Written Assignment #1: Transformation and Viewing

15-462 Graphics I, Fall 2003
Doug James
Due: Thursday, September 18, 2003 (before lecture)
60 POINTS

September 11, 2003

- The work must be all your own.
- The assignment is due before lecture on Thursday, September 18.
- Be explicit, define your symbols, and explain your steps. This will make it a lot easier for us to assign partial credit.
- Use geometric intuition together with trigonometry and linear algebra.
- Verify whether your answer is meaningful with a simple example.

1 Angel, Chapter 4, Exercise 4.2

Two transformations, $A$ and $B$, are said to commute if $AB = BA$. Show that the following transformation sequences commute:

1. A rotation and a uniform scaling;
2. Two rotations about the same axis;
3. Two translations.

2 Angel, Chapter 4, Exercise 4.7

Show that any sequence of rotations and translations can be replaced by a single rotation about the origin, followed by a translation.

3 Angel, Chapter 4, Exercise 4.8

Derive the shear transformation from the rotation, translation, and scaling transformations.
4 Angel, Chapter 4, Exercise 4.9

In two dimensions, we can specify a line by the equation \( y = mx + b \).

1. Find an affine transformation to reflect two-dimensional points about this line.

2. Extend your result to reflection about a plane in three dimensions.

5 Angel, Chapter 4, Exercise 4.20

Given two nonparallel three-dimensional vectors, \( u \) and \( v \), how can we form an orthogonal coordinate system in which \( u \) is one of the basis vectors?

6 Angel, Chapter 4, Exercise 4.22

Find the quaternions for 90-degree rotations about the \( x \)- and \( y \)-axes. Determine their product. What rotation is this?

7 Angel, Chapter 5, Perspective Projection

In §5.19, it is shown that the OpenGL perspective transformation can be factored as

\[
P = \begin{bmatrix}
\frac{2z_{\text{min}}}{x_{\text{max}} - x_{\text{min}}} & 0 & \frac{x_{\text{max}} + x_{\text{min}}}{x_{\text{max}} - x_{\text{min}}} & 0 \\
0 & \frac{2z_{\text{min}}}{y_{\text{max}} - y_{\text{min}}} & \frac{y_{\text{max}} + y_{\text{min}}}{y_{\text{max}} - y_{\text{min}}} & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & \frac{2z_{\text{max}} + z_{\text{min}}}{z_{\text{max}} - z_{\text{min}}} & 0 \\
\end{bmatrix}.
\]

1. State each of the homogeneous matrix factors, \( N \), \( S \) and \( H \).

2. In your own words and pictures, explain the role of each of the factors.