

# Announcements

Assignment 1 due Friday at midnight

Written Assignment 1 out Thursday on the web

Questions on Assignment 1?

# Hierarchical Modeling

A lesson in stick person anatomy.  
or  
Choosing the right parameters.  
Hierarchical transformations.  
The matrix stack.

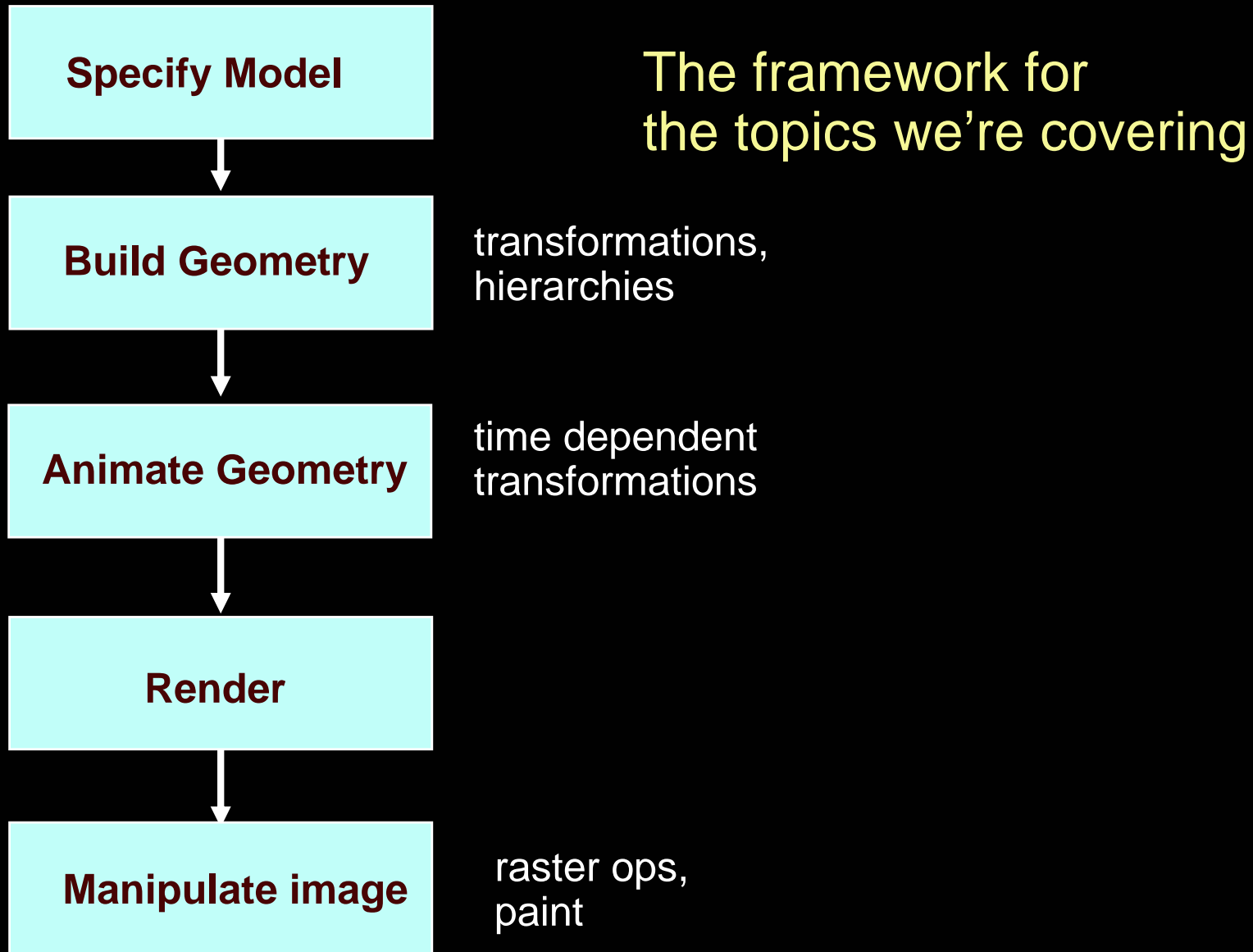
See Angel 9.1-9.7



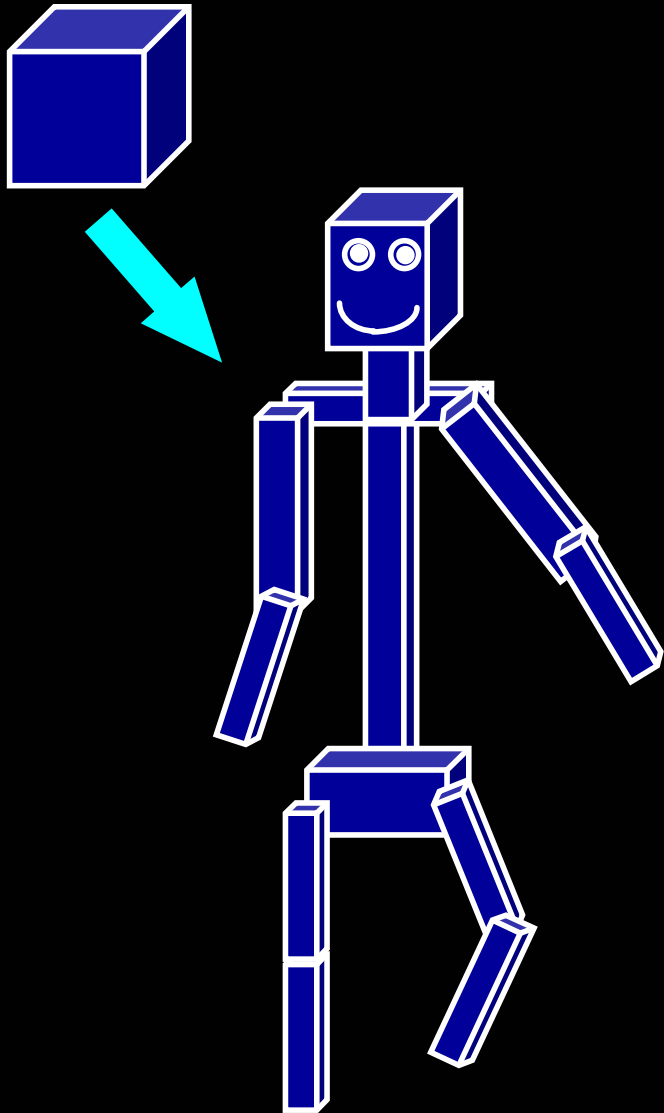
**COMPUTER GRAPHICS 1**  
**15-462**

13 Sept 2001

# Staying Oriented (in the course)

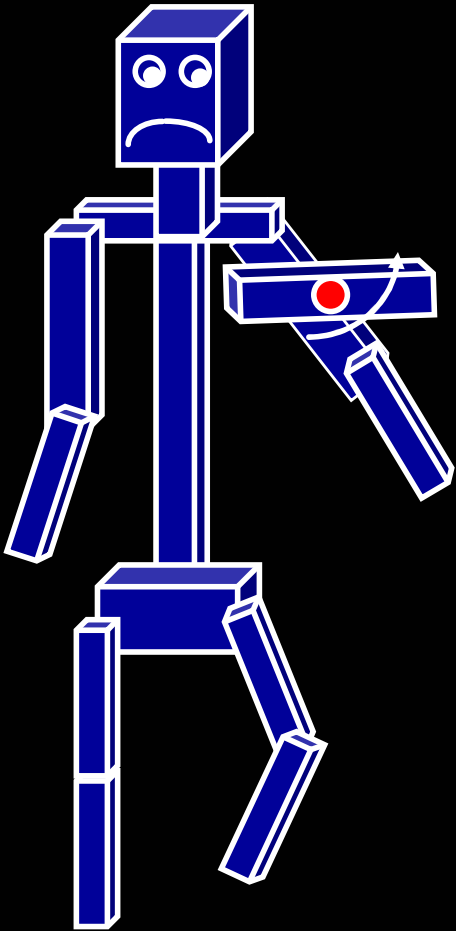


# Modeling with Transformations



- You've learned everything you need to know to make a stick person out of cubes.
- Just translate, rotate, and scale each one to get the right size, shape, position, and orientation.
- Looks great--until you try to make it move.

# The Right Control Knobs



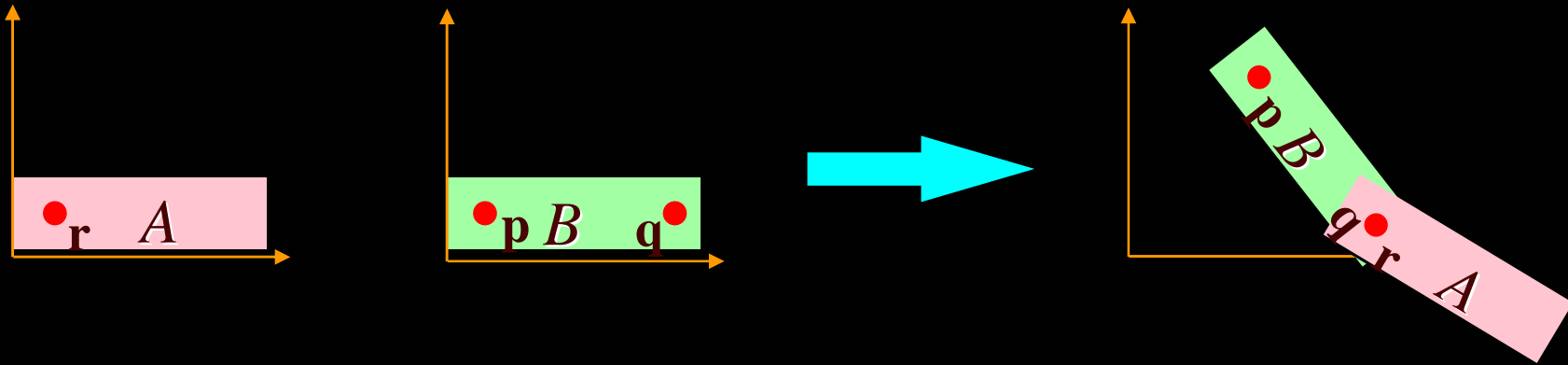
- As soon as you want to change something, the model falls apart
- Reason: the thing you're modeling is *constrained* but your model doesn't know it
- What we need:
  - some sort of representation of *structure*
  - a set of “control knobs” (parameters) that make it easy to move our stick person through *legal configurations*
- This kind of control is convenient for static models, and *vital* for animation!
- Key is to structure the transformations in the right way: **using a hierarchy**

# Hierarchical Modeling Example



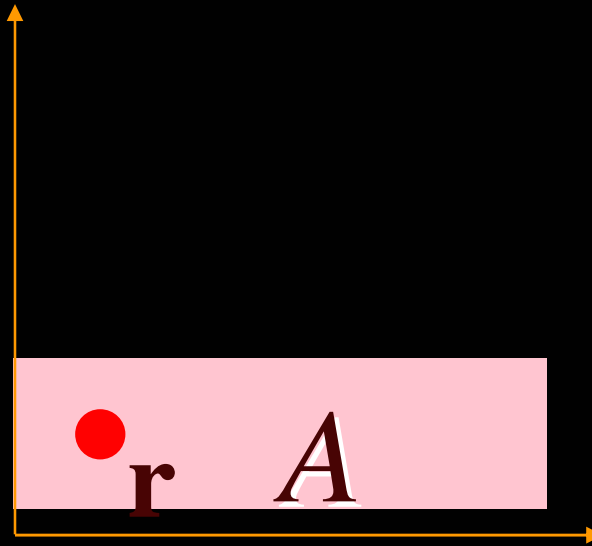
**"Number One" Playgroup - Duran Duboi**  
**Issue 141: SIGGRAPH 2002 Electronic Theater Program**

# Making an Articulated Model



- A minimal 2-D jointed object:
  - Two pieces,  $A$  (“forearm”) and  $B$  (“upper arm”)
  - Attach point  $q$  on  $B$  to point  $r$  on  $A$  (“elbow”)
  - Desired control knobs:
    - »  $T$ : shoulder position (point at which  $p$  winds up)
    - »  $u$ : shoulder angle ( $A$  and  $B$  rotate together about  $p$ )
    - »  $v$ : elbow angle ( $A$  rotates about  $r$ , which stays attached to  $q$ )

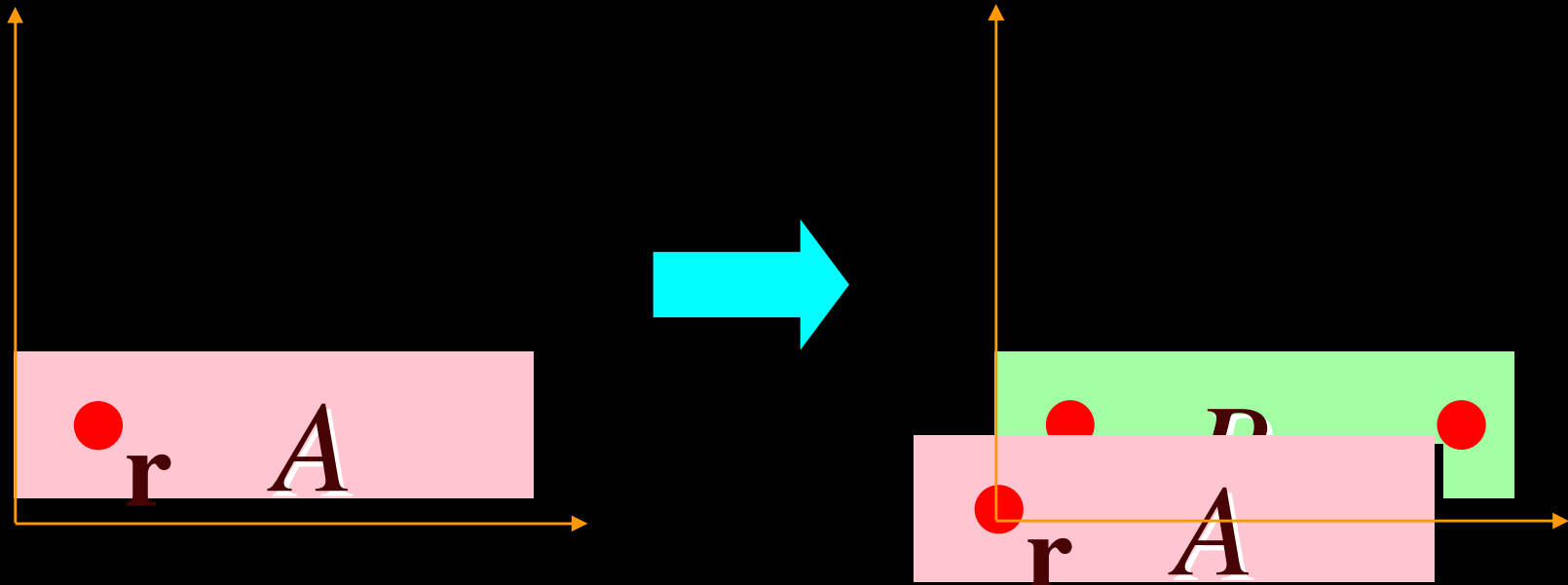
## Making an Arm, step 1



- Start with  $A$  and  $B$  in their untransformed configurations ( $B$  is hiding behind  $A$ )
- First apply a series of transformations to  $A$ , leaving  $B$  where it is...

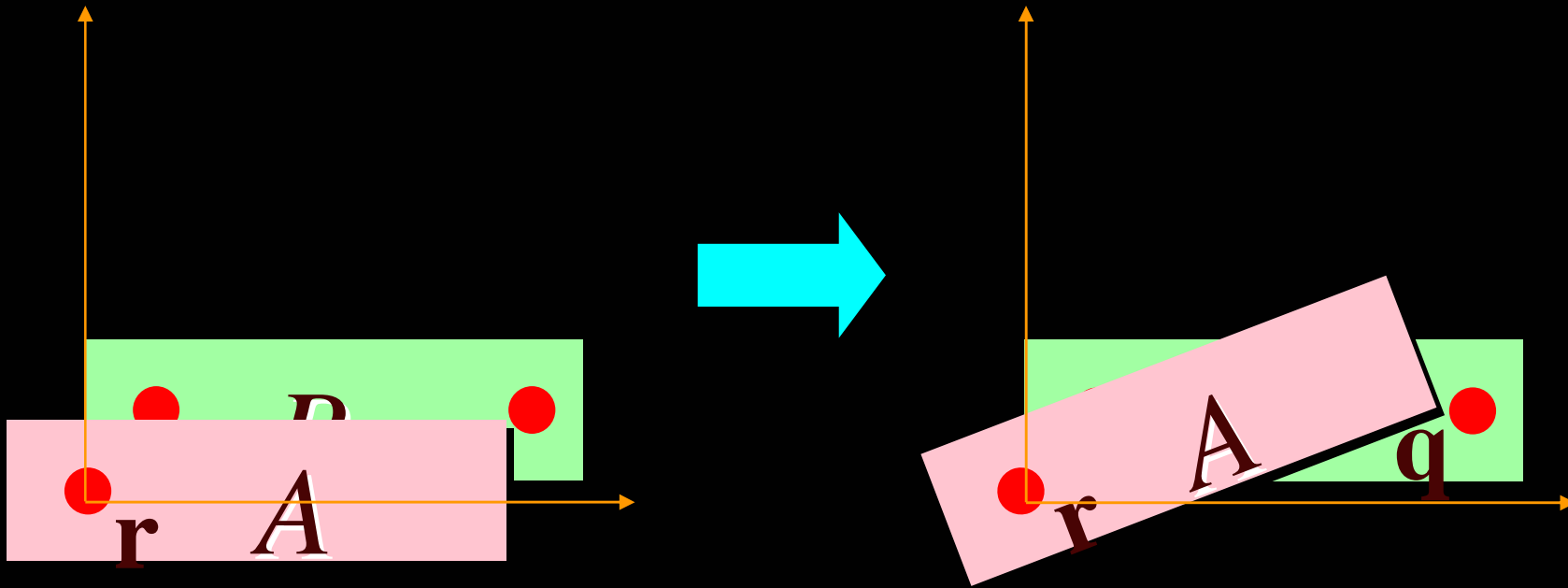


## Making an Arm, step 2



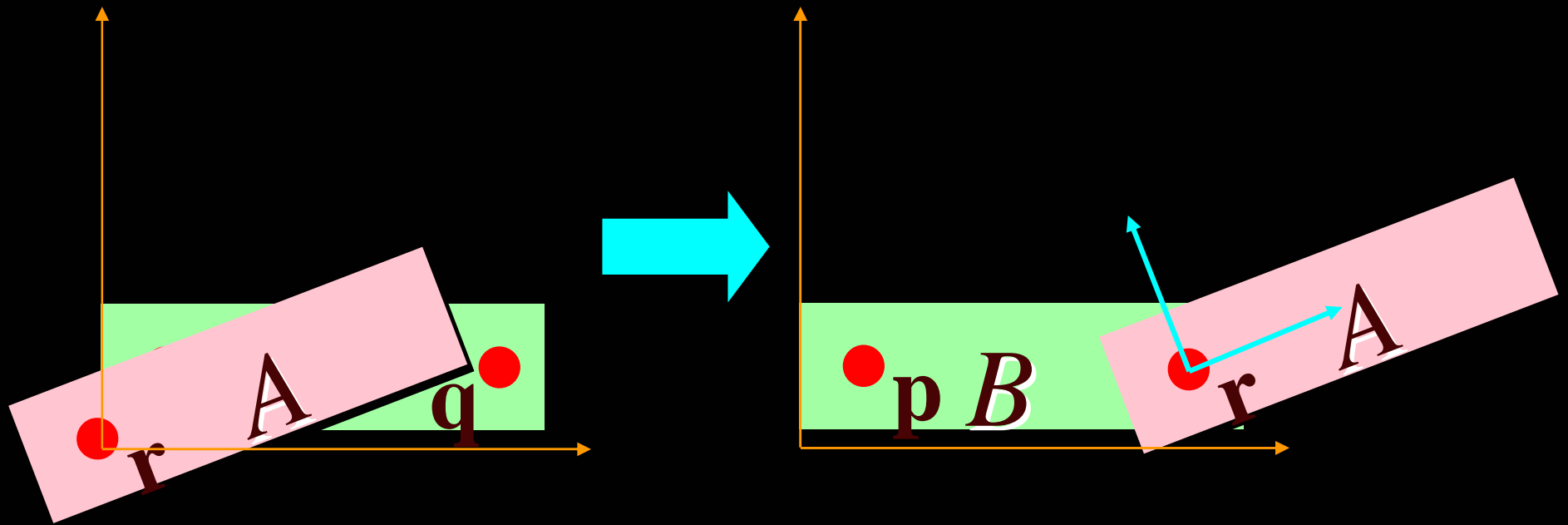
- Translate by  $-r$ , bringing  $r$  to the origin
- You can now see  $B$  peeking out from behind  $A$

## Making an Arm, step 3



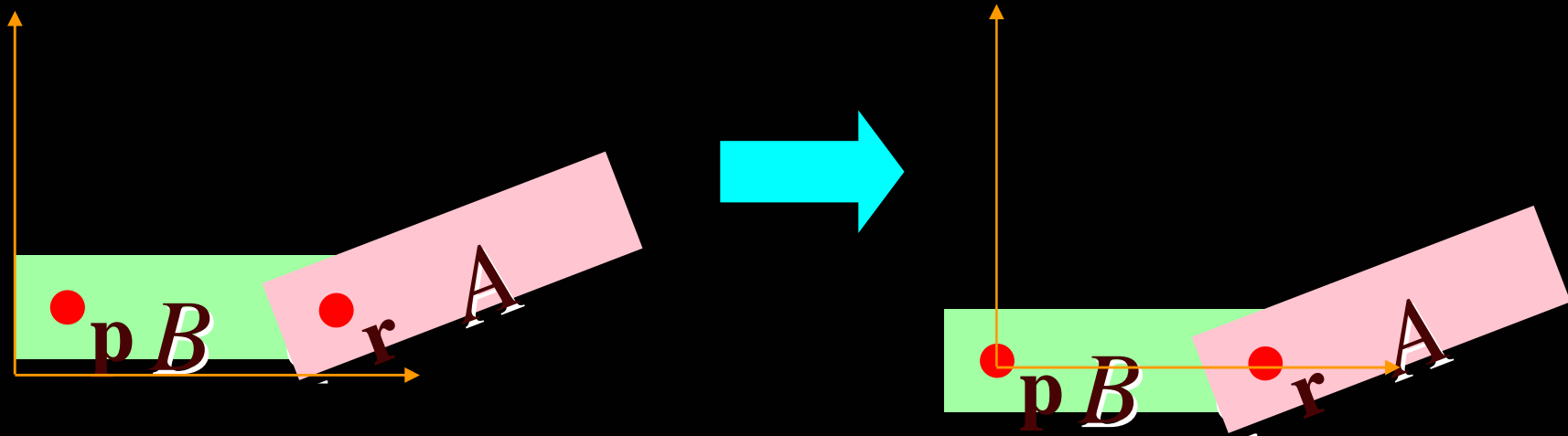
- Next, we rotate  $A$  by  $v$  (the “elbow” angle)

## Making an Arm, step 4



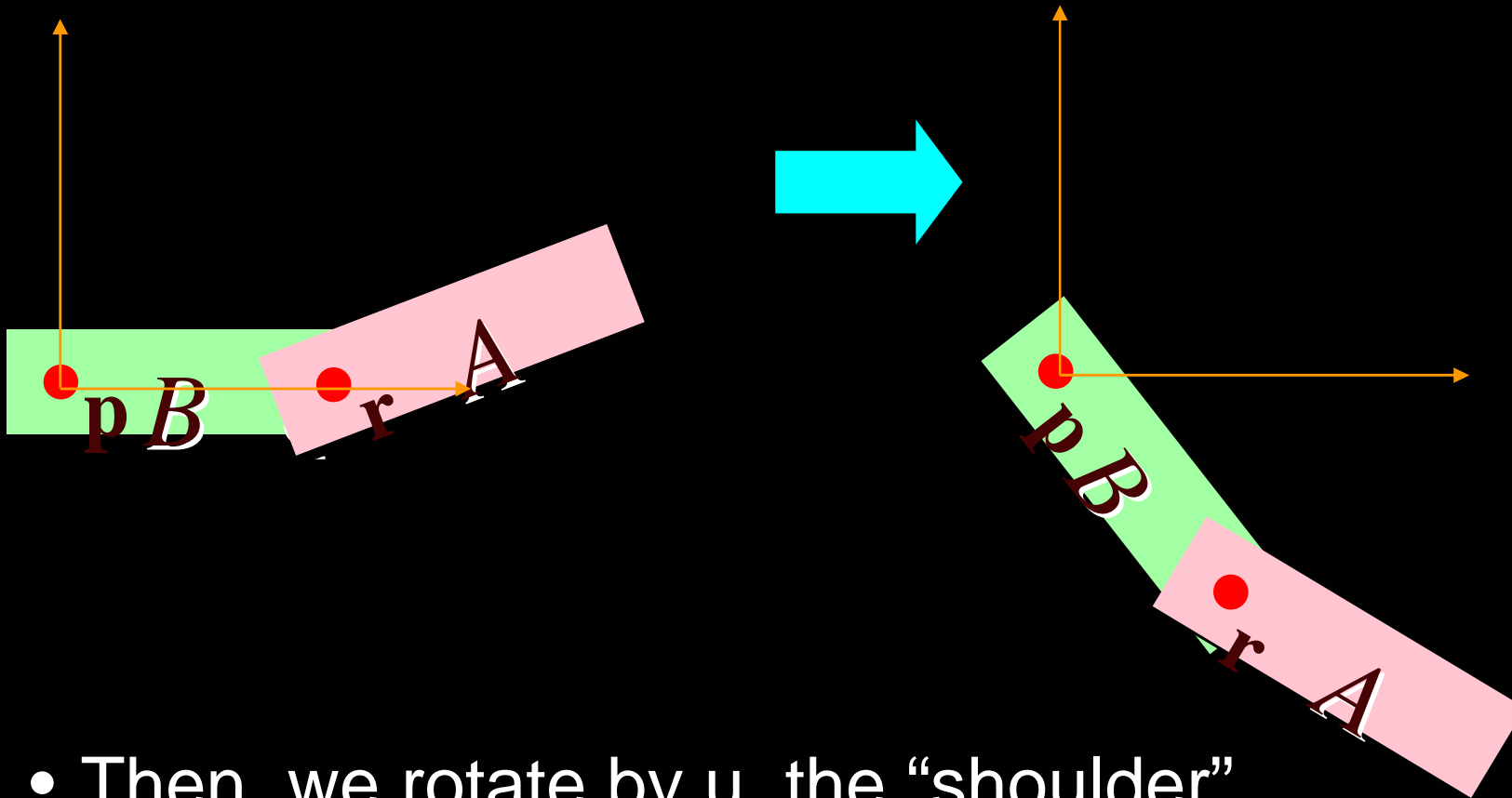
- Translate  $A$  by  $q$ , bringing  $r$  and  $q$  together to form the elbow joint
- We can regard  $q$  as the origin of the *lower arm coordinate system*, and regard  $A$  as being in this coordinate system.

## Making an Arm, step 5



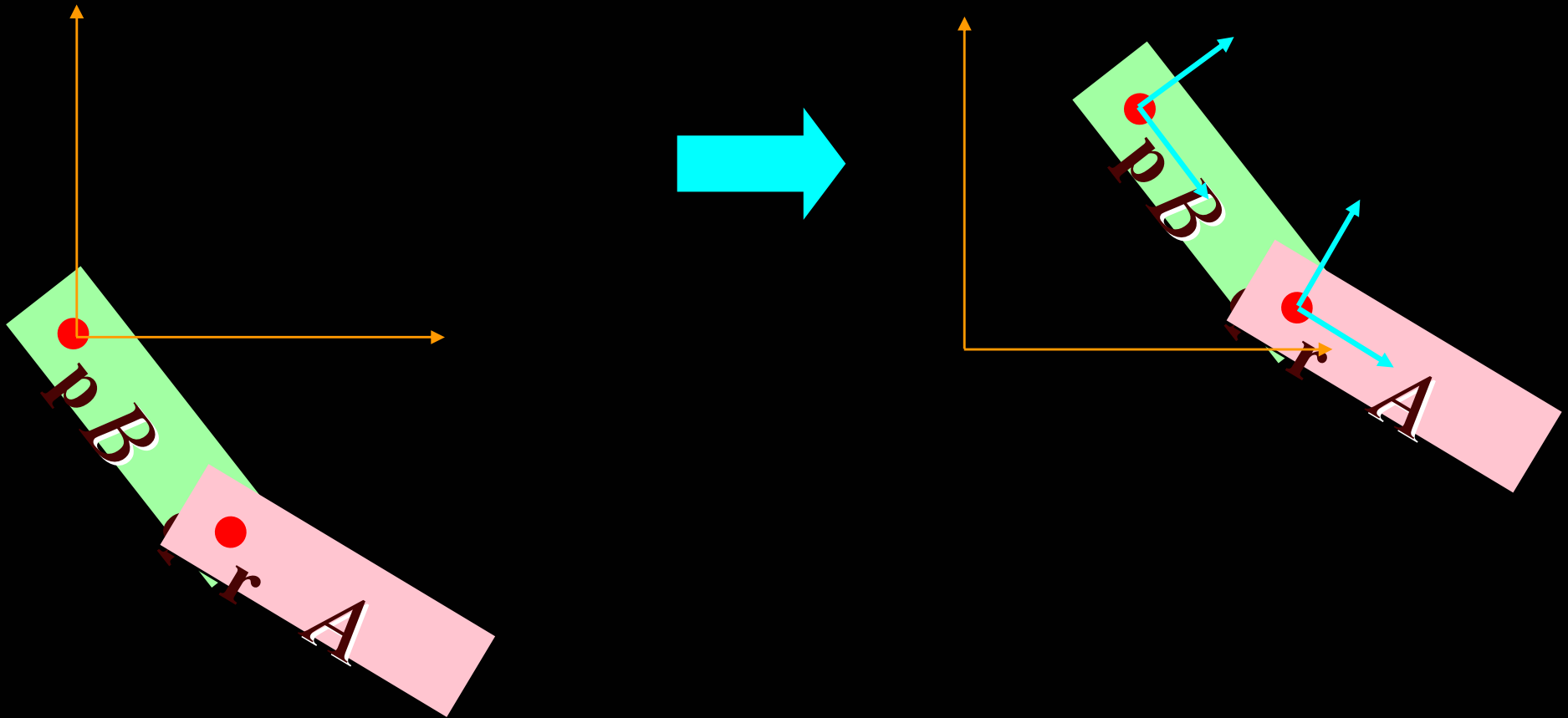
- From now on, each transformation applies to *both*  $A$  and  $B$  (This is important!)
- First, translate by  $-p$ , bringing  $p$  to the origin
- $A$  and  $B$  both move together, so the elbow doesn't separate!

## Making an Arm, step 6



- Then, we rotate by  $u$ , the “shoulder” angle
- Again,  $A$  and  $B$  rotate together

## Making an Arm, step 7

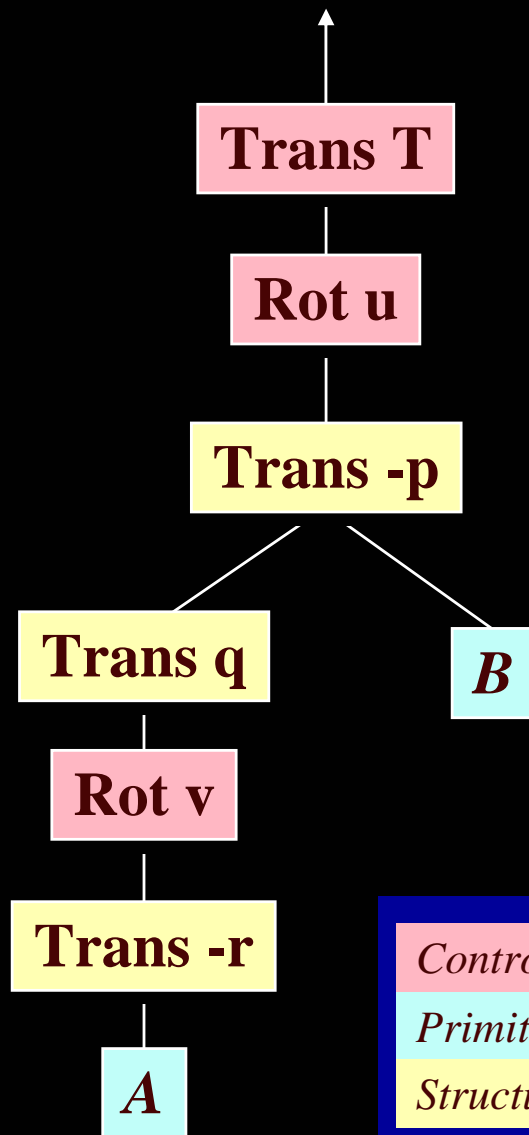


- Finally, translate by  $T$ , bringing the arm where we want it
- $p$  is at origin of *upper arm coordinate system*

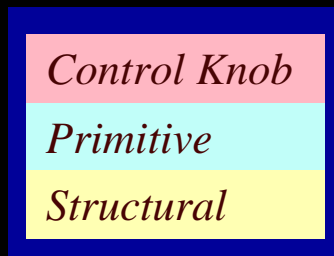
## So What Have We Done?

- Seems more complicated than just translating and rotating each piece separately
- But the model is easy to modify/animate:
  - Remember the transformation sequence, and the parameters you used—they're part of the model.
  - Whenever the parameters change, reapply all of the transformations and draw the result
    - » The model will not fall apart!!!
- Note:
  - $u$ ,  $v$ , and  $T$  are parameters of the model.
  - but  $p$ ,  $q$ , and  $r$  are *structural constants*.
  - Changing  $u, v$ , or  $T$  wiggles the arm
  - Changing  $p, q$ , or  $r$  *dismembers it* (useful only in video games!)

# Transformation Hierarchies

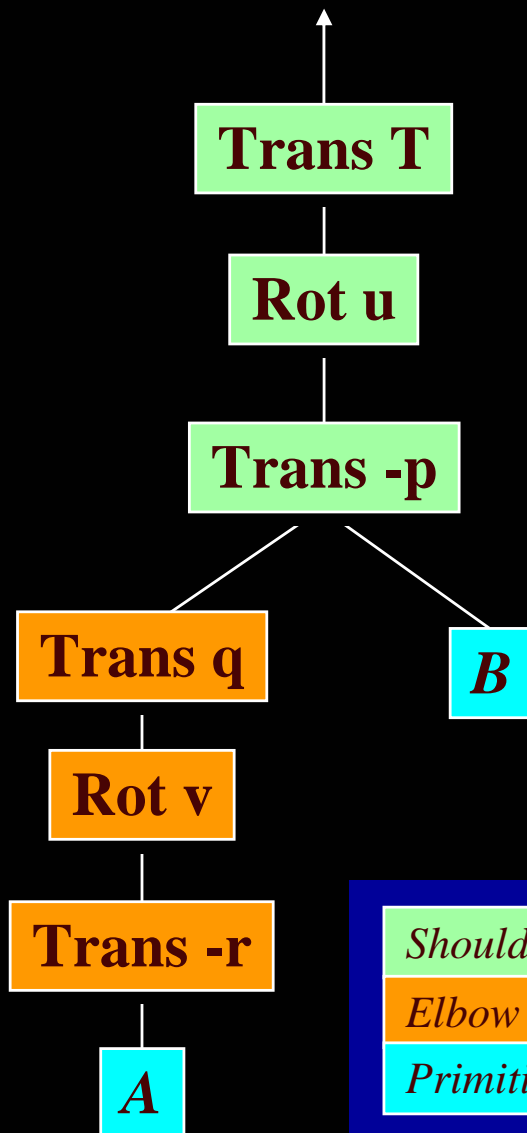


- This is the build-an-arm sequence, represented as a tree
- Interpretation:
  - Leaves are geometric primitives
  - Internal nodes are transformations
  - Transformations apply to everything under them—start at the bottom and work your way up
- You can build a wide range of models this way

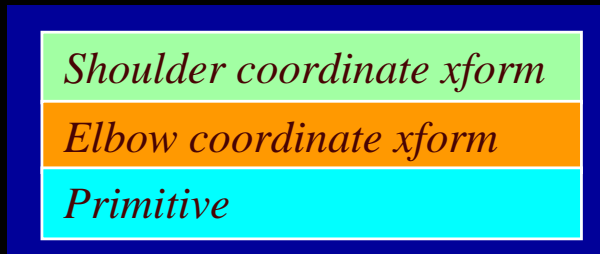




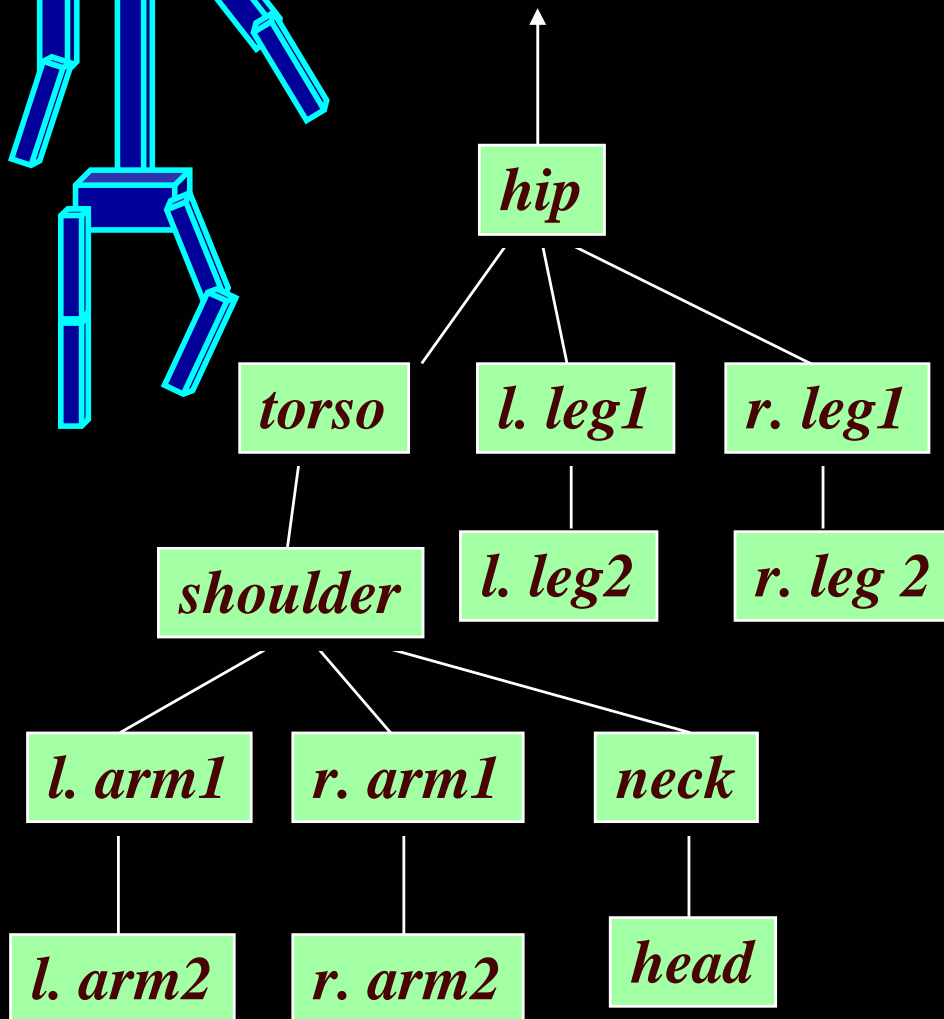
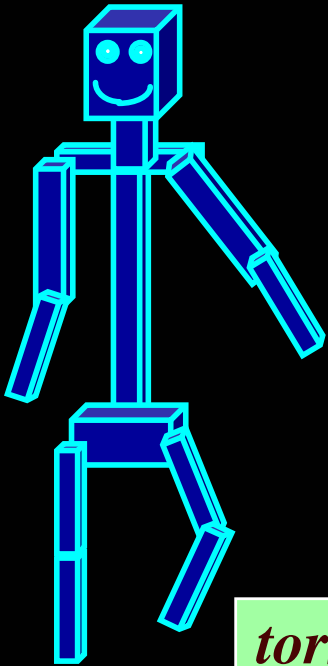
# Transformation Hierarchies



- Another point of view:
- The shoulder coordinate transformation moves everything below it with respect to the shoulder:
  - B
  - A and its transformation
- The elbow coordinate transformation moves A with respect to the shoulder coordinate transform

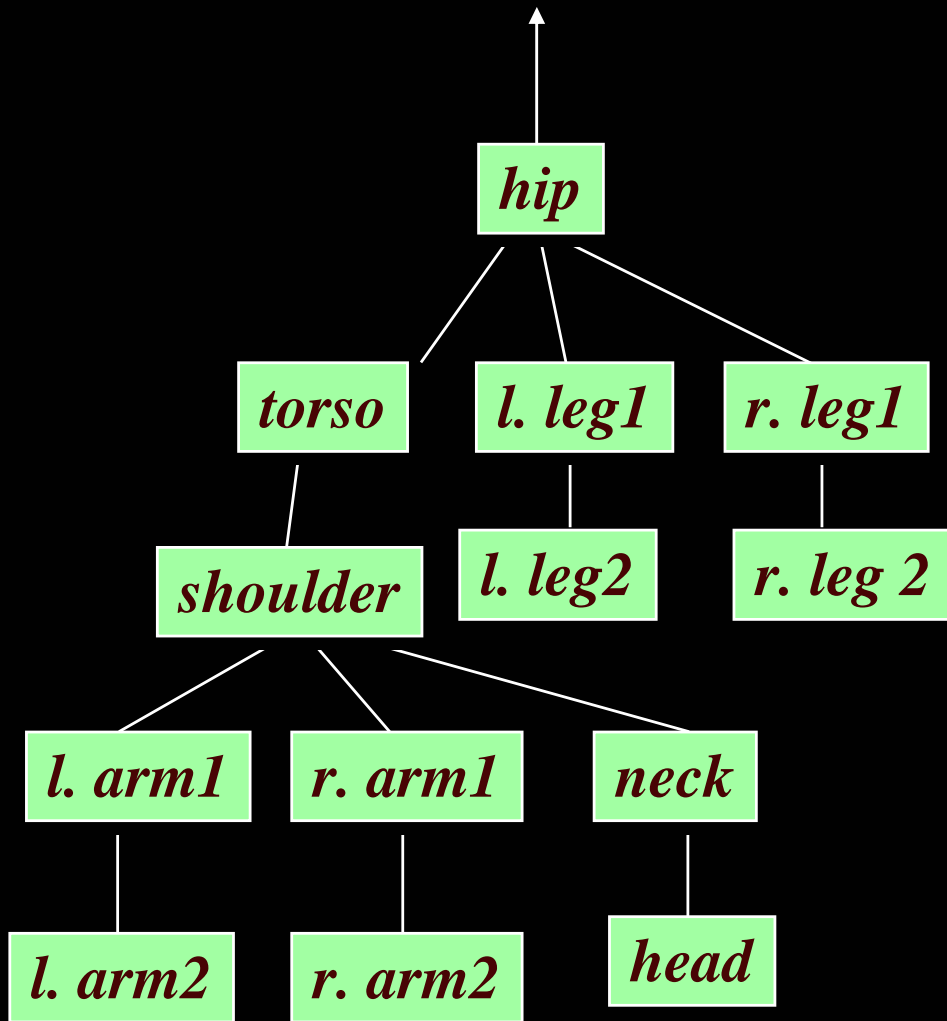


# A Schematic Humanoid



- Each node represents
  - rotation(s)
  - geometric primitive(s)
  - struct. transformations
- The root can be anywhere. We chose the hip (*can re-root*)
- Control knob for each joint angle, plus global position and orientation
- A realistic human would be *much* more complex

# Directed Acyclic Graph



This is a graph, so you can re-root it (make head the root)

It's *directed*, rendering traversal only follows links one way.

It's *acyclic*, to avoid infinite loops in rendering.

Not necessarily a tree.

e.g. l.arm2 and r.arm2 primitives might be two instantiations (one mirrored) of the same geometry

# What Hierarchies Can and Can't Do

- Advantages:
  - Reasonable control knobs
  - Maintains structural constraints
- Disadvantages:
  - Doesn't always give the "right" control knobs trivially
    - » e.g. hand or foot position - re-rooting may help
  - Can't do closed kinematic chains easily (keep hand on hip)
  - Missing other constraints: do not walk through walls
- Hierarchies are a vital tool for modeling and animation

# So What Have We Done?

- Forward Kinematics
  - Given the model and the joint angles, where is the end effector?
    - » In graphics compute this so you know where to draw
    - » In robotics compute this to know how to control the end effector
- Inverse Kinematics
  - Given a desired location of the end effector, what are the required joint angles to put it there.
    - » In robotics, required to place the end effector near to objects in real world

**Inverse Kinematics is useful in animation as well**

**Kinematics is easy, IK is hard because of redundancy.**

# Implementing Hierarchies

- Building block: a *matrix stack* that you can push/pop
- Recursive algorithm that descends your model tree, doing transformations, pushing, popping, and drawing
- Tailored to OpenGL's state machine architecture (or vice versa)
- Nuts-and-bolts issues:
  - What kind of nodes should I put in my hierarchy?
  - What kind of interface should I use to construct and edit hierarchical models?
- Extensions:
  - expressions, languages.

# The Matrix Stack

- Idea of Matrix Stack:
  - LIFO stack of matrices with push and pop operations
  - *current transformation matrix* (product of all transformations on stack)
  - transformations modify matrix at the top of the stack
- Recursive algorithm:
  - load the identity matrix
  - for each internal node:
    - » push a new matrix onto the stack
    - » concatenate transformations onto current transformation matrix
    - » recursively descend tree
    - » pop matrix off of stack
  - for each leaf node:
    - » draw the geometric primitive using the current transformation matrix

# Relevant OpenGL routines

`glPushMatrix()`, `glPopMatrix()`

*push and pop the stack. push leaves a copy of the current matrix on top of the stack*

`glLoadIdentity()`, `glLoadMatrixd(M)`

*load the Identity matrix, or an arbitrary matrix, onto top of the stack*

`glMultMatrixd(M)`

*multiply the matrix C on top of stack by M.  $C = CM$*

`glOrtho (x0,y0,x1,y1,z0,z1)`

*set up parallel projection matrix*

`glRotatef(theta,x,y,z)`, `glRotated(...)`

*axis/angle rotate. “f” and “d” take floats and doubles, respectively*

`glTranslatef(x,y,z)`, `glScalef(x,y,z)`

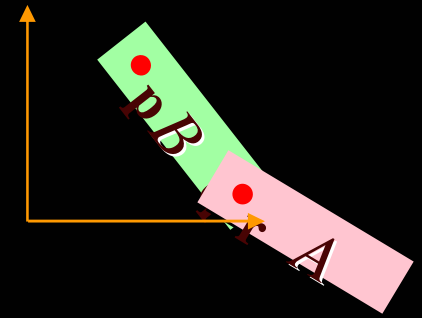
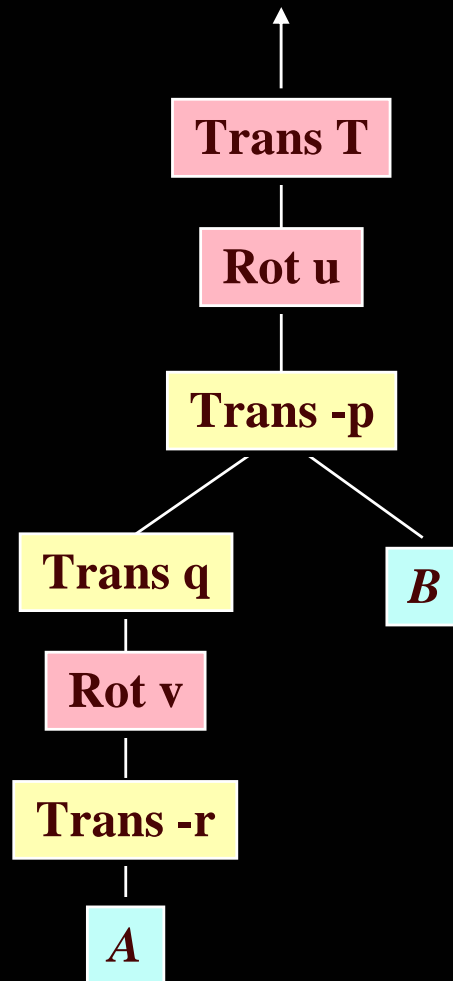
*translate, rotate. (also exist in “d” versions.)*



# Two-link arm, revisited, in OpenGL

## Trace of OpenGL calls

```
glLoadIdentity();
glOrtho(...);
glPushMatrix();
glTranslatef(Tx, Ty, 0);
glRotatef(u, 0, 0, 1);
glTranslatef(-px, -py, 0);
glPushMatrix();
glTranslatef(qx, qy, 0);
glRotatef(v, 0, 0, 1);
glTranslatef(-rx, -ry, 0);
Draw(A);
glPopMatrix();
Draw(B);
glPopMatrix();
```



# Building and Editing Hierarchies

Three approaches:

- Edit the boxes-and-arrows diagram
  - +easy to use
  - hard to visualize effect of a change
- Edit the picture (select and group)
  - +easy to visualize (WYSIWYG)
  - confusing, no view of the graph, limited control
- Textual description (declarative or code)
  - +precise
  - +easy to implement
  - hard to visualize, unintuitive

## Building and Editing, continued

- Two aspects to a model
  - structure: nodes, connectivity, primitives
  - parameters: trans, rot, scale, primitive attributes...
- Hard to build model by point-and-click on a rendering of the model (*but point-and-click on a graph view is OK*)
- Hard to set/edit parameters by typing in numbers
- Best: a hybrid (used by Maya and other anim packages)
  - Build structure in a graph view
  - Attach parameter values to sliders
  - Render result to show effects of parameter changes

## Select-and-Group Interface

- A common method of building a hierarchy
  - Select a set of objects (click on them)
  - Group command creates a new top-level “group” node with the objects as children
  - Grouping groups forms a hierarchy
- Ungrouping a group makes all its children top-level nodes
- Editing options are group, ungroup, delete

# What Should Transformation Nodes Do?

- Separate nodes for translation, rotation and scale
  - +lots of flexibility
  - many nodes making select-and-click difficult
- Nodes perform multiple transformations in hard-wired sequence, e.g. rotate-translate-scale
  - +less complex tree
  - hard-wired sequences are less flexible

# Hardwired Group Transformation Sequence

- Must select a good hard-wired sequence that the user will think is intuitive
  - Rule of thumb: scale before rotate
    - » avoid object shearing during rotation
  - Rule of thumb: rotate before translate
    - » make sure rotation occurs about correct point
- Occasionally this sequence won't be enough - a more flexible scheme is required

# Group Parameters and Transformations

- Parameters (2D)
  - (cx, cy): center of rotation and scaling
  - (sx, sy): scaling
  - theta: rotation
  - (tx, ty): translation
- Full sequence of primitive transformations:
  - trans(-cx, -cy)      *move center to origin*
  - scale(sx,sy)      *scale*
  - rot(theta)      *rotate*
  - trans(cx,cy)      *move center back*
  - trans(tx,ty)      *translate (can combine with*  
*previous)*

# Variables and Expressions

- Better control can come from the transformation parameters being functions of other variables
- Simple example:
  - a clock with second, minute and hour hands
  - hands should rotate together
  - express all the motions in terms of a “seconds” variable
  - whole clock is animated by varying the seconds parameter

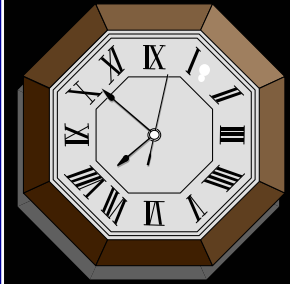
$$m = \frac{s}{60}$$

$$h = \frac{m}{60}$$

$$\theta_s = \frac{2\pi s}{60}$$

$$\theta_m = \frac{2\pi m}{60}$$

$$\theta_h = \frac{2\pi h}{12}$$



Or arms and legs of a walking human figure



## Getting Expressions into Your Models

- Some commercial systems (e.g. Maya ) have expression-evaluating facilities.
- Some high-end systems (e.g. Pixar's in-house system) contain full-blown embedded interpreted languages — most of their models are really programs.
- If you write your models in a general-purpose language, interpreted or not, you get this for free.
- The trick is to avoid losing too much speed in the process.
- The example on the next slide shows (very schematically) how you might go about writing C code to draw a complex hierarchical model.

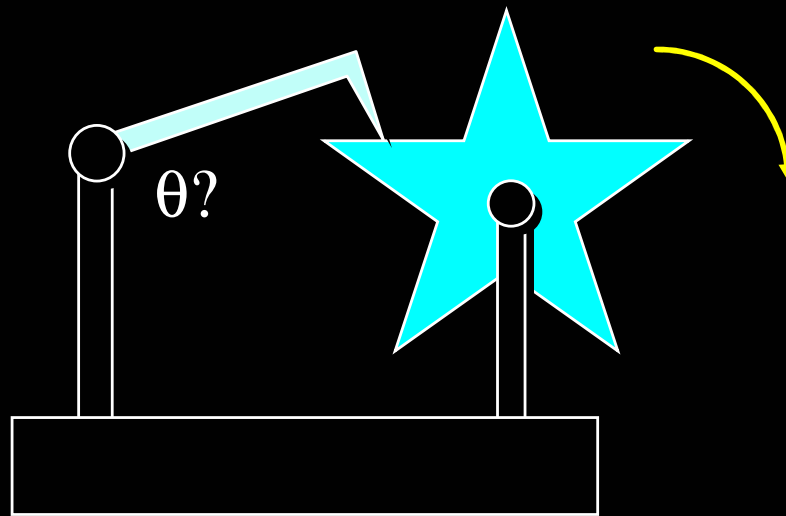
## Models as Code: draw-a-bug.

```
void draw_bug(walk_phase_angle, xpos, ypos, zpos){
    pushmatrix
    translate(xpos, ypos, zpos)
    calculate all six sets of leg angles based on
        walk phase angle.
    draw bug body
    for each leg:
        pushmatrix
        translate(leg pos relative to body)
        draw_bug_leg(theta1 & theta2 for that leg)
        popmatrix
    popmatrix
}
```

```
void draw_bug_leg(float theta1, float theta2){
    glPushMatrix();
    glRotatef(theta1, 0, 0, 1);
    draw_leg_segment(SEGMENT1_LENGTH)
    glTranslatef(SEGMENT1_LENGTH, 0, 0);
    glRotatef(theta2, 0, 0, 1);
    draw_leg_segment(SEGMENT2_LENGTH)
    glPopMatrix();
}
```

## Hard Examples

- A walking humanoid that swings its arms and bobs its head, under control of a single variable, so it walks when you “turn the crank.” (you’d have extra parameters for walking style, of course.)
- In the figure below, what expression would you use to calculate the arm’s rotation angle to keep the tip on the star-shaped wheel as the wheel rotates???
- This gets arbitrarily hard. There’s got to be a better way to do constraints. We’ll get back to this topic when we do animation.



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