Puzzle: Communicating to Plan Noam Nisan's 60th Birthday Workshop

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Please send solutions to the author by e-mail, with the title of this puzzle in the subject header. The best solution will be published in the next issue of SIGecom Exchanges, provided that this solution is of sufficiently high quality. Quality is judged by the author, taking into account at least soundness, completeness, and clarity of exposition. (Incidentally, there is another birthday puzzle for which we still need a solution [1]!)

This is a puzzle in honor of Noam Nisan's 60th birthday and the June 2022 workshop associated with it (and perhaps also a bit in honor of one of its distinguished attendees, Hervé Moulin). This workshop was held at the Hebrew University of Jerusalem in Israel.

Michael, Moshe, and Shahar—i.e., a constant number of organizers—are planning the workshop for Noam's 60th birthday, and are trying to predict who, out of n people, will attend. Whether a person wants to attend is a function of who else attends. "The more the merrier," so for each person i, if i would attend when S is the set of other attendees, and $S \subseteq S'$, then i would attend when S' is the set of other attendees. Let $\mathcal{S}_i$ be the set of sets S of other people for which i would attend (so, $\mathcal{S}_i$ is upward closed).

To split the work, the organizers partition the set of n people among themselves. Subsequently, each of them figures out, for every player i in his own part, what $\mathcal{S}_i$ is. (Note that each organizer thus still needs to think about how much "his" people like the people in the other parts. But each organizer knows $\mathcal{S}_i$ only for people i in his own part.) At this point, the organizers, who of course want the workshop to be successful, must communicate with each other to find the *largest* possible set of people S^* that can consistently attend (i.e., the largest set with the property such that every person in it will attend given that everyone else in the set attends: i.e., for each $i \in S^*$, we have $S^* \setminus \{i\} \in \mathcal{S}_i$, and S^* is the largest set with that property).

Up to a constant factor, how many bits of communication do the organizers need to figure this out?

REFERENCES

Vincent Conitzer. Puzzle: The AI Circus. (Puzzle in honor of Tuomas Sandholm's 50th birthday.) *SIGecom Exchanges*, 17(2):76–77, October 2019.

[1]I thank Shahar Dobzinski for helpful feedback.

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