

Constructive Logic (15-317), Fall 2019

Assignment 6: Admissibility and Derivability

Instructor: Karl Crary
TAs: Avery Cowan, David Kahn, Siva Somayyajula

Due: Friday, October 11, 2019, 11:59 pm

This assignment must be submitted electronically via Gradescope. Submit your homework as a pdf containing your written solutions.

1 Admissibility and Derivability

Task 1 (24 points). For each of the following rules (with A, B, C atomic) in the cut-free sequent calculus, indicate which are derivable, admissible or neither. If a rule is derivable, you must supply the derivation; if it is not derivable but is admissible, you must include a proof that it is admissible.¹ If it is neither admissible or derivable, please just indicate why you believe this (but no rigorous proof is required).

$$\frac{\Gamma \Longrightarrow A \quad \Gamma, B \Longrightarrow \perp}{\Gamma \Longrightarrow A \wedge \neg B} \quad (1)$$

$$\frac{\Gamma, A \Longrightarrow B \vee C}{\Gamma, A \wedge \neg B \Longrightarrow C} \quad (2)$$

$$\frac{\Gamma, B \Longrightarrow A}{\Gamma \Longrightarrow B \supset (C \wedge A)} \quad (3)$$

$$\frac{}{\Gamma, A \vee (B \supset (C \wedge B)) \Longrightarrow A \vee (B \supset (C \wedge B))} \quad (4)$$

2 Admissibility of Cut

Task 2 (16 points). Extend the proof of the admissibility of cut from class by filling in the following inductive case. A valid proof should include a detailed english explanation—not only notation.

Case: \mathcal{D} ends in $\vee R_2$ and \mathcal{E} ends in $\vee L$, where $\vee L$ is applied on the principal formula of the cut.

¹You may use lemmas that we have proved in class, including the admissibility of cut.

3 Practicing Sequent Calculus

Task 3 (15 pts). Derive each of the following judgments in the cut-free sequent calculus, with A, B, C atomic.

a. $\cdot \Rightarrow A \wedge (B \vee C) \supset (A \wedge B) \vee (A \wedge C)$

b. $\cdot \Rightarrow (A \supset B) \supset (\neg B \supset \neg A)$

c. $\cdot \Rightarrow (A \vee \neg A) \supset \neg\neg A \supset A$