02-713 Review

Midterm 1

* I took some images from Prof Kingsford's slides

- *Disclaimer*: I have not seen the midterm you will take.
- But we would like to recap some important points and try our best to answer questions you may have.

Anything is fair game, but ...

- I always like using slides as a starting point for what could be asked.
- See what theorems could be handy w/ each algorithm: they are useful tools in the design of algorithms.
- Branch out to the book for sup info.

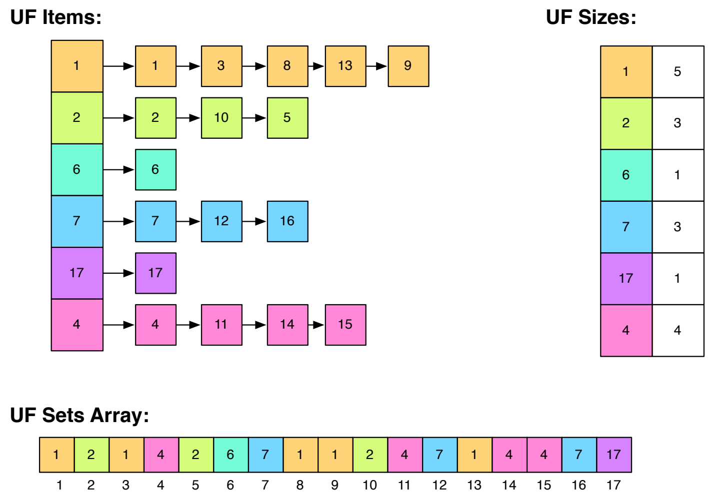
For the test ...

- On anything that's more open-ended, describe your primary strategy.
- Pseudocode is good for being more specific.

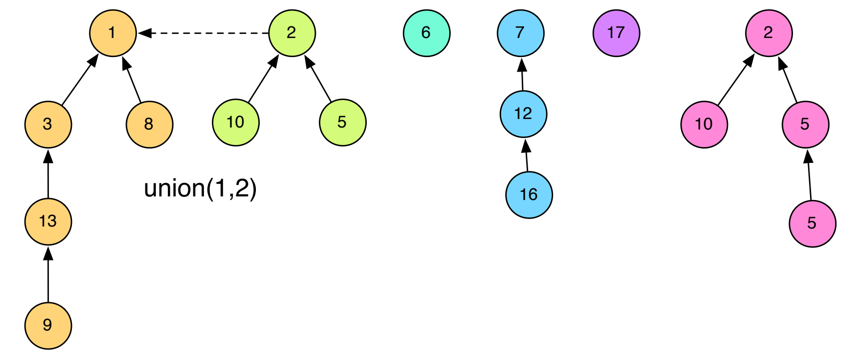
- Basic data structures
  (Arrays, Lists, Trees, Graphs, Heaps, Union-Find)
- Analyze asymptotic run-time of an algorithm (Big Oh)
- 2 sorting algorithms
  (HeapSort, MergeSort)
- 3 Optimal Algorithms for MST
  (Prim's, Reverse-Delete, Kruskal's)
- 5 Algorithms based on Tree Growing
  (Prim's, BFS, DFS, Dikjstra's, A*)
- Preview of dynamic programming
  (Bellman-Ford)
- Intro to Divide and Conquer
  (Counting Inversions, Closest Points)

Heaps

- Finding the minimum item takes constant time.
- Tree property: children are always greater in value than the parent.
- Insertion and deletion take log(n) time and preserve this property via “sift ups” and “sift downs”.
- Heapsort: Keep grabbing the minimum element from the top of the heap and delete it.

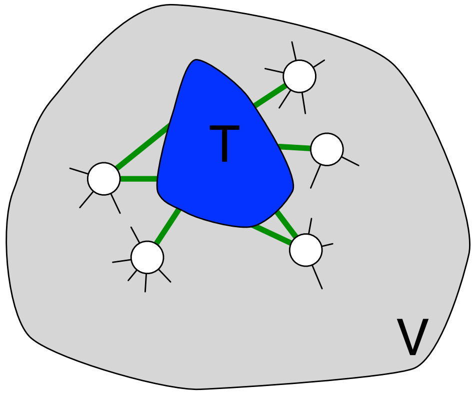


Union-Find



- Given: a partition of a set (collection of sets are disjoint)
- Goal: efficiently find which set an element belongs to and efficiently merge them together.
- During a union, updating the array that says which set an element is a member of can loop through entire array.
- However, *k* union operations is bounded by *k*log(*k*) (set sizes “double”)

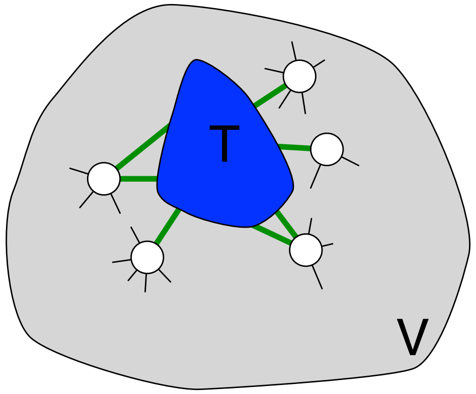
MST Key Points (1)



- Find the tree in a graph with the minimum total edge cost.
- Prim's algorithm: tree grow by adding the lowest cost edge to the tree.
- Cut property can be used to prove their correctnesses.

Theorem (MST Cut Property). *Let S be a subset of nodes, with $|S| \geq 1$ and $|S| < n$. Every MST contains the edge $e = \{v, w\}$ with $v \in S$ and $w \in V - S$ that has minimum weight.*

MST Key Points (2)



- Reverse-Delete algorithm: Remove edges and avoid disconnecting the graph.
- Kruskal's algorithm: Add edges avoiding cycles.
- Cycle property can be used to prove the correctness of both of these.

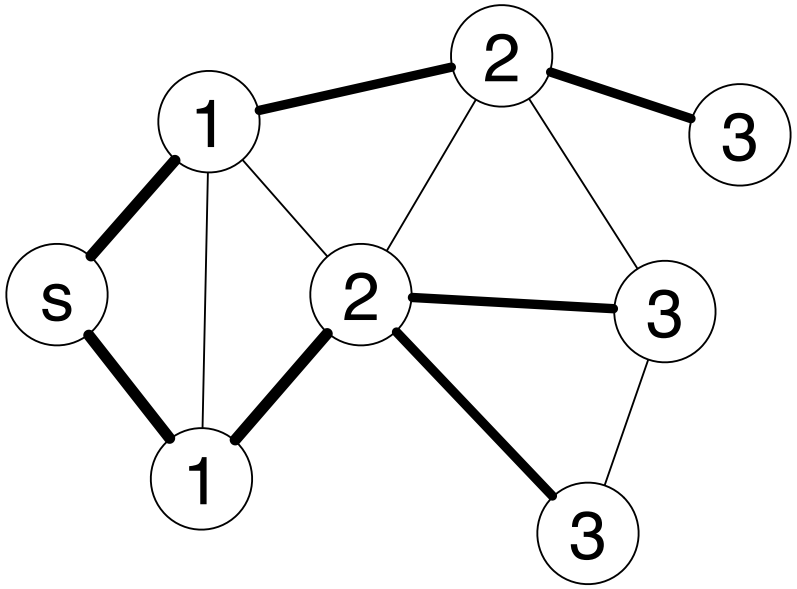
Theorem (Cycle Property). *Let C be a cycle in G. Let $e = (u, v)$ be the edge with maximum weight on C. Then e is not in $\boxed{\text{any}}$ MST of G.*

Big Oh

- Big Oh used often in practice to describe runtime of algorithms
- Ideally find the "best" Oh
(i.e. the *lowest* upper bound)
- When proving statements in asymptotic analysis, start with the definition and make sure you specify constants

Definition (O). A runtime $T(n)$ is $O(f(n))$ if there exist constants $n_0 \geq 0$ and $c > 0$ such that:

$$T(n) \leq cf(n) \qquad \text{for all } n \geq n_0$$



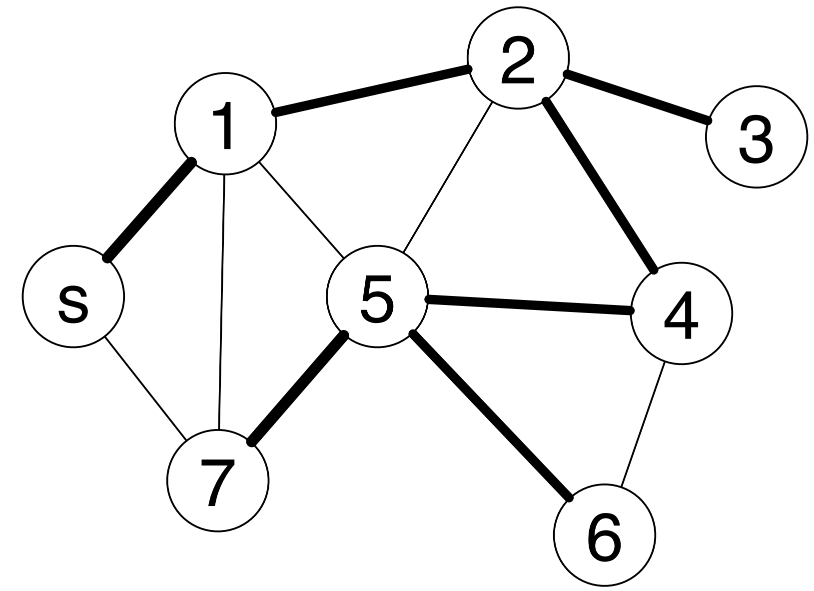
BFS

- Traverse the graph in 'layers'
- Add visited nodes to queue (FIFO, TreeGrowing process *earliest* discovered node)

Non-tree edge property:

Theorem. *Choose $x \in L_i$ and $y \in L_j$ such that $\{x, y\}$ is an edge in undirected graph G. Then i and j differ by at most 1.*

DFS

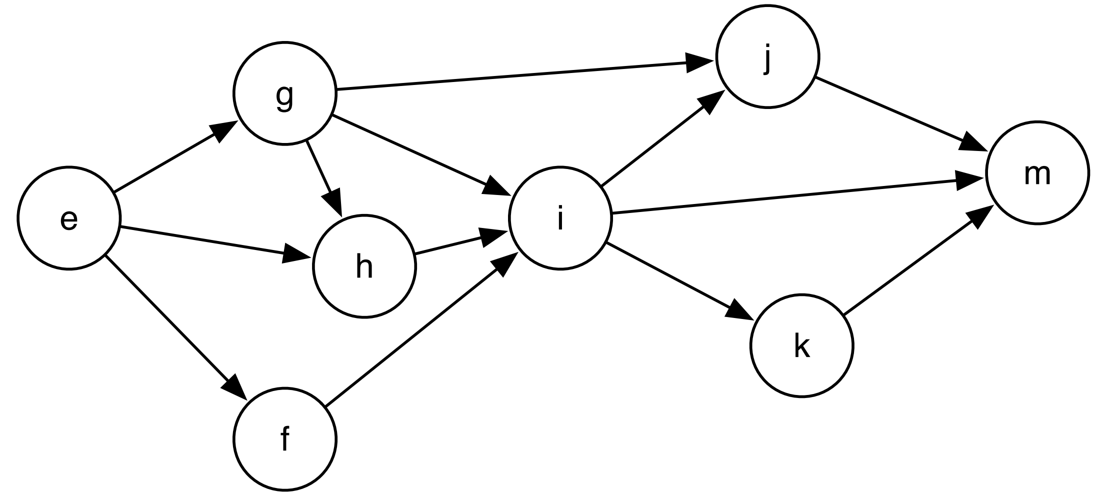


- Keep walking down a path until you are forced to backtrack.
- Add edges to a stack (LIFO, TreeGrowing: process *most recently* discovered node)

Non-tree edge property:

Theorem. *Let x and y be nodes in the DFS tree T_G such that $\{x, y\}$ is an edge in undirected graph G. Then one of x or y is an ancestor of the other in T_G.*

DAG

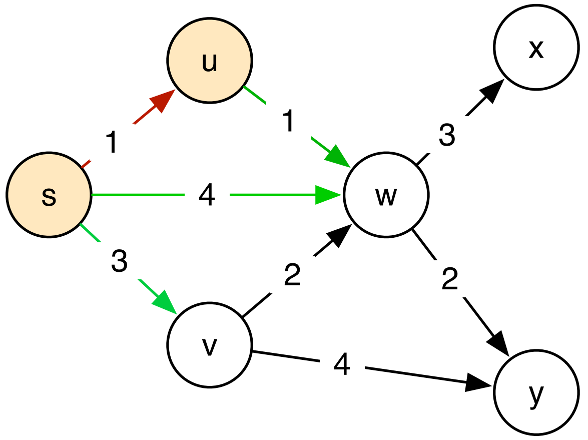


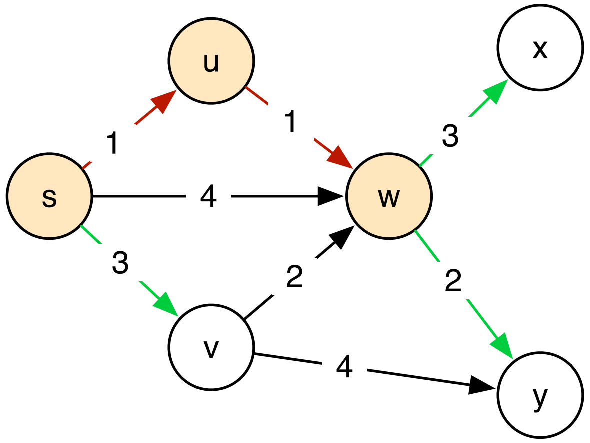
- Key property: Directed, no cycles!
- You can ‘topologically sort a DAG’ so that there are no ‘backwards’ edges.

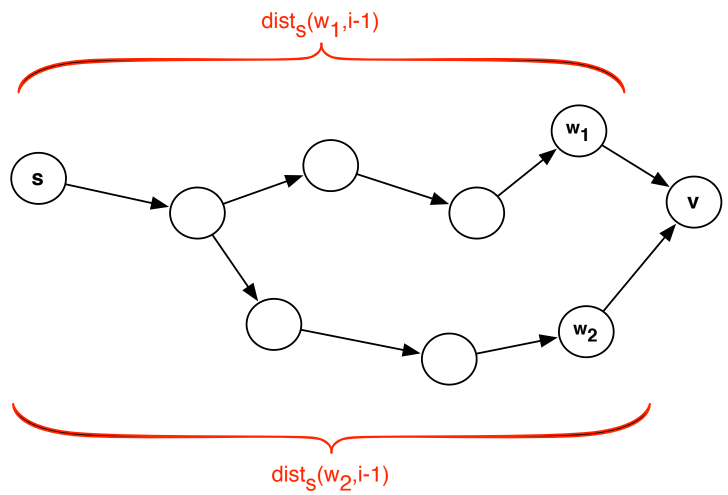
Theorem. *Every DAG contains a vertex with no incoming edges.*

Dikjstra

- Tree Growing: instead of choosing min-weight edge like Prim's, choose the minimum of d(u) + length(u,v)
- Only positive weights







Bellman-Ford

- Handles negative weights (assume no negative cycles)
- 1 pass applies the Ford rule to all edges
- Can make at most *n*-1 passes
- Not 'greedy' like Dikjstra's algorithm

Ford step. Find an edge (u, v) such that

$$dist_s(u) + d(u, v) \leq dist_s(v)$$

and set $dist_s(v) = dist_s(u) + d(u, v)$.

A*

- Another Tree Growing, but with a goal node and a heuristic to help get us there
- Heuristic guarantees we will find optimal solution
- Heuristic *must* be less than true distance
- Ideally it is closer to the real distance

Divide and Conquer

- Try to divide into ‘equal’ parts to get O($n\log n$)
- Assume the problem is solved for each part and the trick is to design an efficient merge step.
- Need a base case for the ‘leaf’ of the recursion tree.