.
  - Substitution matrices should be parameterized by evolutionary divergence.
  - Given the greater number and variety of the amino acids, compared with nucleotides, amino acid substitution models rely more heavily on learning parameters from data than nucleotide models.
We considered two families of amino acid substitution matrices: the PAM matrices and the BLOSUM matrices. Both families were derived according to the following general approach, although the details of each step differ between the two methods.

1. Use a set of “trusted” multiple sequence alignments (ungapped) to infer model parameters.
2. Count observed amino acid pairs in the trusted alignments, correcting for sample bias.
3. Estimate substitution frequencies from amino acid pair counts.
4. Construct a log odds scoring matrix from substitution frequencies.

• The PAM model
  
  – The Dayhoff Markov model of amino acid replacement.
    
    * Dayhoff’s PAM matrices are derived from a Markov model of amino acid replacement. What is the basic structure of this model?
    * The unit of divergence used is the PAM or “percent accepted mutation”. How is the PAM defined?
    * What are the properties of the data that Dayhoff used to obtain amino acid pair counts for her model? How are those properties related to the underlying assumptions of the Markov chain strategy that she used?
    * How did Dayhoff derive counts from that data set and how did she account for potential sample bias in her data?
    * How did Dayhoff use the amino acid counts to derive the PAM transition matrix? How does this derivation account for differences in amino acid frequency and amino acid mutability?
    * How did Dayhoff ensure that her basic model corresponds to one PAM of divergence?
    * How is the PAM-N model derived from the PAM-1 model?
    * How are multiple substitutions accounted for in the PAM framework?
  
  – The PAM substitution matrices
    
    * How are the PAM log odds substitution matrices derived from the Dayhoff Markov model transition matrices?
    * The transition matrices are not symmetric. The substitution matrices are symmetric. What is the biological intuition associated with this observation?

• BLOSUM matrices
  
  – What are the properties of the data that the Henikoffs used to obtain amino acid pair counts for the BLOSUM matrices? What are the major differences between the data used for the BLOSUM matrices and the data used for the PAM matrices?
  
  – Partitioning sequences into clusters based on percent identity is a key aspect of the BLOSUM method.
• Log odds substitution matrices: Both the PAM and BLOSUM substitution matrices are log-odds matrices. You should understand and be able to work with the log odds substitution matrix framework.
  – When a log odds substitution matrix is used to score an alignment, the alignment score corresponds to a log likelihood ratio; what does this mean?
  – How should a positive element in a substitution matrix be interpreted in this context?
  – How should a negative element in a substitution matrix be interpreted in this context?
  – When comparing the main diagonal elements of matrices representing different amounts of divergence, what trends would you expect to see?
  – When comparing the off-diagonal elements of matrices representing different amounts of divergence, what trends would you expect to see?
• What are the similarities and differences
  – between the Jukes Cantor, Kimura 2 Parameter, and Felsenstein models?
  – between the Jukes Cantor and PAM models?
  – between the PAM and BLOSUM models/matrices?
• Information content of a substitution matrix
  – What is the relative entropy of a matrix?
  – How is the relative entropy of a matrix related to the log-odds formalism?
  – How does the information content of a matrix vary with evolutionary divergence?