More Applications of Suffix Trees

02-714

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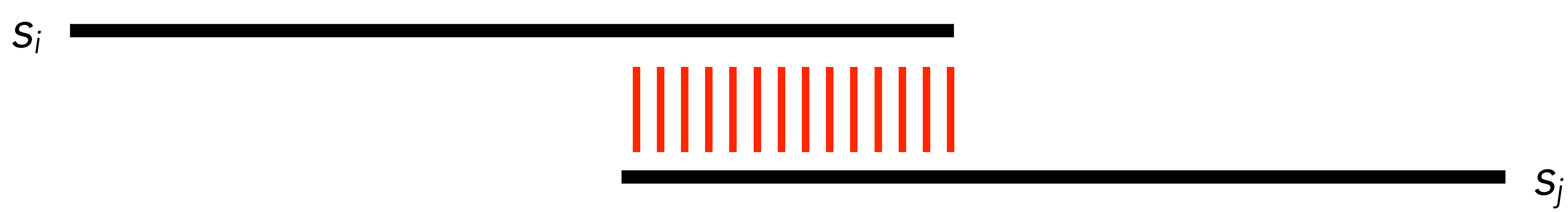
All Pairs Suffix-Prefix Matches

Problem. *Given a set of strings $S = \{s_1,\ldots,s_k\}$ of total length m, find **all** of the longest suffix-prefix exact matches.*



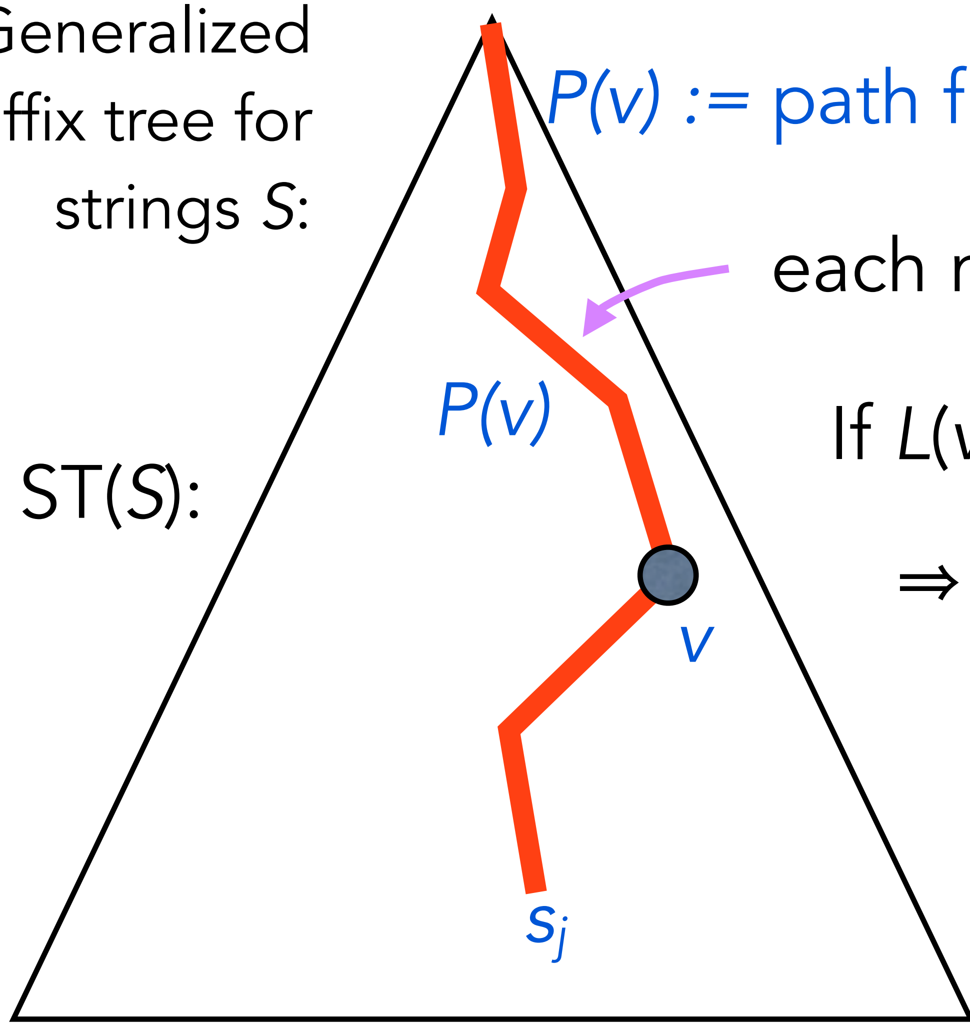
- Can represent these matches by $\binom{k}{2}$ integers.

Notation & Main Idea

Def. $L(v)$:= list of strings (indices) from S such that P(v) is a complete suffix.

Generalized suffix tree for strings S:

ST(S):



$P(v)$:= path from root to v.

each node on this path represents a prefix of s_j

If $L(v)$ on $P(s_j)$ contains i then $P(v)$ is a suffix of s_i.

⇒ deepest node v on $P(s_j)$ s.t. $L(v)$ contains i gives the longest prefix of s_j that matches a suffix of s_i.

Algorithm: find these deepest nodes for all s_i at once with a single traversal of the tree.

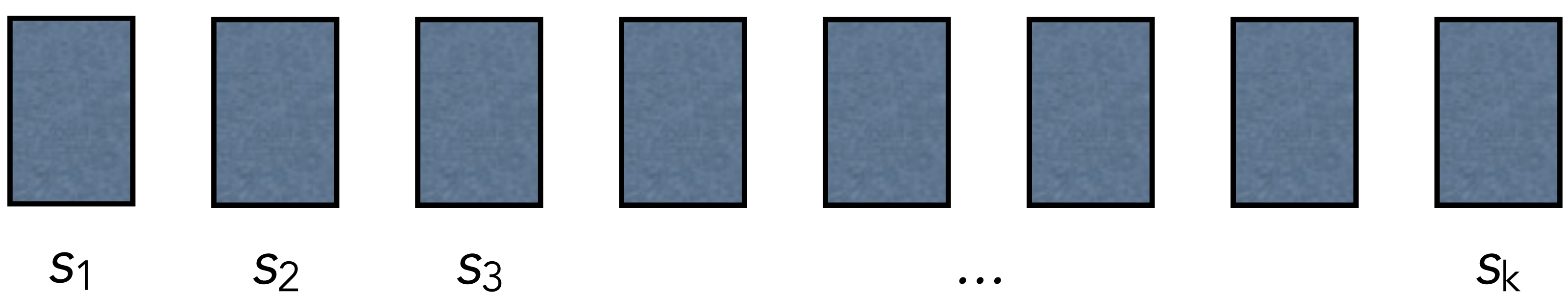
Algorithm

1. Do a DFS on the generalized suffix tree

2. Maintain k stacks during the DFS that you update as follows:

When entering node v, push v onto all stacks in L(v).

When backtracking out of v, pop all stacks in L(v).



each stack *i* will therefore contain the deepest node with $i \in L(v)$ on the current DFS path from the root.

3. When you reach a node corresponding to a full string, output the depth of the nodes at the top of each stack.

Runtime

Thm. *Runtime of the preceding algorithm is $O(m + k^2)$*

Proof. Building the generalized suffix tree takes O(m) time.

The L(v) sets can be saved as you are building the suffix tree (when you add in a suffix for i, add it to L(v)).

There are O(m) indices in the union of the L(v) lists, so the total pop/push events take O(m).

There are k nodes at which you will output, and each output takes O(k) time, leading to $O(k^2)$ time for output.

Ziv-Lempel Compression

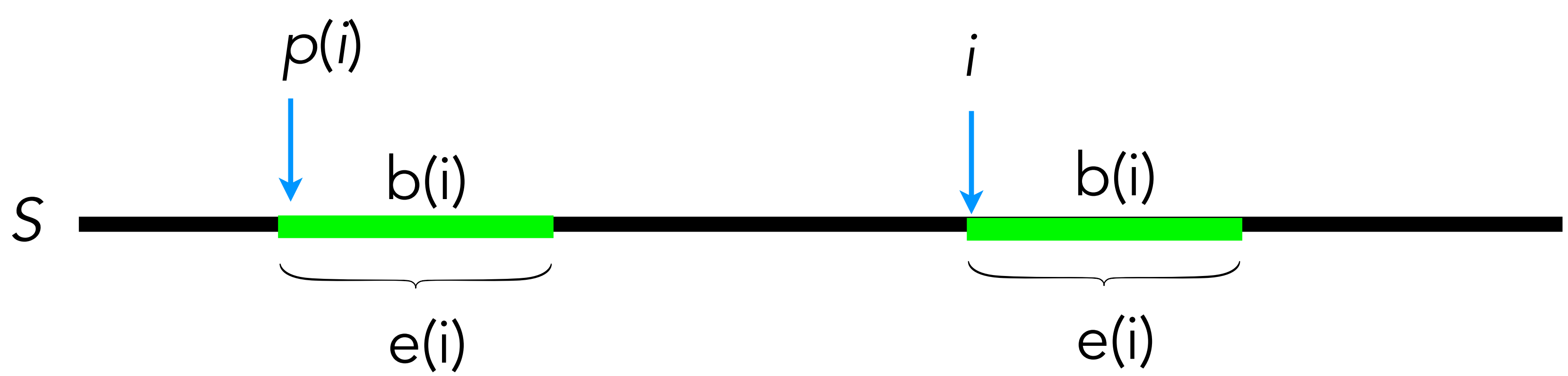
Let S be a string of length *m* that we want to compress.

Notation.

b(i) := the longest string in S[1...i-1] that matches a prefix of S[i...m]

e(i) := |b(i)|

p(i) := the position of b(i) in S[1...i-1]



Algorithm:

1. walk down string from left to right.
2. when at position *i*, output (p(*i*), e(*i*)) instead of S[*i*...i+e(*i*)-1]
3. skip ahead to i := i + e(i)

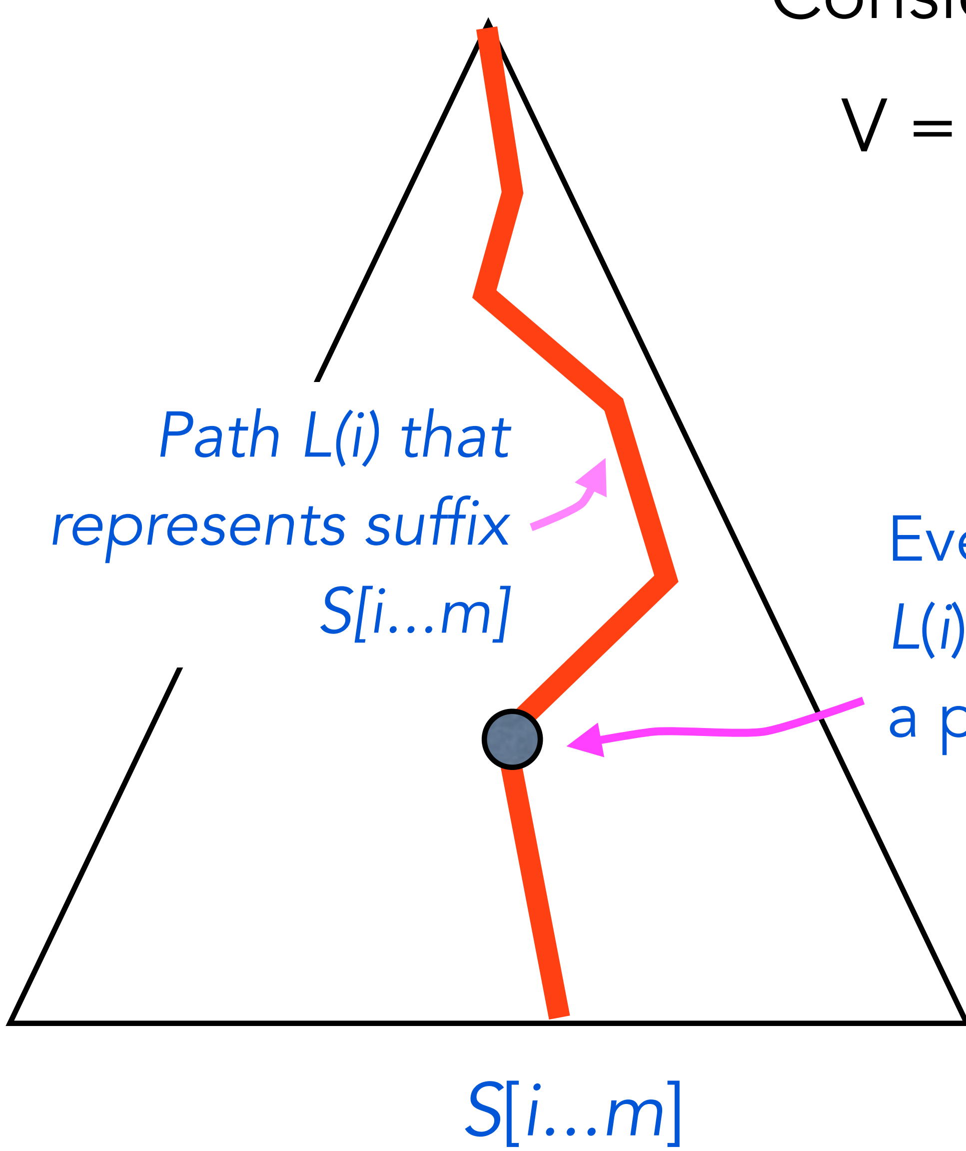
Computing p(i), e(i)

Build suffix tree on S, and at every node v, store:

$c(v)$:= minimum suffix # in the subtree rooted at v

Consider the set

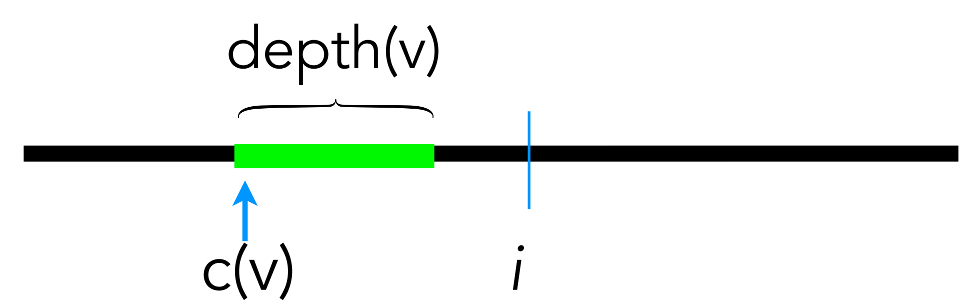
$$V = \{v \in L(i) : c(v) + \text{depth}(v) < i\}$$



Every node v on $L(i)$ corresponds to a prefix of $S[i\ldots m]$.

Every $v \in V$ matches a prefix of $S[i\ldots m]$ **and** is contained in $S[1\ldots i\text{-}1]$ (because $c(v) < i$)

The deepest such node represents $b(i)$.



Running time

Algorithm for p(*i*), e(*i*):

1. To compute p(i): walk down path spelling out S[*i*...*m*]
2. Stop when c(*v*) + depth(*v*) = *i*.
3. p(*i*) = c(*v*)
4. e(*i*) = depth(*v*)

Total running time to compress a string of length *m*:

- O(*m*) to built the suffix tree
- O(*m*) time (bottom up traversal) to compute c(v)
- O(e(*i*)) to search at position i, but each time you spend O(e(*i*)) time, you skip e(*i*) letters ⇒ O(*m*) total time.