

Constructive Logic (15-317), Fall 2025

Assignment 6: Cut Admissibility

Constructive Logic Staff
(Instructor: Karl Crary)

Due: Wednesday, October 8, 2025, 11:59 pm

This assignment will have a written portion and a coding portion. You will submit both portions through Gradescope, to the assignments labelled “Homework 6 (written)” and “Homework 6 (code).” Please submit a file named “**hw.pdf**” to the former, and a file named “**hw.deriv**” to the latter.

We recommend that you typeset your written solutions. Most students use L^AT_EX, but other software is acceptable. (Please put each task on its own page to speed up grading.) If you choose not to typeset your solutions, be aware that you are answerable for your handwriting. Any that the grader has difficulty reading (in the sole judgement of the grader), will be marked wrong.

For the coding portion you will use Dcheck. You can find documentation on Dcheck at cs.cmu.edu/~crary/dcheck/dcheck.pdf and a sample file at cs.cmu.edu/~crary/dcheck/example.deriv. (Be aware that the sample file uses several logics that we have not seen yet in class.)

1 Cut for a New Connective

Recall the $\diamond$ connective from HW3:

$$\begin{array}{c}
 [A \text{ true}]_v \quad [A \text{ true}]_w \\
 \vdots \quad \vdots \\
 \frac{B \text{ true} \quad C \text{ true}}{\diamond(A, B, C) \text{ true}} \diamond I^{v,w}
 \end{array}$$

$$\begin{array}{c}
 [B \text{ true}]_u \quad [C \text{ true}]_u \\
 \vdots \quad \vdots \\
 \frac{\diamond(A, B, C) \text{ true} \quad A \text{ true} \quad D \text{ true}}{D \text{ true}} \diamond E1^u \quad \frac{\diamond(A, B, C) \text{ true} \quad A \text{ true} \quad D \text{ true}}{D \text{ true}} \diamond E2^u
 \end{array}$$

In sequent calculus, $\diamond$ would have the following rules:

$$\begin{array}{c}
 \frac{\Delta, A \implies B \quad \Delta, A \implies C}{\Delta \implies \diamond(A, B, C)} \diamond R \quad \frac{\Delta, \diamond(A, B, C) \implies A \quad \Delta, \diamond(A, B, C), B \implies D}{\Delta, \diamond(A, B, C) \implies D} \diamond L1 \\
 \frac{\Delta, \diamond(A, B, C) \implies A \quad \Delta, \diamond(A, B, C), C \implies D}{\Delta, \diamond(A, B, C) \implies D} \diamond L2
 \end{array}$$

Task 1 (5 points). If we wanted to add this connective (and the sequent calculus rules describing it above) to our logic, what would be the additional cases we have to prove for cut to remain

admissible? Please list the additional cut cases that would need to be added to the cut proof from class. For each of these cases, specify A , $\mathcal{D}$ and $\mathcal{E}$ where A is the cut formula. Use as few distinct cases as possible. For example, when the final rule of $\mathcal{D}$ is *init*, it does not matter what $\mathcal{E}$'s final rule is, so it would be listed as one case, not as several cases. You do not need to prove any of these additional cases; you need only list them.

2 Cut Admissibility

Extend the proof of the admissibility of cut from class by filling in the following inductive cases. A valid proof should include a detailed English explanation—not only notation. Make sure to cite weakening explicitly if it is used, and to justify any use of the induction hypothesis.

Task 2 (10 points). $\mathcal{D}$ ends in $\vee R2$ and $\mathcal{E}$ ends in $\vee L$, where $\vee L$ operates on the cut formula.

Task 3 (10 points). $\mathcal{E}$ ends in $\supset L$, where $\supset L$ *does not* operate on the cut formula.

Task 4 (2 points). What category of cuts do the cases in tasks 2 and 3 fall into?

3 Sequent Calculus Mastery

Task 5 (5 points). Using Dcheck, give a derivation of the following sequent, using the original sequent calculus (*i.e.*, not the reduced sequent calculus).

$$\implies \neg P \vee \neg Q \supset \neg(P \wedge Q)$$

Name your derivation `task10`. (Remember that Dcheck takes the propositions P and Q to be atomic.) **Instant feedback is turned off for this task, so be extra careful.**

Since instant feedback is turned off, **pay special care to the assumption numbers**. Dcheck numbers assumptions from right to left, starting at zero. Thus, the rightmost assumption is numbered 0. No partial credit will be granted for submissions that would be correct except for assumption numbers.