Written Assignment #1: Transformation and Viewing

15-462 Graphics I, Fall 2003
Doug James
Due: Thursday, September 18, 2003 (**before lecture**)
60 POINTS

September 11, 2003

- The work must be all your own.
- The assignment is due before lecture on Thursday, September 18.
- Be explicit, define your symbols, and explain your steps. This will make it a lot easier for us to assign partial credit.
- Use geometric intuition *together* with trigonometry and linear algebra.
- Verify whether your answer is meaningful with a simple example.

1 Angel, Chapter 4, Exercise 4.2

Two transformations, A and B, are said to commute if $AB = BA$. Show that the following transformation sequences commute:

1. A rotation and a uniform scaling;
2. Two rotations about the same axis;
3. Two translations.

2 Angel, Chapter 4, Exercise 4.7

Show that any sequence of rotations and trasnlations can be replaced by a single rotation about the origin, followed by a translation.

3 Angel, Chapter 4, Exercise 4.8

Derive the shear transformation from the rotation, translation, and scaling transformations.

4 Angel, Chapter 4, Exercise 4.9

In two dimensions, we can specify a line by the equation $y = mx + b$.

1. Find an affine transformation to reflect two-dimensional points about this line.
2. Extend your result to reflection about a plane in three dimensions.

5 Angel, Chapter 4, Exercise 4.20

Given two nonparallel three-dimensional vectors, u and v, how can we form an orthogonal coordinate system in which u is one of the basis vectors?

6 Angel, Chapter 4, Exercise 4.22

Find the quaternions for 90-degree rotations about the x- and y-axes. Determine their product. What rotation is this?

7 Angel, Chapter 5, Perspective Projection

In §5.19, it is shown that the OpenGL perspective transformation can be factored as

$$\mathbf{P} = \mathbf{NSH} = \begin{bmatrix} \left(\frac{2z_{min}}{x_{max}-x_{min}}\right) & 0 & \left(\frac{x_{max}+x_{min}}{x_{max}-x_{min}}\right) & 0 \\ 0 & \left(\frac{2z_{min}}{y_{max}-y_{min}}\right) & \left(\frac{y_{max}+y_{min}}{y_{max}-y_{min}}\right) & 0 \\ 0 & 0 & \left(-\frac{z_{max}+z_{min}}{z_{max}-z_{min}}\right) & \left(\frac{2z_{max}z_{min}}{z_{max}-z_{min}}\right) \\ 0 & 0 & -1 & 0 \end{bmatrix}.$$

1. State each of the homogeneous matrix factors, $\mathbf{N}$, $\mathbf{S}$ and $\mathbf{H}$.
2. In your own words and pictures, explain the role of each of the factors.