Constructive Logic (15-317), Fall 2019 Assignment 7: Classical Logic

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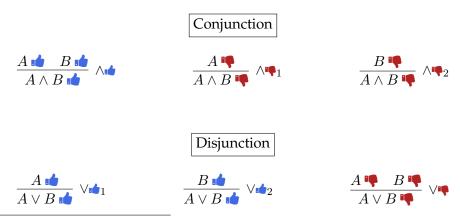
Due: Friday, October 18, 11:59 pm

Submit your homework via GradeScope as a file named hw7.pdf.

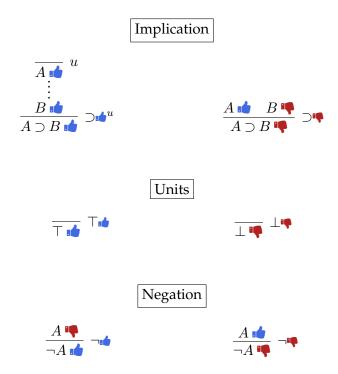
1 A New Constructive Logic: Classical Logic

Intuitionistic logic is based on the idea that the fundamental mathematical activity is to *affirm* the truth of something using evidence. Classical logic should be understood as a different, *dialectical* model of mathematical activity, in which one party tries to affirm and the other party tries to deny. Whereas the central duality of intuitionistic natural deduction was between the *introduction* and *elimination* rules for truth A true, in classical natural deduction, each proposition is explained through the interaction between rules for affirmation $A \bowtie (i.e. A true)$ and rules for denial $A \blacktriangleleft (i.e. A false)$, a contest governed by the nullary form of judgment (i.e. A true) (contradiction).

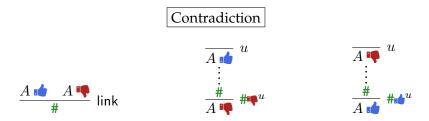
To be precise, each connective comes equipped with introduction rules for *both* affirmation and denial; classical negation $\neg A$ implements the involutive "change of perspective" between player (affirmation) and opponent (denial).



¹The symbols $A \stackrel{\bullet}{\bullet}$ and $A \stackrel{\bullet}{\bullet}$ can be written using the provided macros; if you have trouble using these symbols, it is also acceptable to write $A \ true$ and $A \ false$, as from lecture.



In classical natural deduction, affirmation and denial compete with each other in a *formal contradiction*, a nullary judgment written #. The rules for contradictions are as follows:



Using the rules **#...** and **#...**, all the usual "elimination rules" for truth can be *derived* in classical natural deduction.

Task 1 (14 pts). Recall the introduction and elimination rules for the universal quantifier in intuitionistic natural deduction:

$$\begin{array}{c} [z:\tau] \\ \vdots \\ A(z) \ true \\ \forall x:\tau. \ A(x) \ true \end{array} \ \forall \mathbf{I}^z \\ \hline \begin{array}{c} t:\tau \quad \forall x:\tau. \ A(x) \ true \\ \hline A(t) \ true \end{array} \ \forall \mathbf{E} \end{array}$$

Now it's your turn: *invent* affirmation and denial rules $\forall \mathbf{d}$, $\forall \mathbf{q}$ for the universal quantifier, as an extension to the classical natural deduction calculus which we have seen so far.

Task 2 (14 pts). Recall the introduction and elimination rules for the existential quantifier in intuitionistic natural deduction:

$$\frac{t:\tau\quad A(t)\ true}{\exists x:\tau.\ A(x)\ true}\ \exists \mathbf{I}$$

$$\frac{\exists x:\tau.\ A(x)\ true\quad C\ true}{C\ true}\ \exists \mathbf{E}^{z,u}$$

As in the previous task, invent affirmation and denial rules $\exists \bullet$, $\exists \bullet$ for the existential quantifier.

Task 3 (22 pts). Using the rules you invented in the previous tasks, show that the following *elimination* rules for the universal and the extistential quantifier are derivable.

