15-462 Computer Graphics I

Lecture 17

Spatial Data Structures

Hierarchical Bounding Volumes
Regular Grids
Octrees
BSP Trees
Constructive Solid Geometry (CSG)

[Angel 9.10]

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Announcements

- Ray tracing handouts online (CMU access only)
  - Supplements text
- Last written assignment out tomorrow
  - Due Tuesday, Nov 18.
- Last programming assignment
  - Ray tracing
  - Out Tuesday, Nov 18.
  - Due Tuesday, Dec 2.

Ray Tracing Acceleration

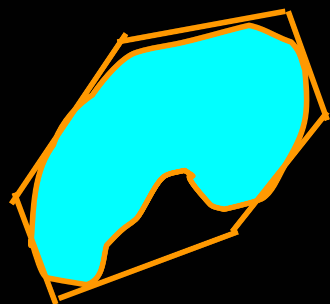
- Faster intersections
  - Faster ray-object intersections
    - Object bounding volume
    - Efficient intersectors
  - Fewer ray-object intersections
    - Hierarchical bounding volumes (boxes, spheres)
    - Spatial data structures
    - Directional techniques
- Fewer rays
  - Adaptive tree-depth control
  - Stochastic sampling
- Generalized rays (beams, cones)

Spatial Data Structures

- Data structures to store geometric information
- Sample applications
  - Collision detection
  - Location queries
  - Chemical simulations
  - Rendering
- Spatial data structures for ray tracing
  - Object-centric data structures (bounding volumes)
  - Space subdivision (grids, octrees, BSP trees)
  - Speed-up of 10x, 100x, or more

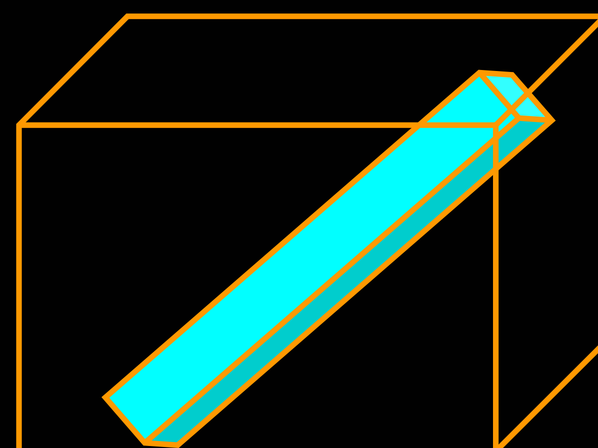
Bounding Volumes

- Wrap complex objects in simple ones
- Does ray intersect bounding box?
  – No: does not intersect enclosed objects
  – Yes: calculate intersection with enclosed objects
- Common types
  – Boxes, axis-aligned
  – Boxes, oriented
  – Spheres
  – Finite intersections or unions of above



Selection of Bounding Volumes

- Effectiveness depends on:
  – Probability that ray hits bounding volume, but not enclosed objects (tight fit is better)
  – Expense to calculate intersections with bounding volume and enclosed objects
- Amortize calculation of bounding volumes
- Use heuristics



Hierarchical Bounding Volumes

- With simple bounding volumes, ray casting still has requires O(n) intersection tests
- Idea: use tree data structure
  - Larger bounding volumes contain smaller ones etc.
  - Sometimes naturally available (e.g. human figure)
  - Sometimes difficult to compute
- Often reduces complexity to O(log(n))

Ray Intersection Algorithm

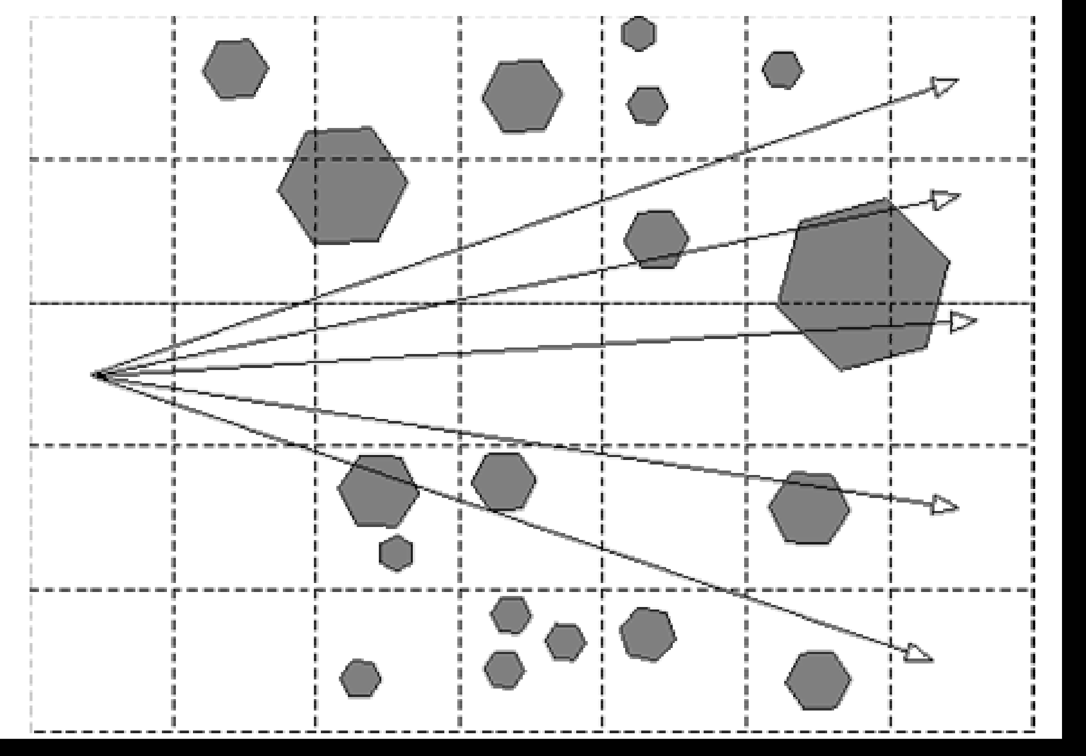
- Recursively descend tree
- If ray misses bounding volume, no intersection
- If ray intersects bounding volume, recurse with enclosed volumes and objects
- Maintain near and far bounds to prune further
- Overall effectiveness depends on model and constructed hierarchy

Spatial Subdivision

- Bounding volumes enclose objects, recursively
- Alternatively, divide space
- For each segment of space keep list of intersecting surfaces or objects
- Basic techniques
  - Regular grids
  - Octrees (axis-aligned, non-uniform partition)
  - BSP trees (recursive Binary Space Partition, planes)

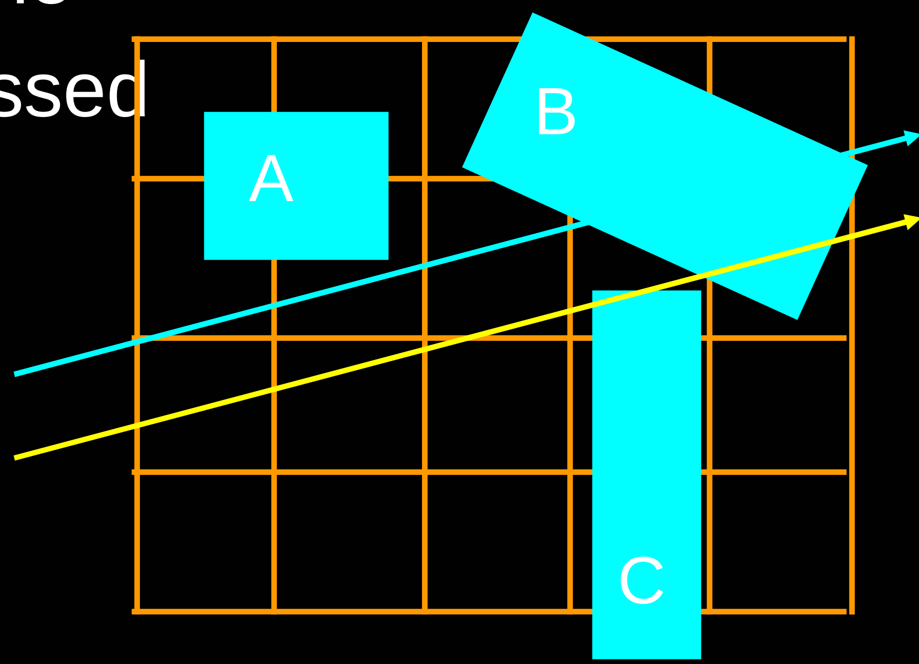
Grids

- 3D array of cells (voxels) that tile space
- Each cell points to all intersecting surfaces
- Intersection alg steps from cell to cell



Caching Intersection points

- Objects can span multiple cells
- For A need to test intersection only once
- For B need to cache intersection and check next cell for closer one
- If not, C could be missed
  (yellow ray)



Assessment of Grids

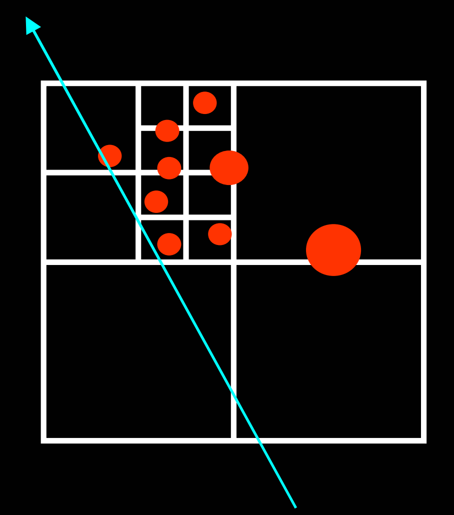
- Poor choice when world is non-homogeneous
- Size of grid
  - Too small: too many surfaces per cell
  - Too large: too many empty cells to traverse
  - Can use alg like Bresenham's for efficient traversal
- Non-uniform spatial subdivision more flexible
  - Can adjust to objects that are present

Outline

- Hierarchical Bounding Volumes
- Regular Grids
- Octrees
- BSP Trees
- Constructive Solid Geometry (CSG)

Quadtrees

- Generalization of binary trees in 2D
  - Node (cell) is a square
  - Recursively split into 4 equal sub-squares
  - Stop subdivision based on number of objects
- Ray intersection has to traverse quadtree
- More difficult to step to next cell



Octrees

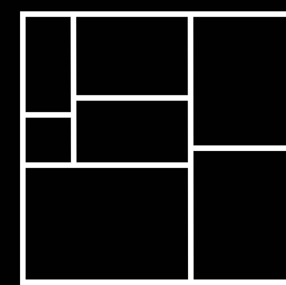
- Generalization of quadtree in 3D
- Each cell may be split into 8 equal sub-cells
- Internal nodes store pointers to children
- Leaf nodes store list of surfaces
- Adapts well to inhomogeneous scenes

Assessment for Ray Tracing

- Grids
  - Easy to implement
  - Require a lot of memory
  - Poor results for inhomogeneous scenes
- Octrees
  - Better on most scenes (more adaptive)
- Alternative: nested grids
- Spatial subdivision expensive for animations
- Hierarchical bounding volumes
  - Natural for hierarchical objects
  - Better for dynamic scenes

Other Spatial Subdivision Techniques

- Relax rules for quadtrees and octrees
- k-dimensional tree (k-d tree)
  - Split at arbitrary interior point
  - Split one dimension at a time



- Binary space partitioning tree (BSP tree)
  - In 2 dimensions, split with any line
  - In k dims. split with k-1 dimensional hyperplane
  - Particularly useful for painter's algorithm
  - Can also be used for ray tracing [see handout]

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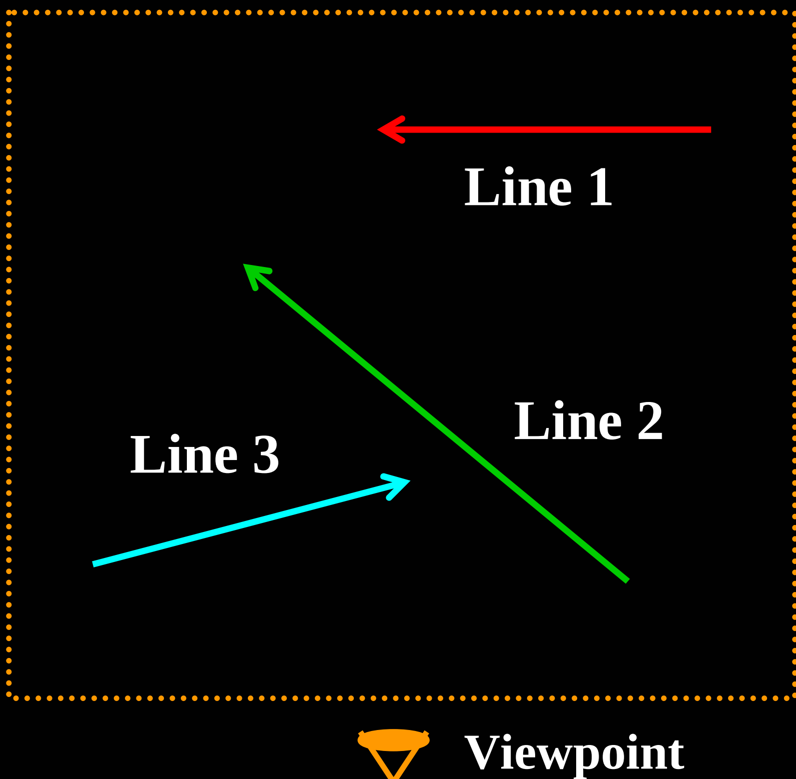
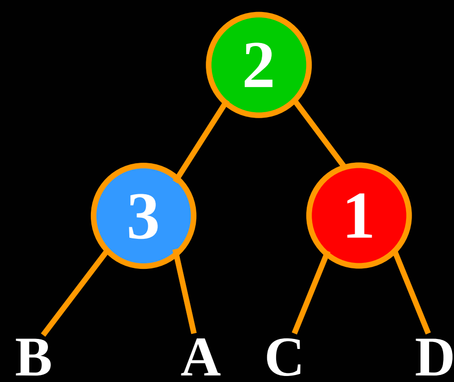
BSP Trees

- Split space with any line (2D) or plane (3D)
- Applications
  - Painters algorithm for hidden surface removal
  - Ray casting
- Inherent spatial ordering given viewpoint
  - Left subtree: in front, right subtree: behind
- Problem: finding good space partitions
  - Proper ordering for any viewpoint
  - Balance tree
- For details, see http://reality.sgi.com/bspfaq/

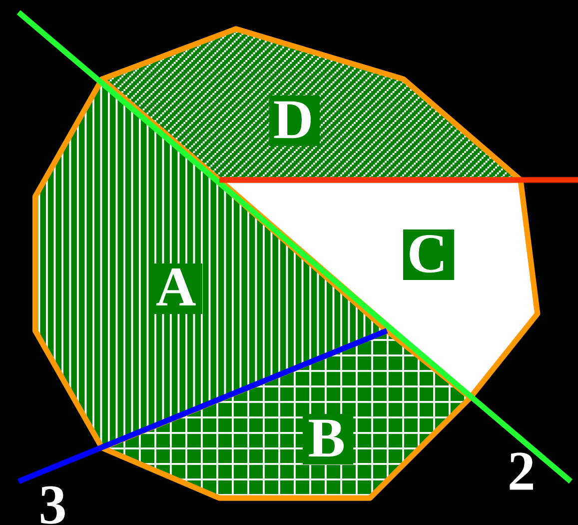
Building a BSP Tree

- Use hidden surface removal as intuition
- Using line 1 or line 2 as root is easy





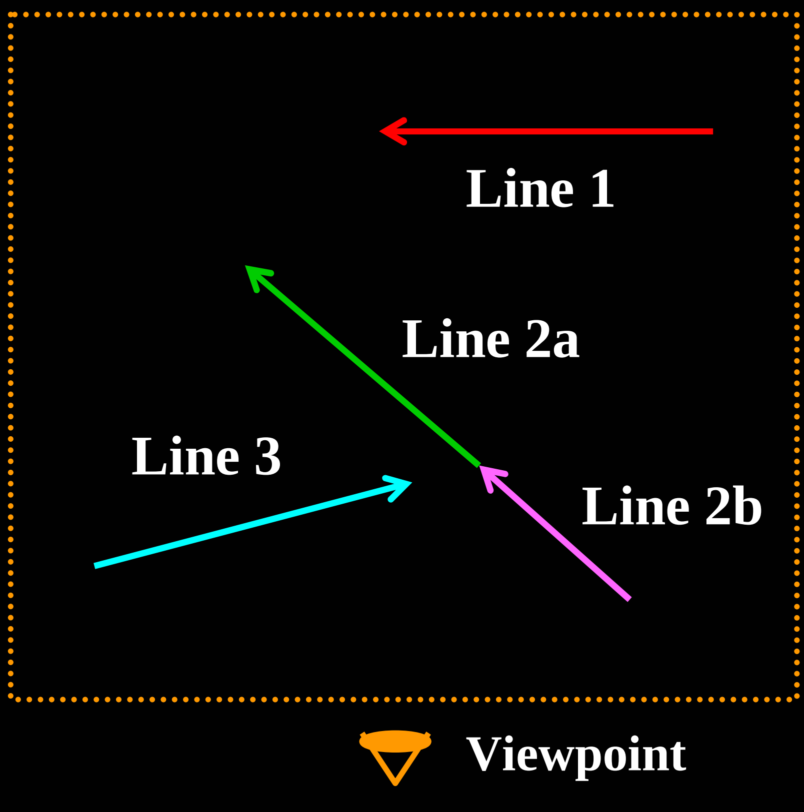
a BSP tree using 2 as root

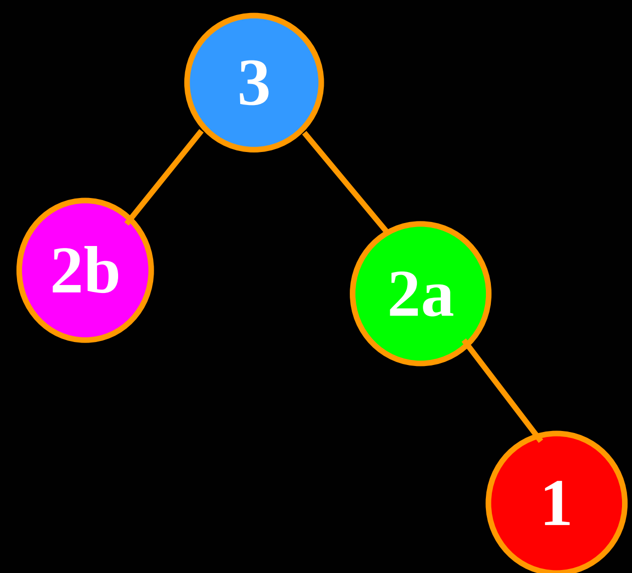


the subdivision of space it implies

Splitting of surfaces

- Using line 3 as root requires splitting





Building a Good Tree

- Naive partitioning of n polygons yields $O(n^3)$ polygons (in 3D)
- Algorithms with $O(n^2)$ increase exist
  - Try all, use polygon with fewest splits
  - Do not need to split exactly along polygon planes
- Should balance tree
  - More splits allow easier balancing
  - Rebalancing?

Painter’s Algorithm with BSP Trees

- Building the tree
  - May need to split some polygons
  - Slow, but done only once
- Traverse back-to-front or front-to-back
  - Order is viewer-direction dependent
  - What is front and what is back of each line changes
  - Determine order on the fly

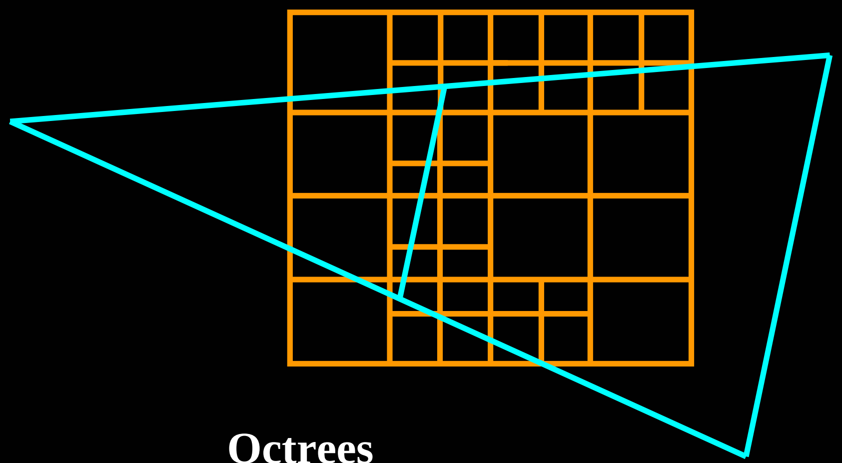
Details of Painter’s Algorithm

- Each face has form Ax + By + Cz + D
- Plug in coordinates and determine
  - Positive: front side
  - Zero: on plane
  - Negative: back side
- Back-to-front: inorder traversal, farther child first
- Front-to-back: inorder traversal, near child first
- Do backface culling with same sign test
- Clip against visible portion of space (portals)

Clipping With Spatial Data Structures

- Accelerate clipping
  - Goal: accept or rejects whole sets of objects
  - Can use an spatial data structures
- Scene should be mostly fixed
  - Terrain fly-through
  - Gaming

Hierarchical bounding volumes



Octrees

Data Structure Demos

- BSP Tree construction
  http://symbolcraft.com/graphics/bsp/index.html
- KD Tree construction
  http://www.rolemaker.dk/nonRoleMaker/uni/algogem/kdtree.htm

Real-Time and Interactive Ray Tracing

- Interactive ray tracing via space subdivision
  http://www.cs.utah.edu/~reinhard/egwr/
- Interactive ray tracing with good hardware
  http://www.cs.utah.edu/vissim/projects/raytracing/

Outline

- Hierarchical Bounding Volumes
- Regular Grids
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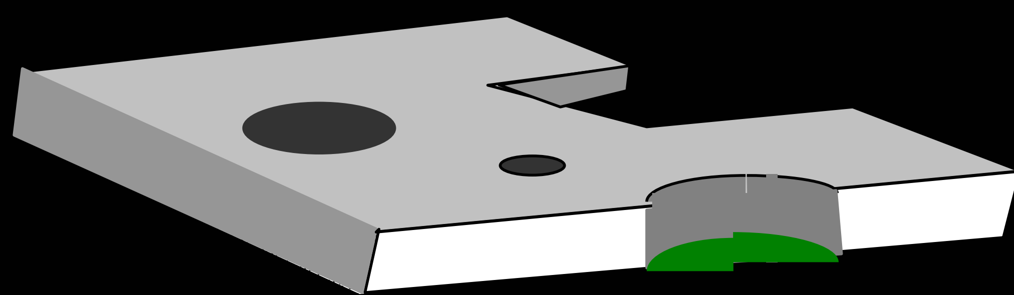
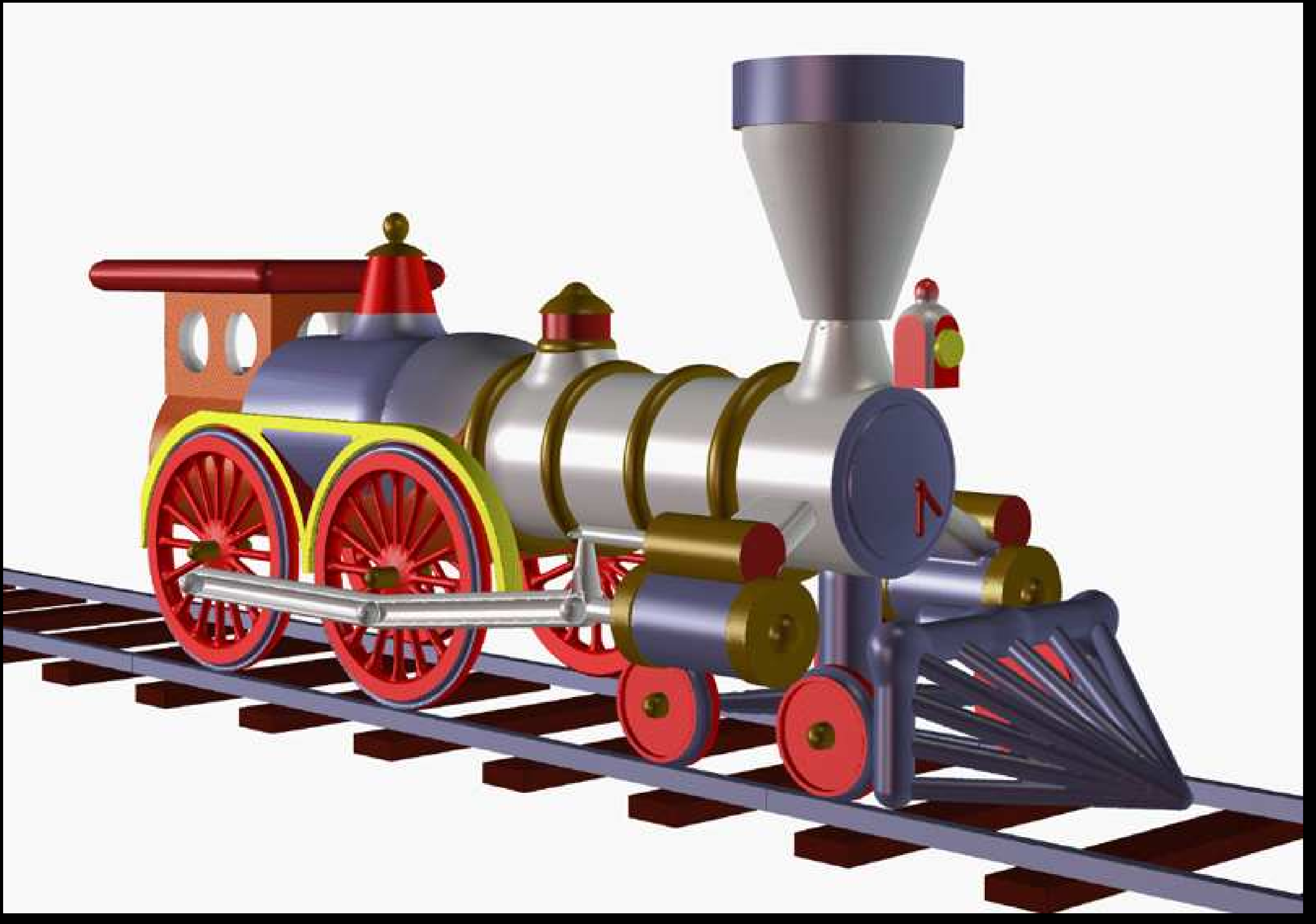
- Constructive Solid Geometry (CSG)

Constructive Solid Geometry (CSG)

- Generate complex shapes with simple building blocks (boxes, spheres, cylinders, cones, ...)
- Particularly applicable for machined objects
- Efficient with ray tracing



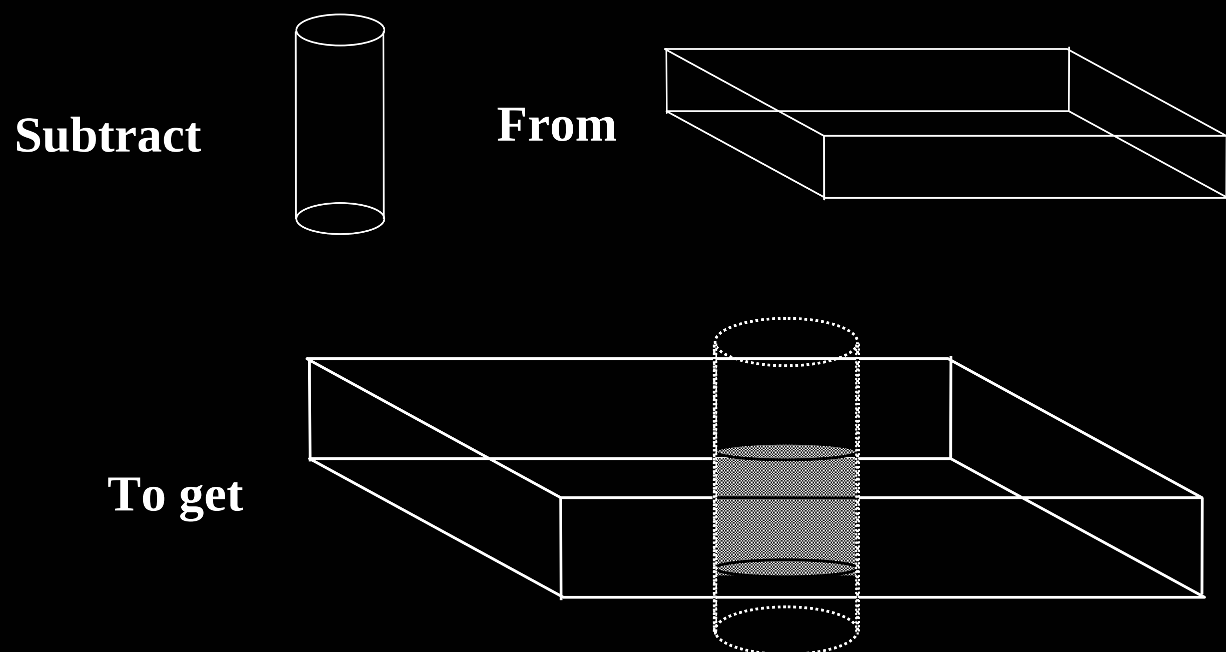
Example: A CSG Train



Brian Wyvill et al., U. of Calgary

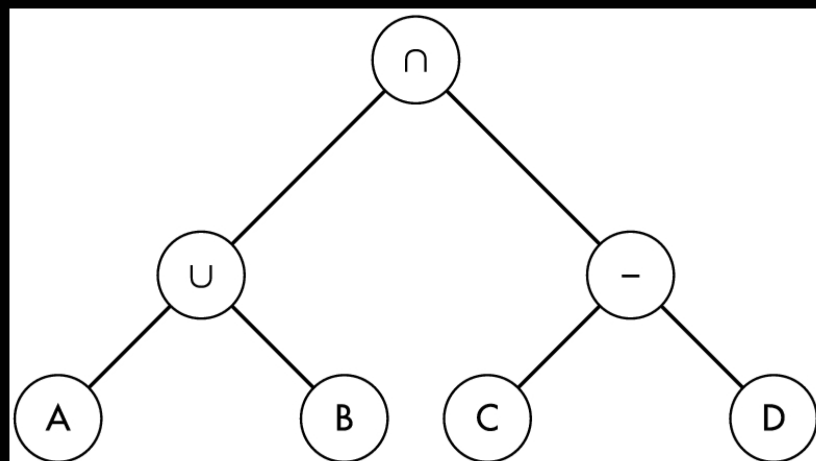
Boolean Operations

- Intersection and union
- Subtraction
  - Example: drilling a hole



CSG Trees

- Set operations yield tree-based representation



- Use these trees for ray/objects intersections
- Think about how!

Implicit Functions for Booleans

- Solid as implicit function, $F(x,y,z)$
  - $F(x, y, z) < 0$ interior
  - $F(x, y, z) = 0$ surface
  - $F(x, y, z) > 0$ exterior
- For CSG, use $F(x, y, z) \in \{-1, 0, 1\}$
- $F_{A\,\mathring{A}\,B}(\mathbf{p}) = \max(F_A(\mathbf{p}), F_B(\mathbf{p}))$
- $F_{A \cup B}(\mathbf{p}) = \min(F_A(\mathbf{p}), F_B(\mathbf{p}))$
- $F_{A - B}(\mathbf{p}) = \max(F_A(\mathbf{p}), -F_B(\mathbf{p}))$

Summary

- Hierarchical Bounding Volumes
- Regular Grids
- Octrees
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- Constructive Solid Geometry (CSG)

Preview

- Radiosity
- Ray tracing handouts online (CMU access only)
  - Supplements text
- Last written assignment out tomorrow
  - Due Tuesday, Nov 18.