B-Trees

CMSC 420: Lecture 9

Another way to achieve "balance"

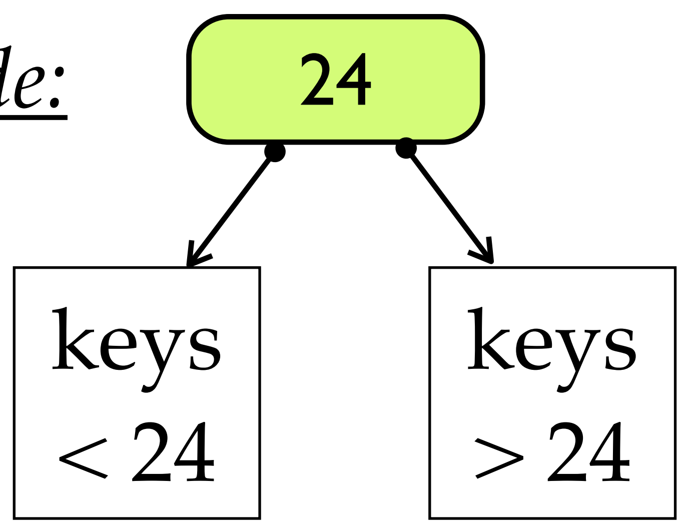
- Height of a perfect binary tree of n nodes is O(log n).
- Idea: Force the tree to be perfect.
  - Problem: can't have an arbitrary # of nodes.
  - Perfect binary trees only have 2^h-1 nodes
- So: relax the condition that the search tree be binary.
- As we'll see, this lets you have any number of nodes while keeping the leaves all at the same depth.

Global balance instead of the local balance of AVL trees.

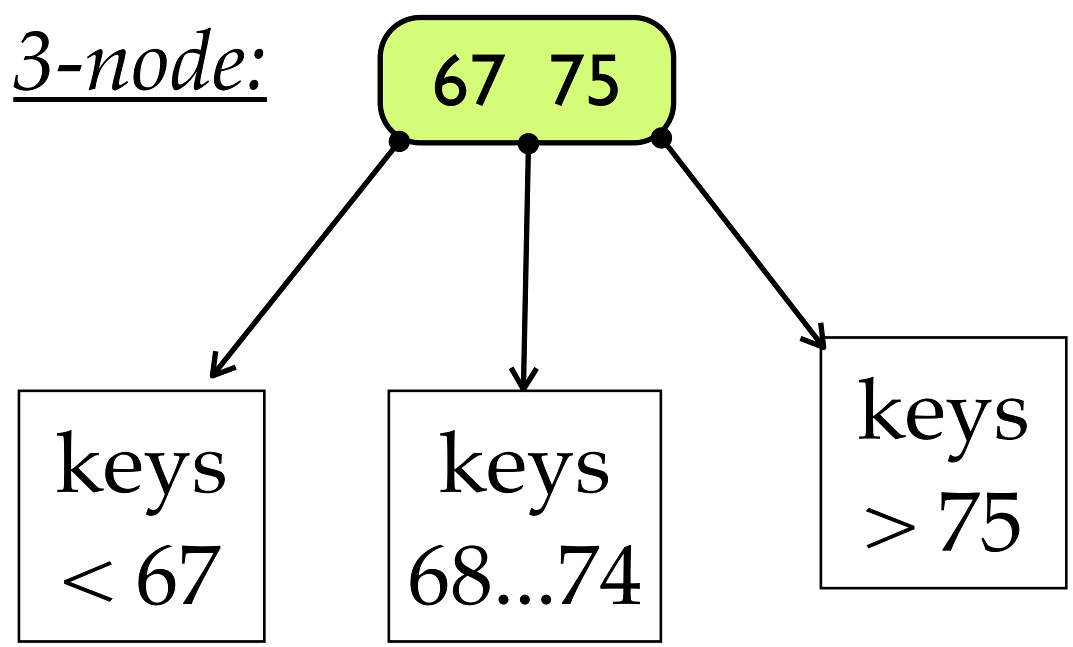
2,3 Trees

- All leaves are at the same level.
- Each internal node has either 2 or 3 children.
- If it has:
  - 2 children => it has 1 key
  - 3 children => it has 2 keys

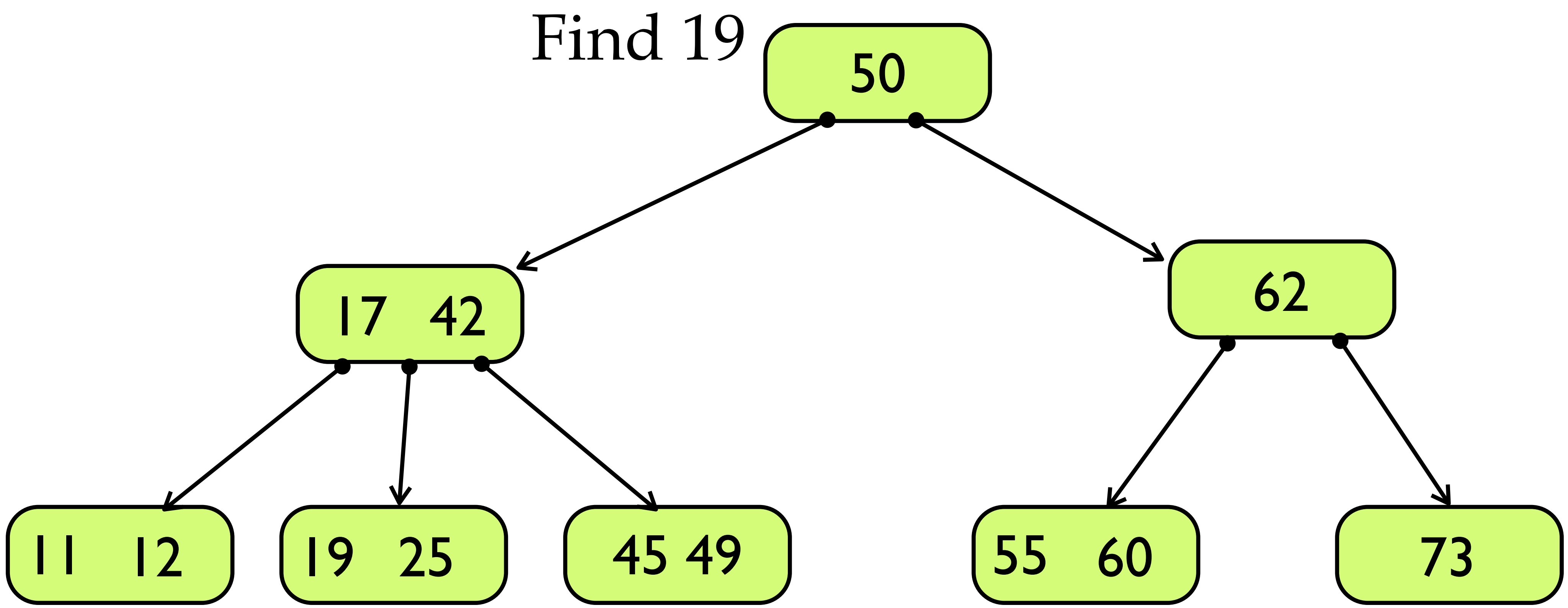
2-node:



3-node:



2,3 Tree Find (multiway searching)



Standard BST-type walk down the tree.
At each node have to examine each key stored there.

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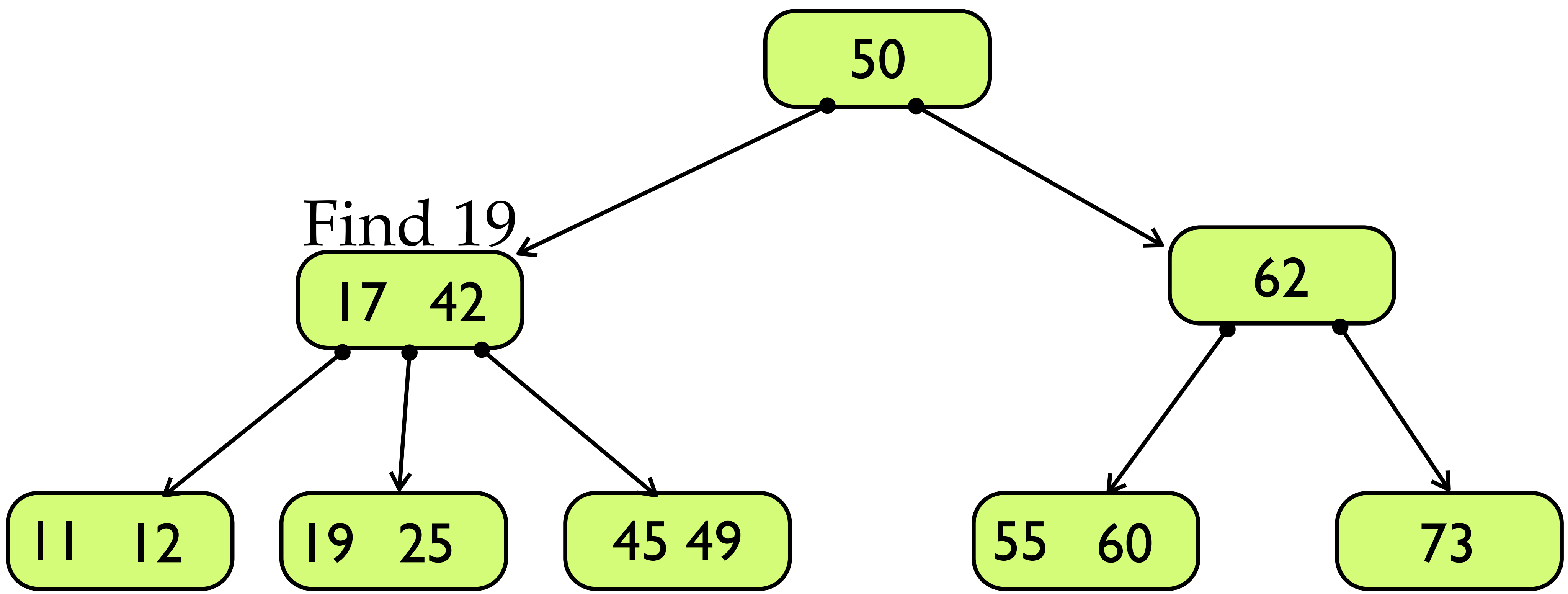

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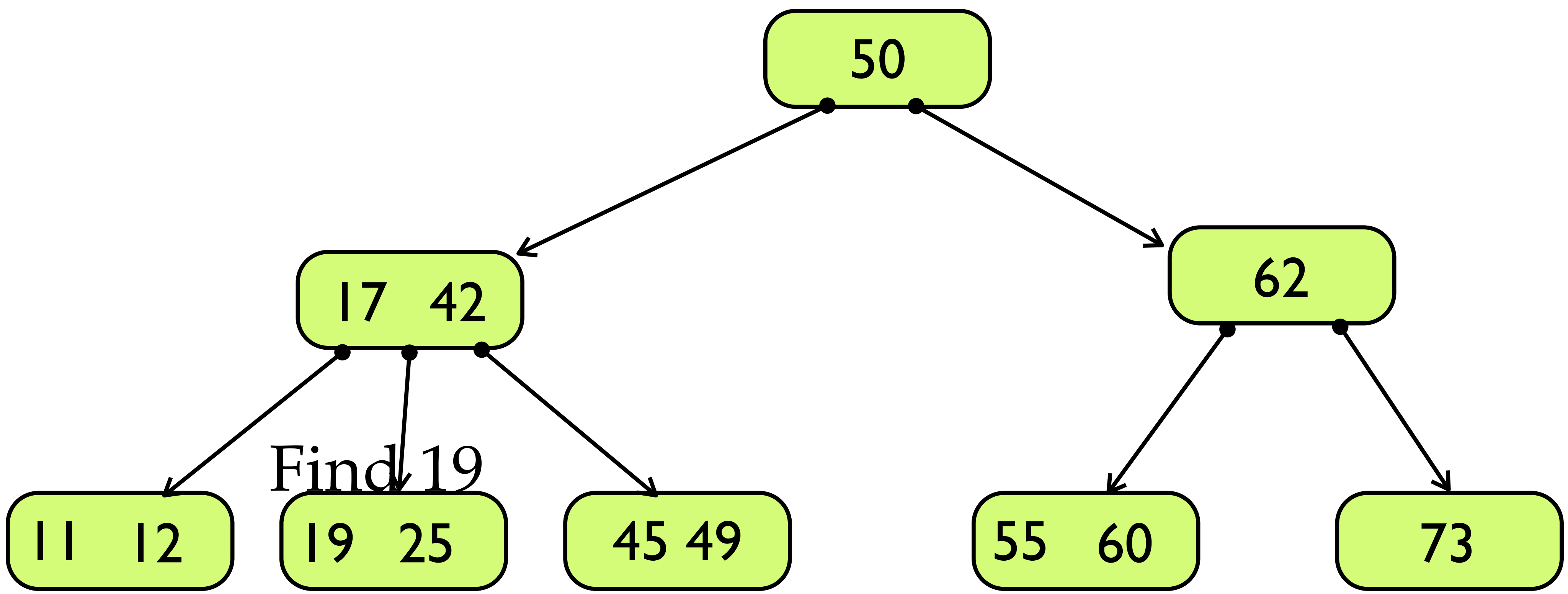

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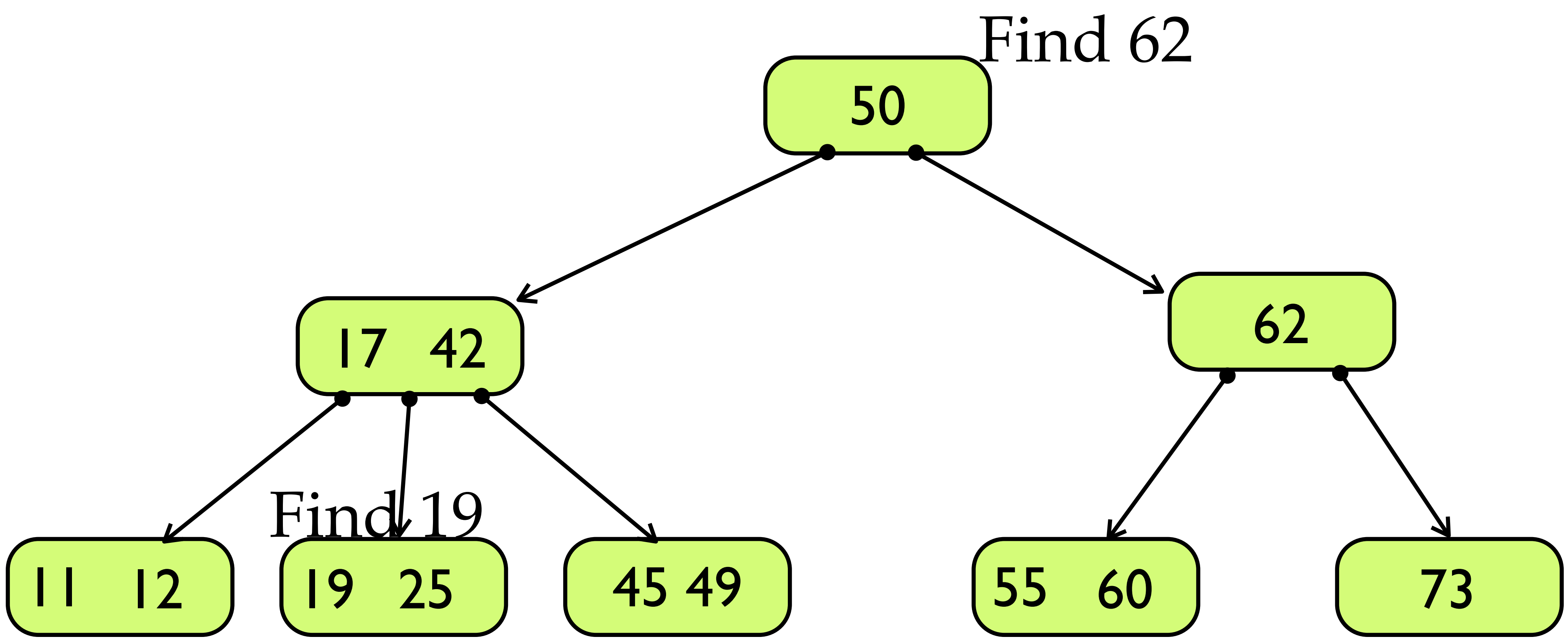

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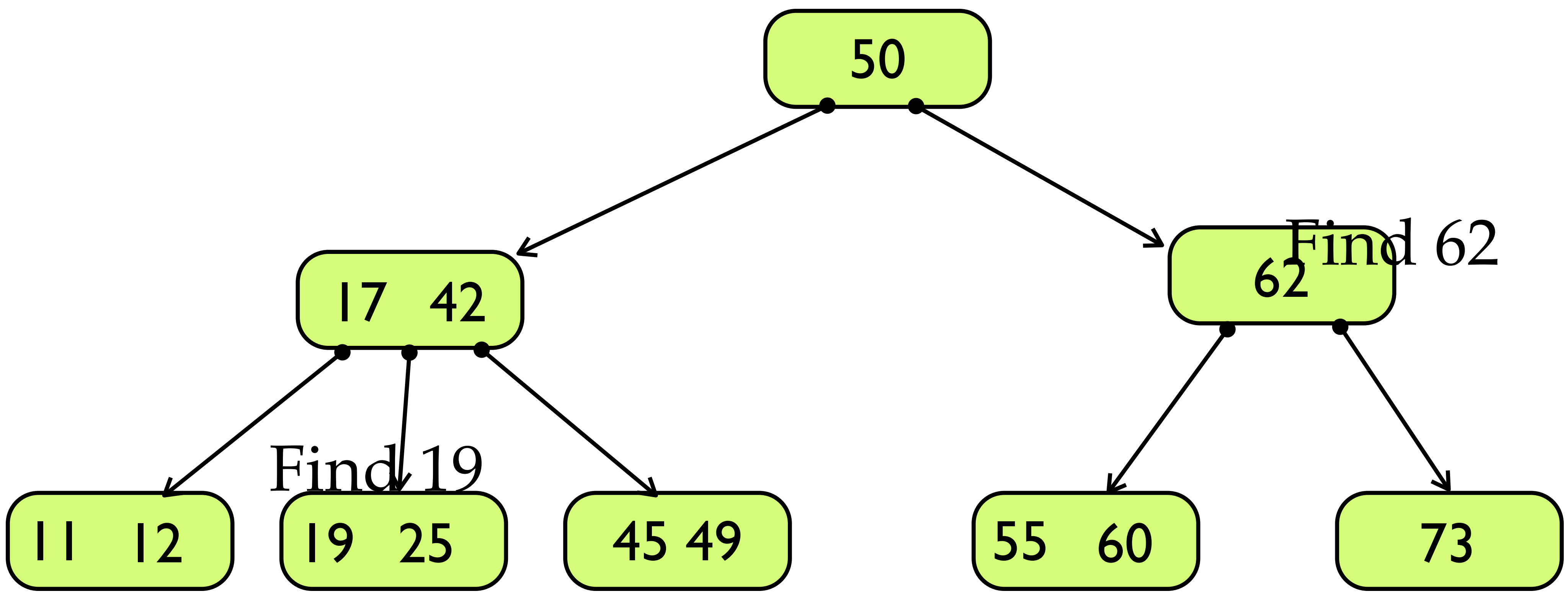

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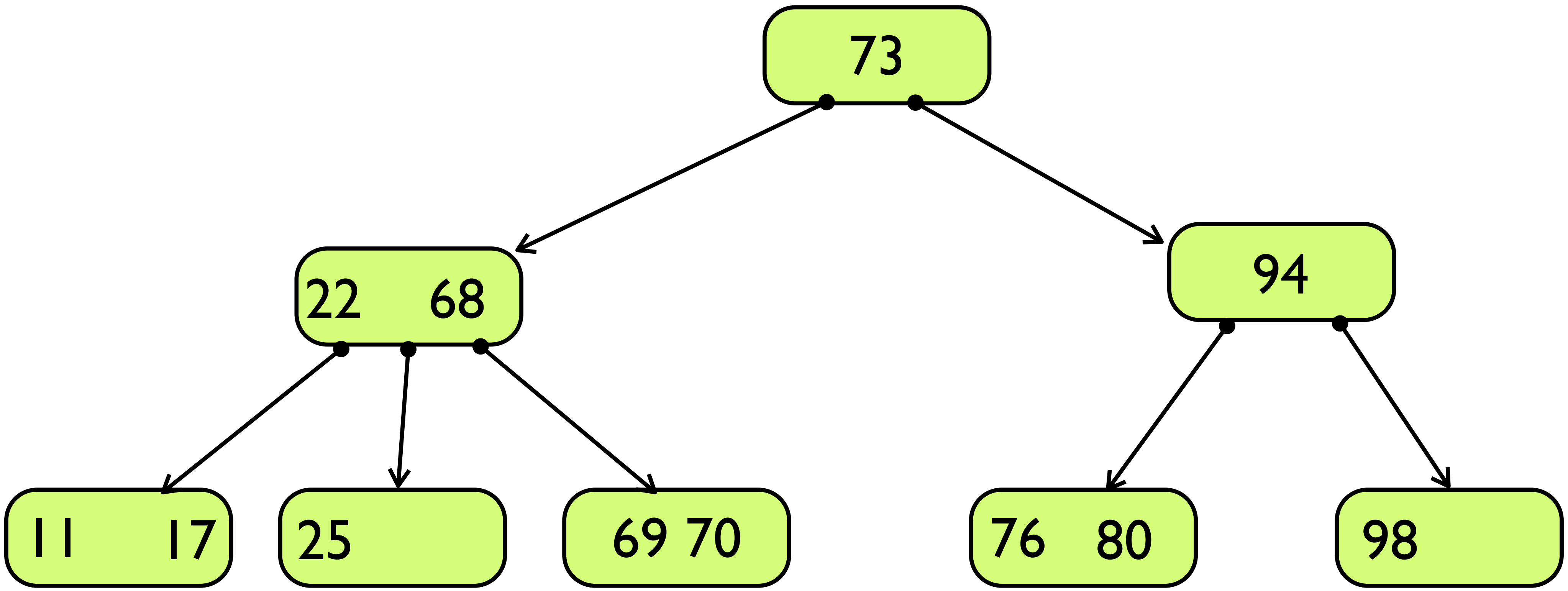

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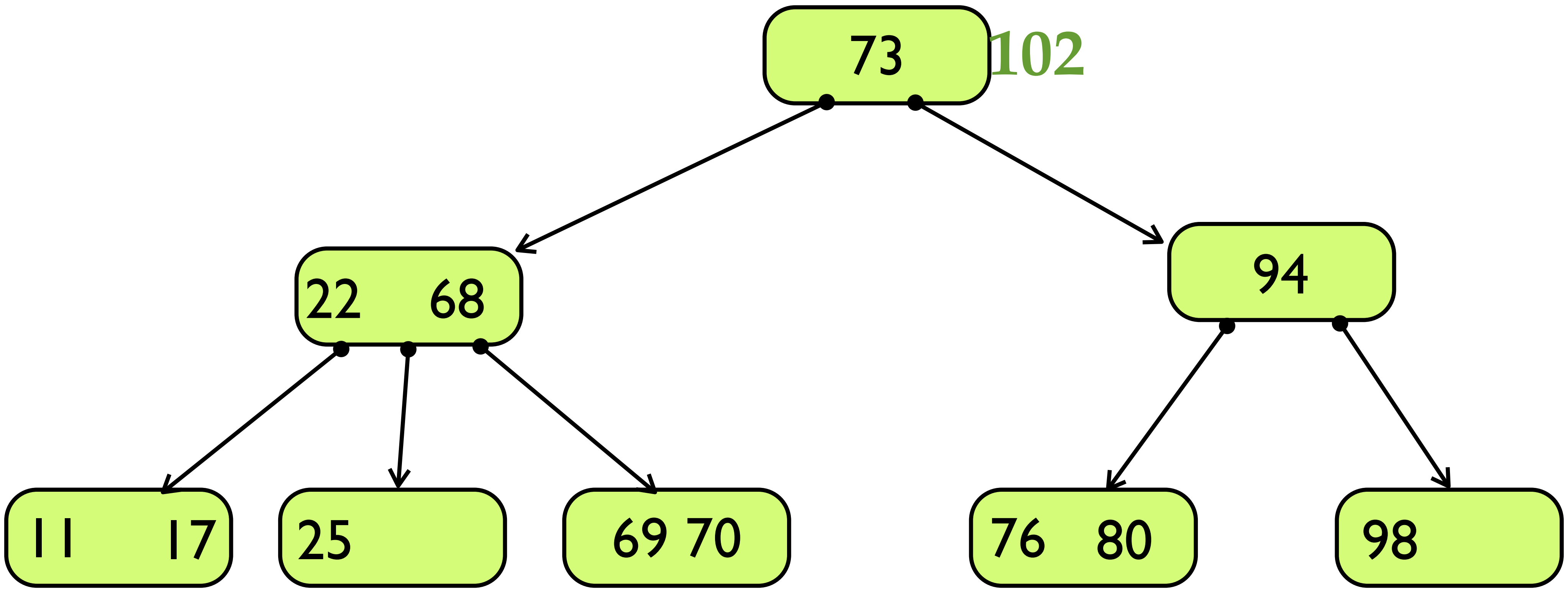

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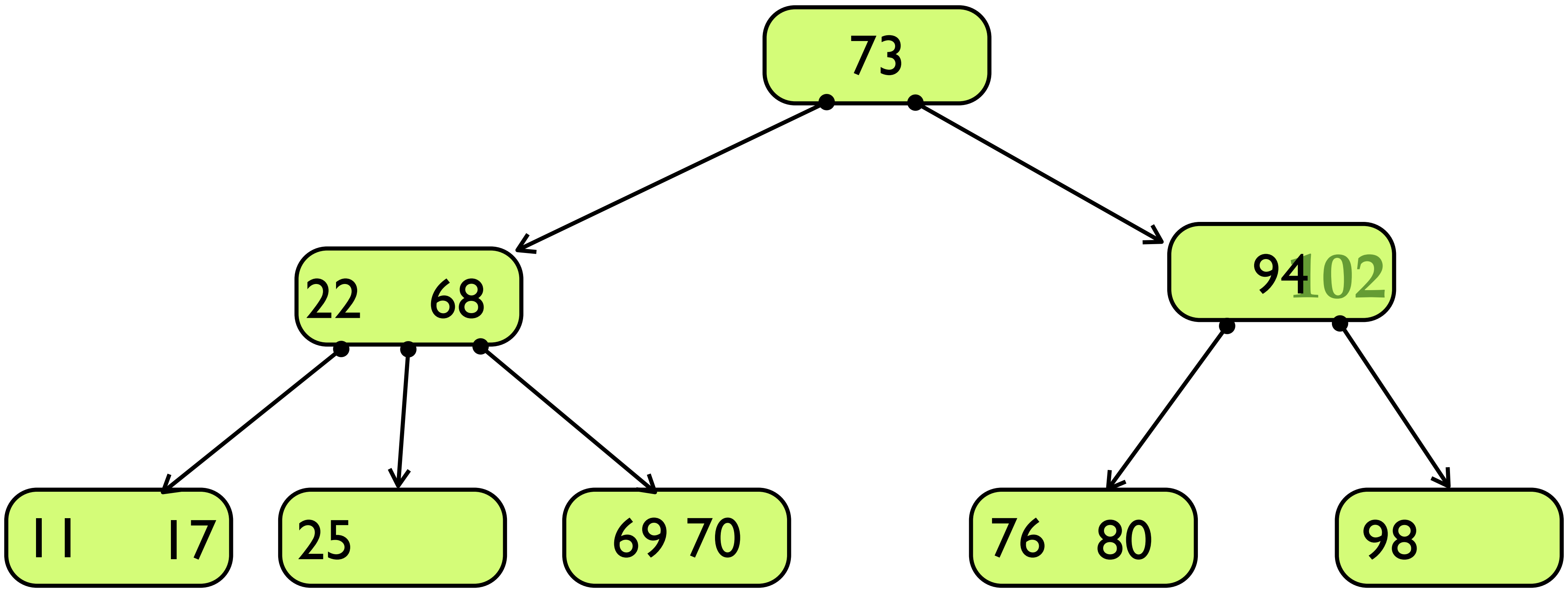

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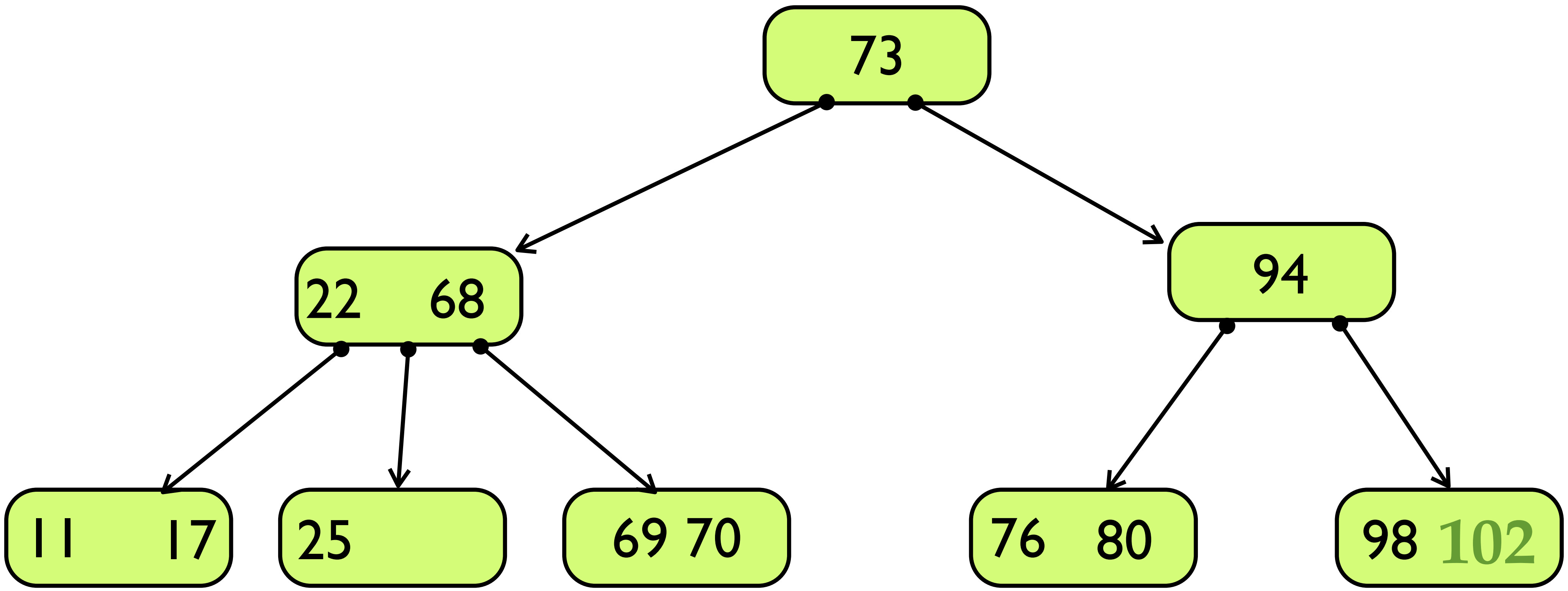

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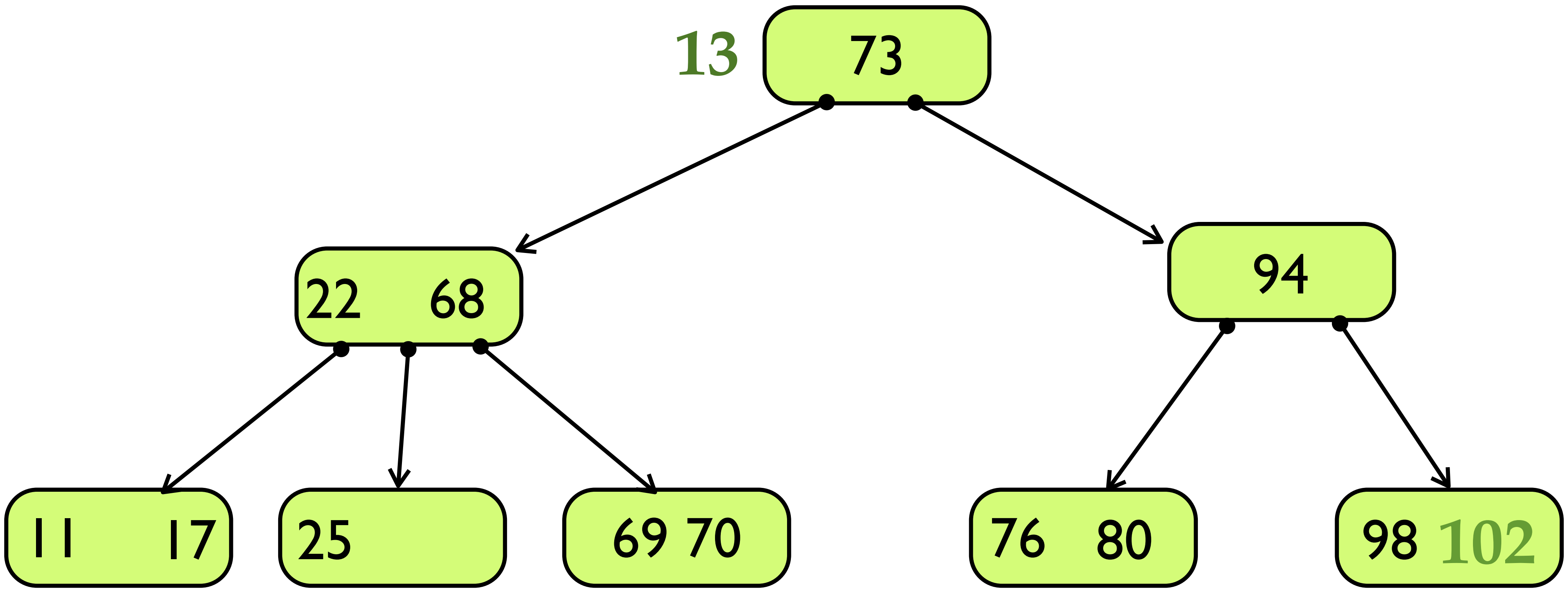

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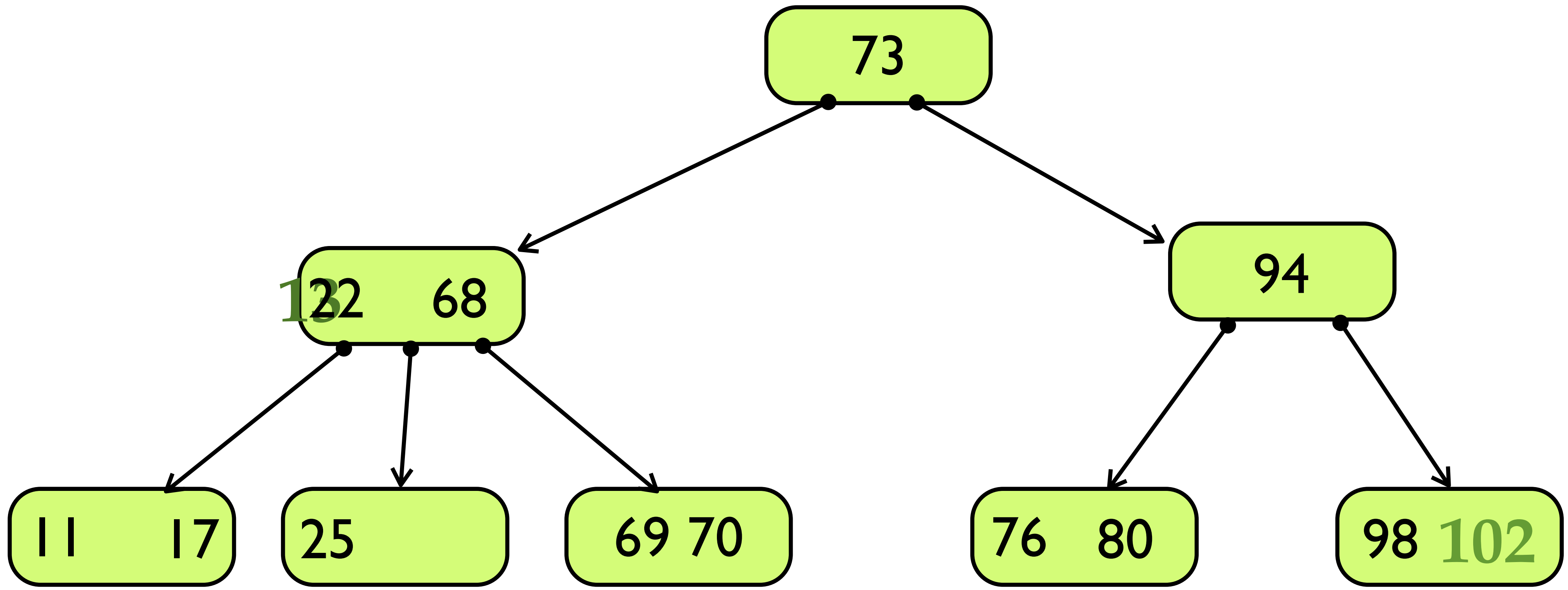

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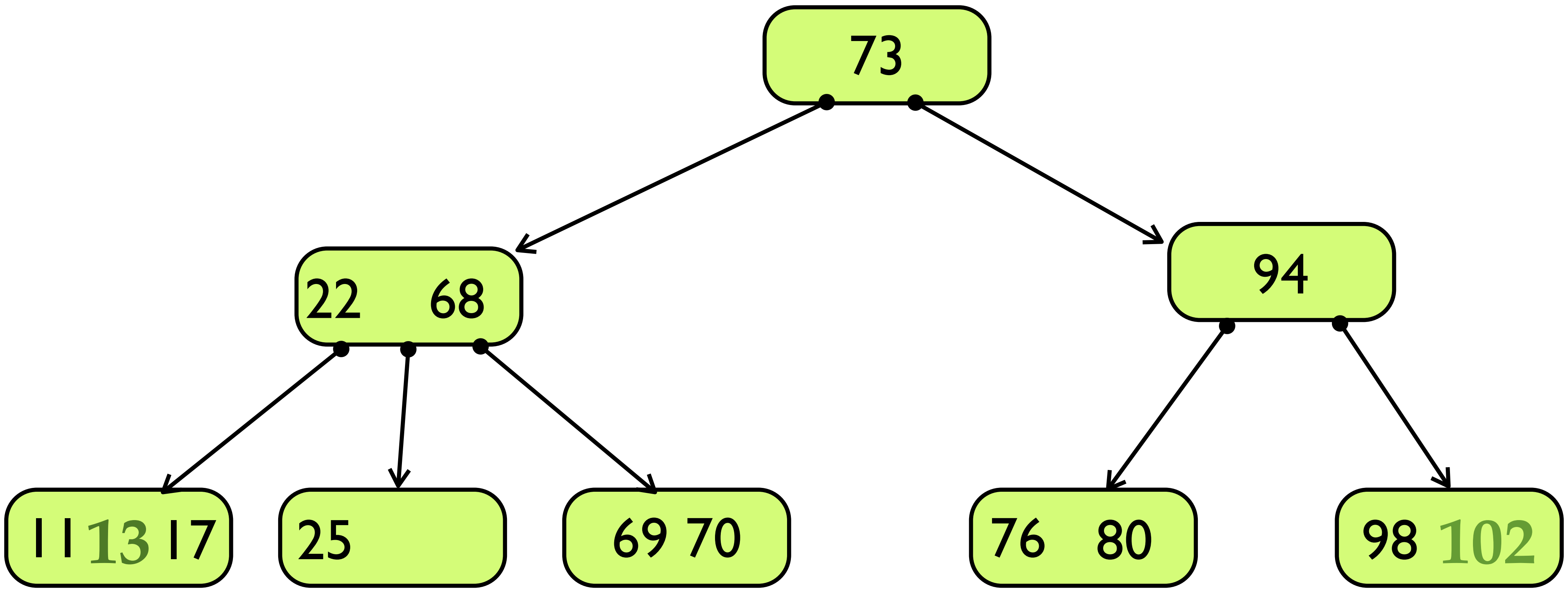

Overflow!

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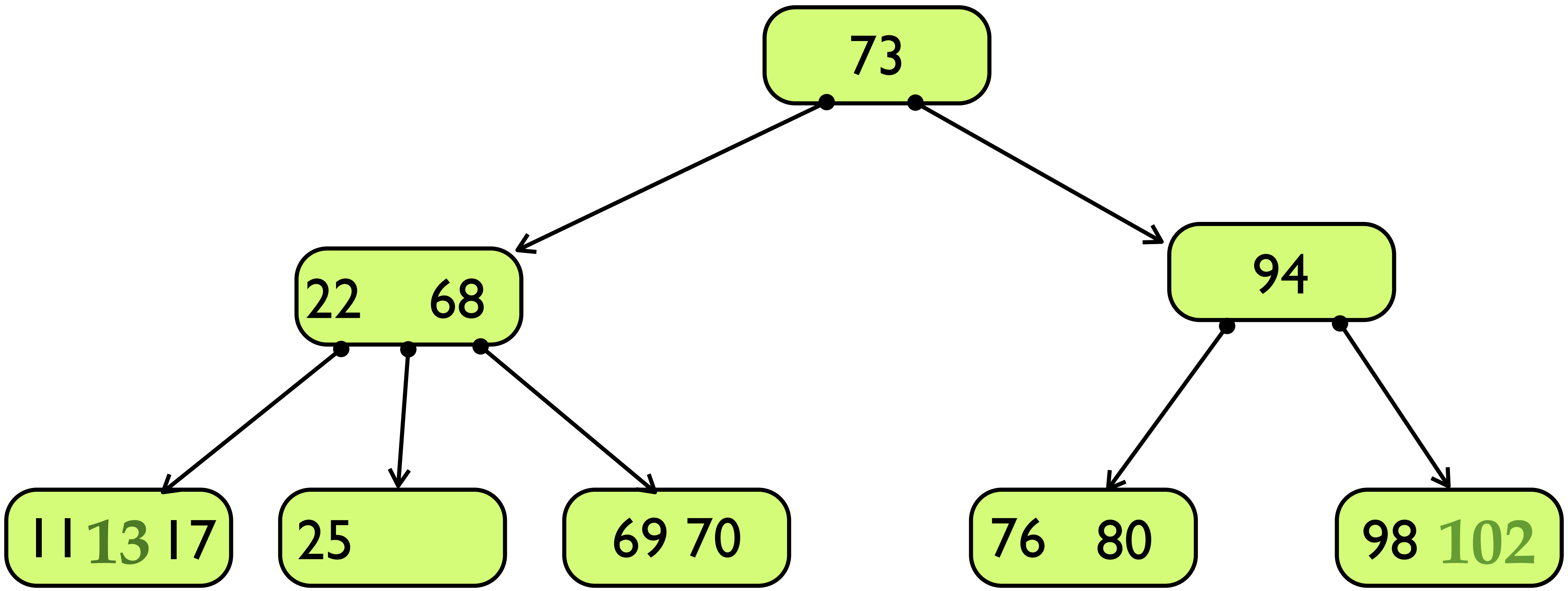

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Try **Key Rotation:** Look for left or right sibling with some space, move a parent key into it, and a child key into the parent

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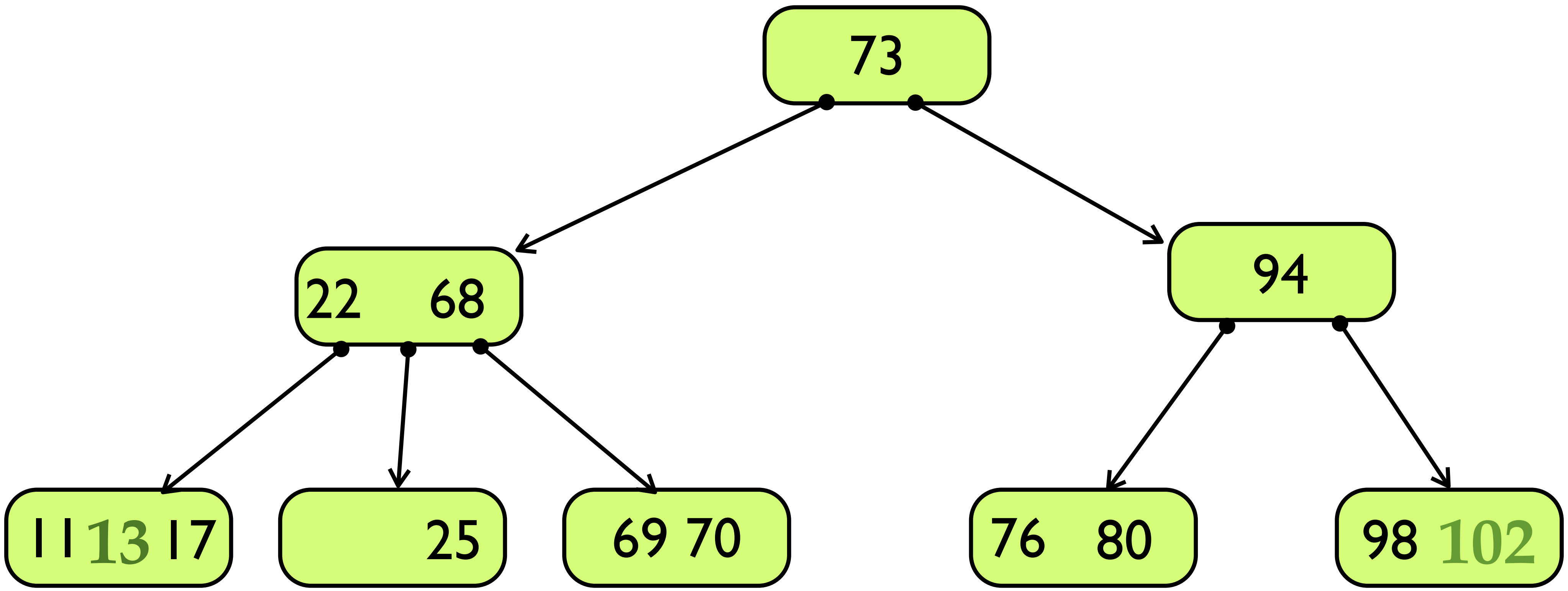

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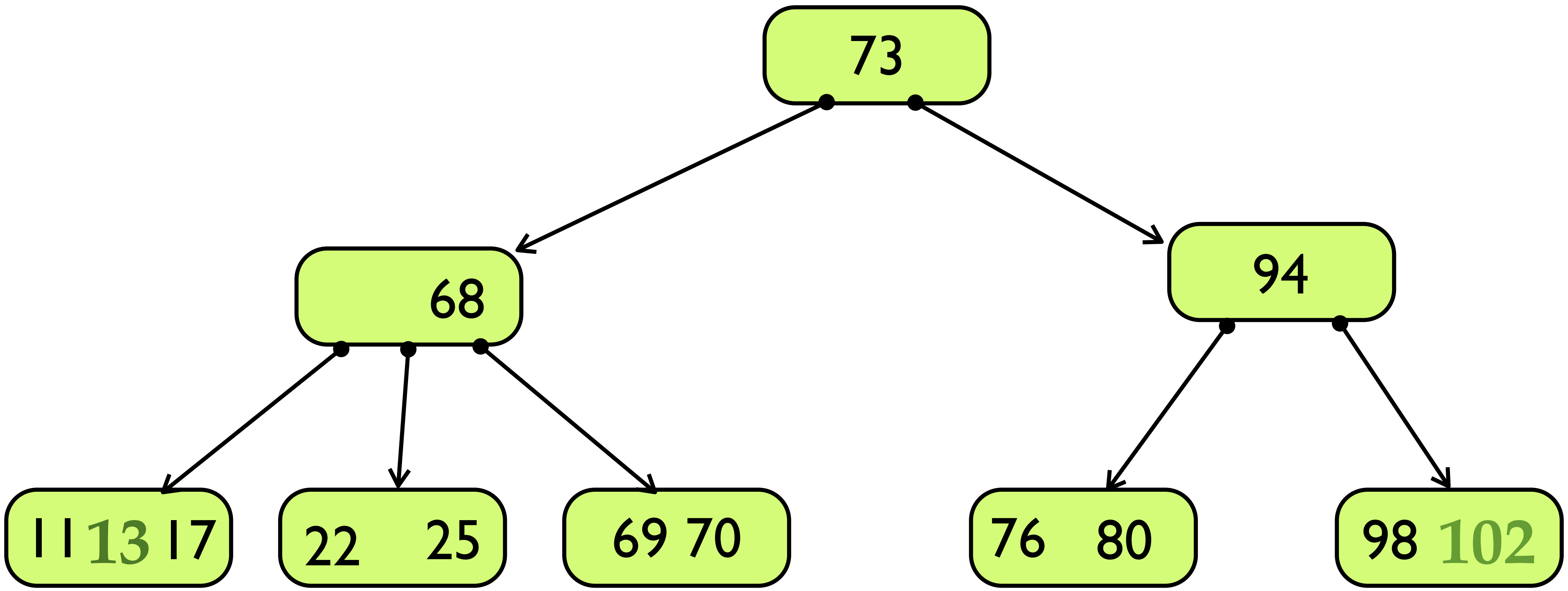

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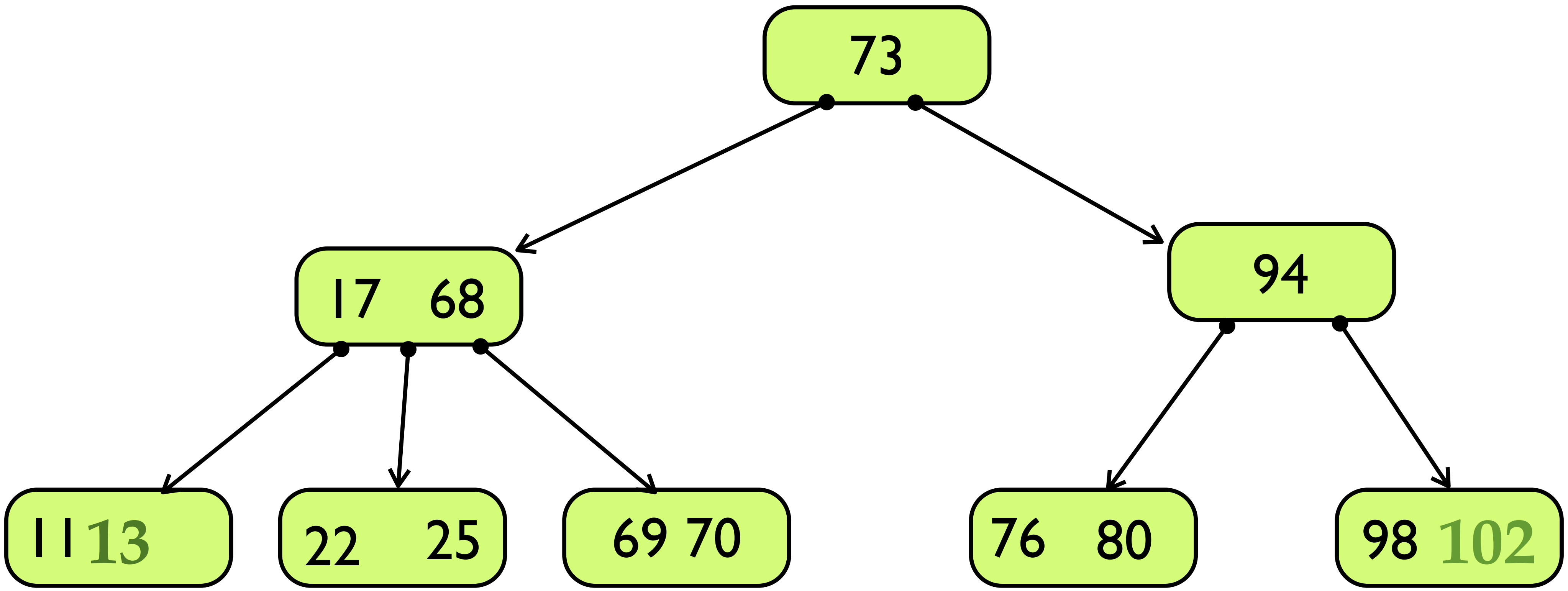

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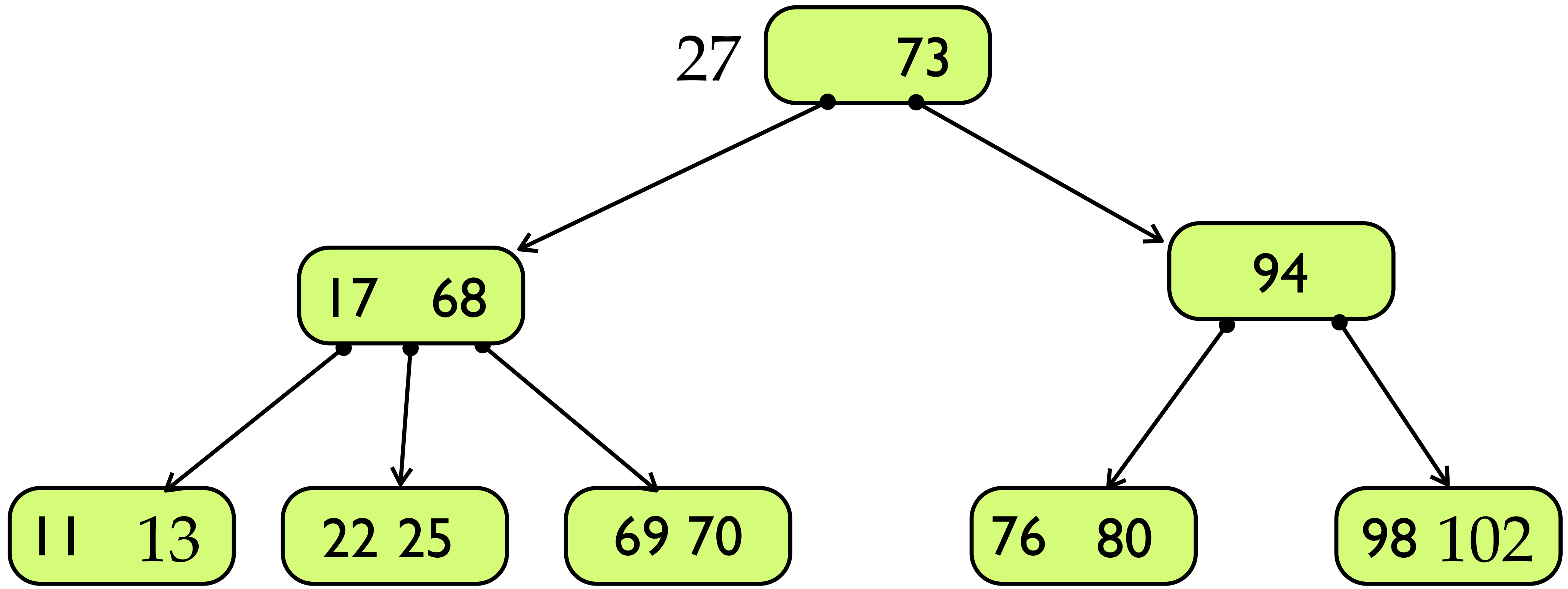

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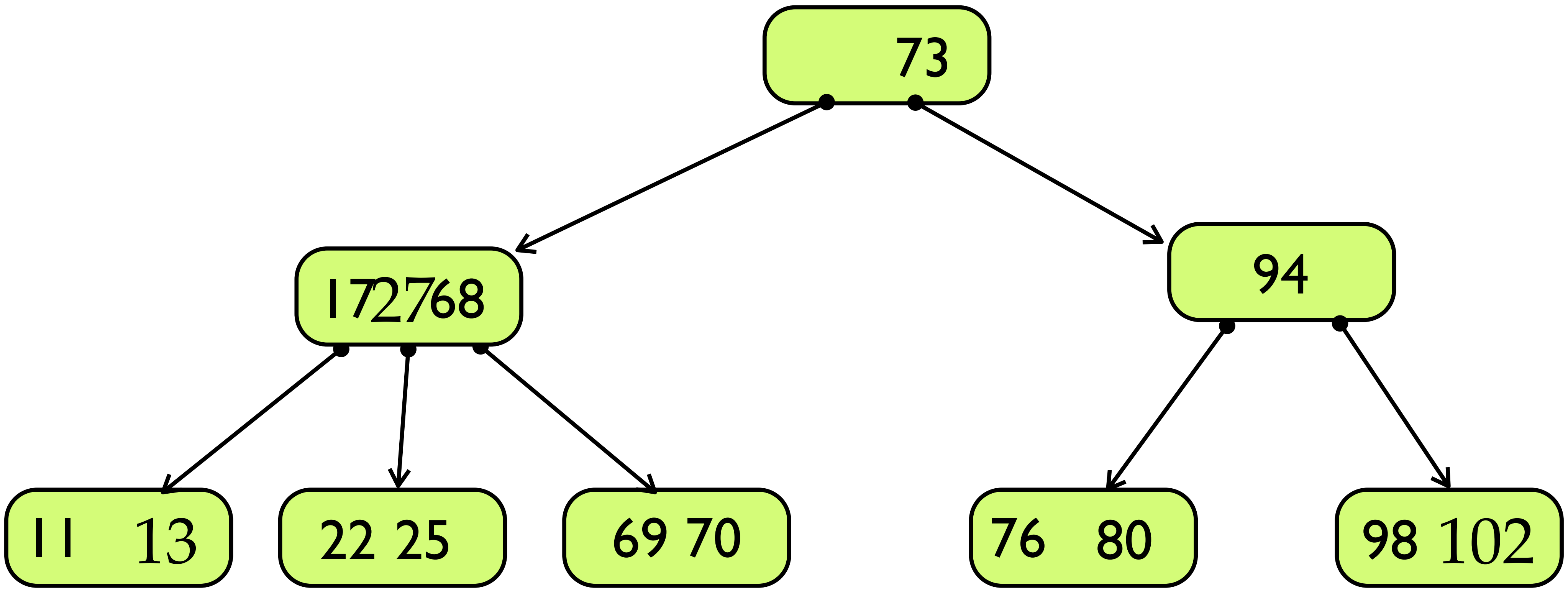

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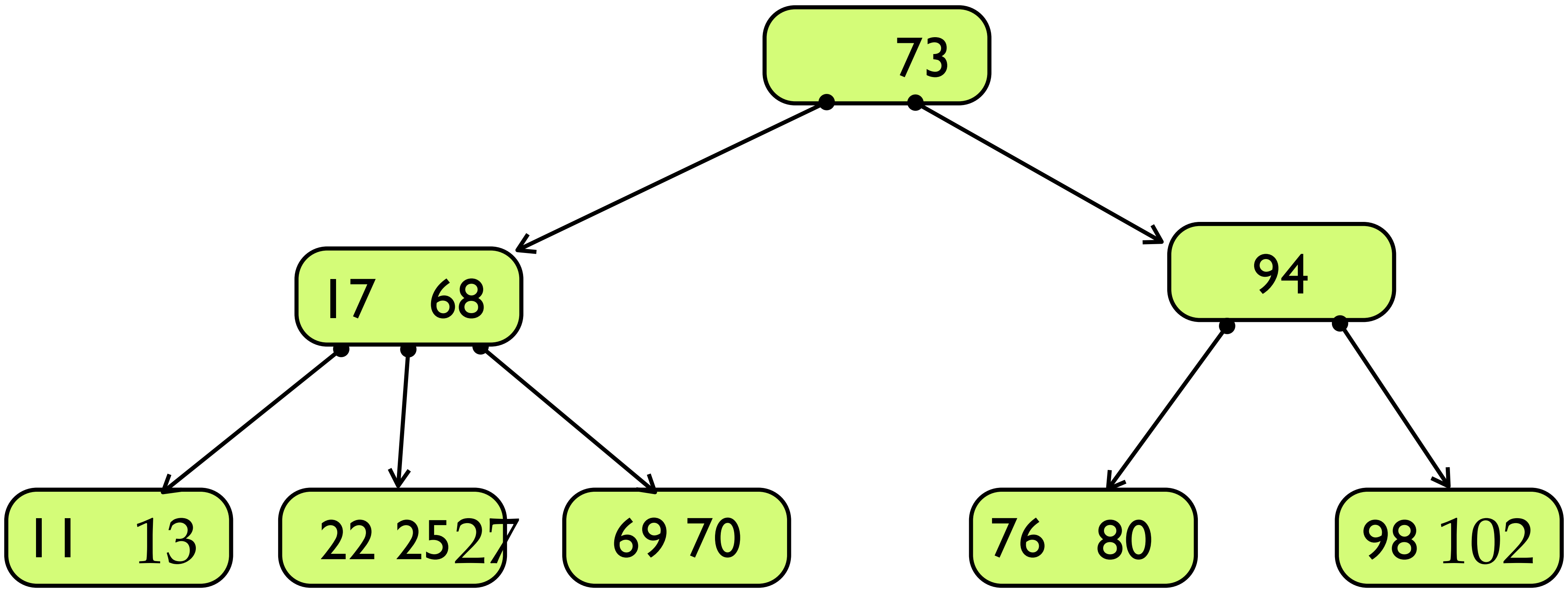

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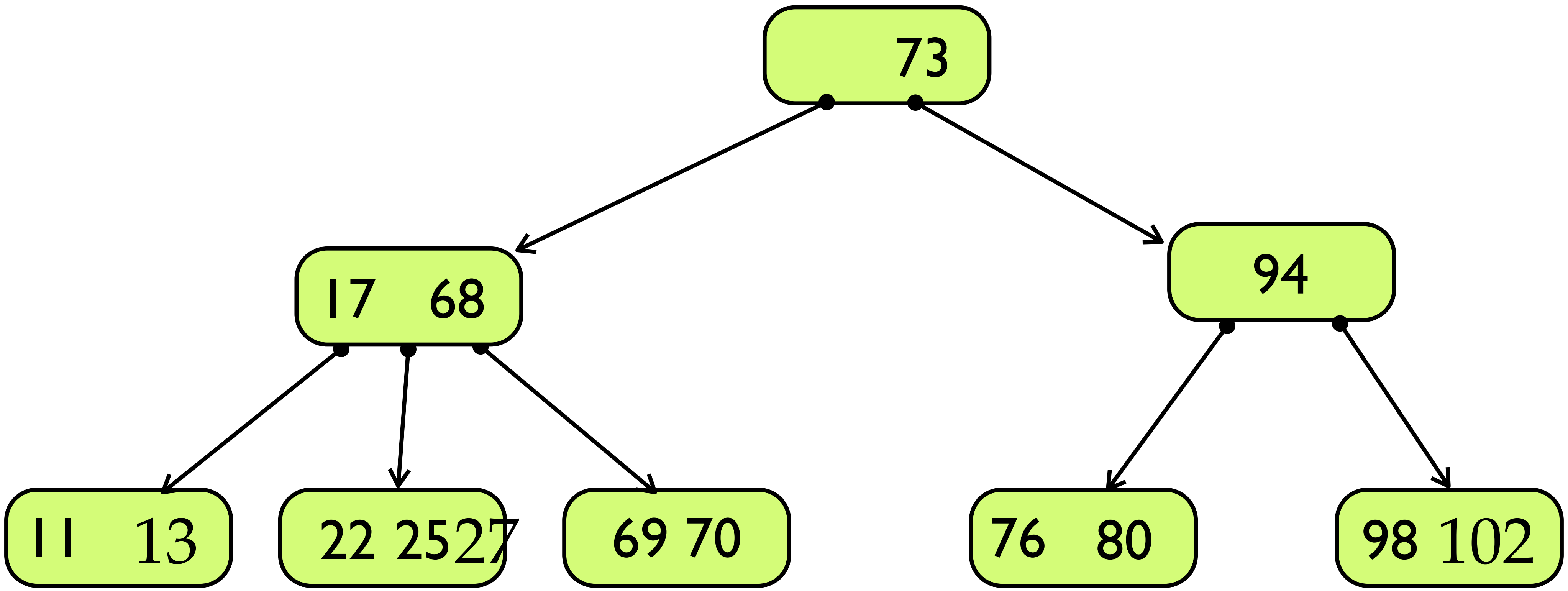

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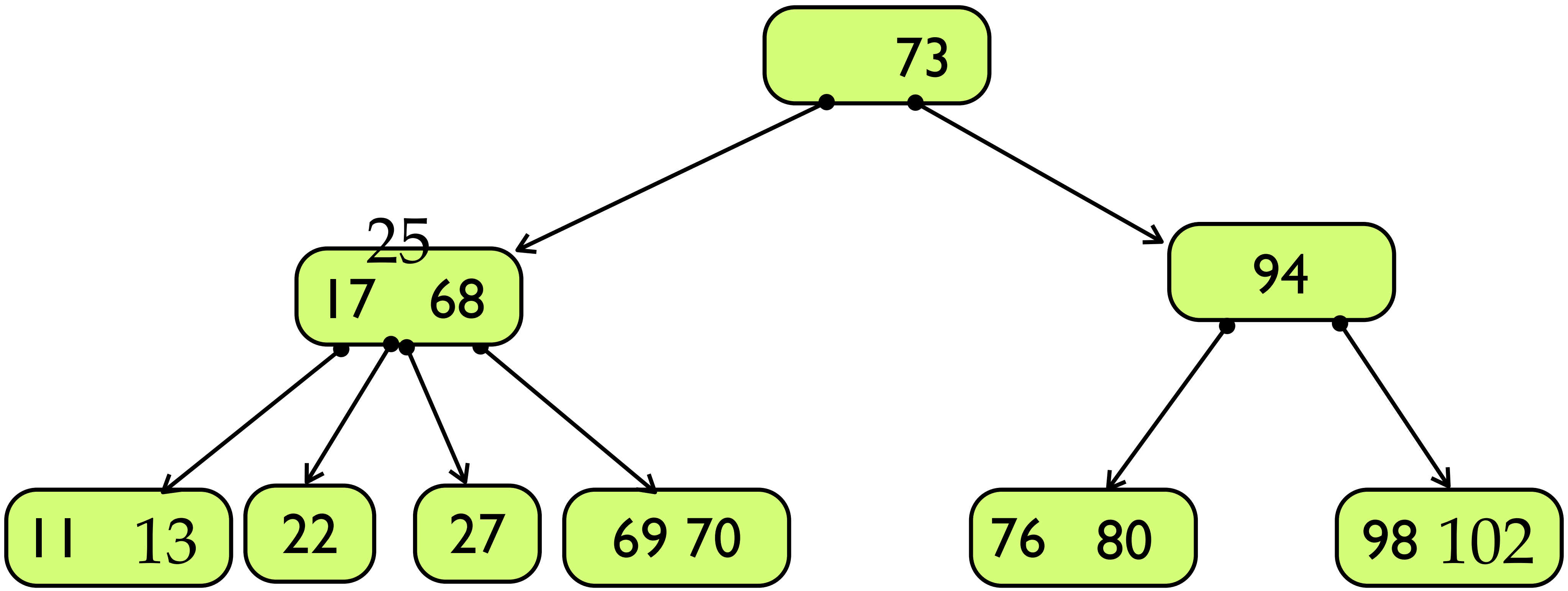

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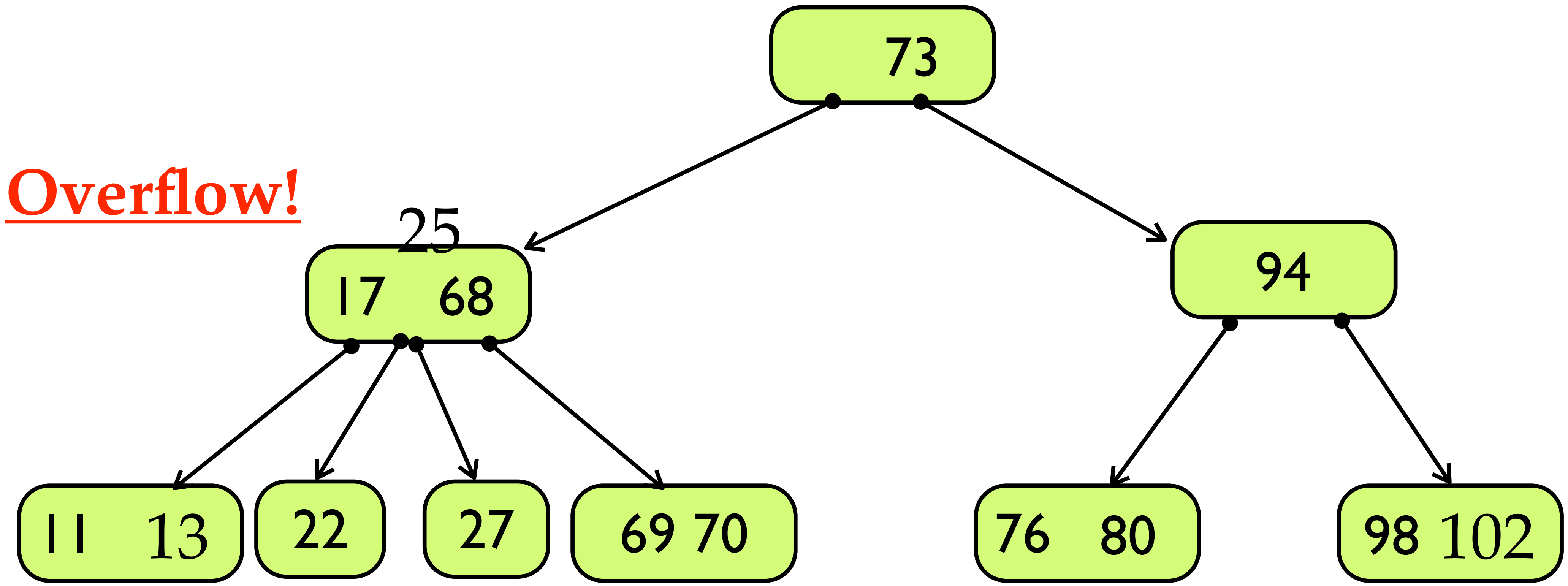

May have to recursively split nodes, working back to the root.

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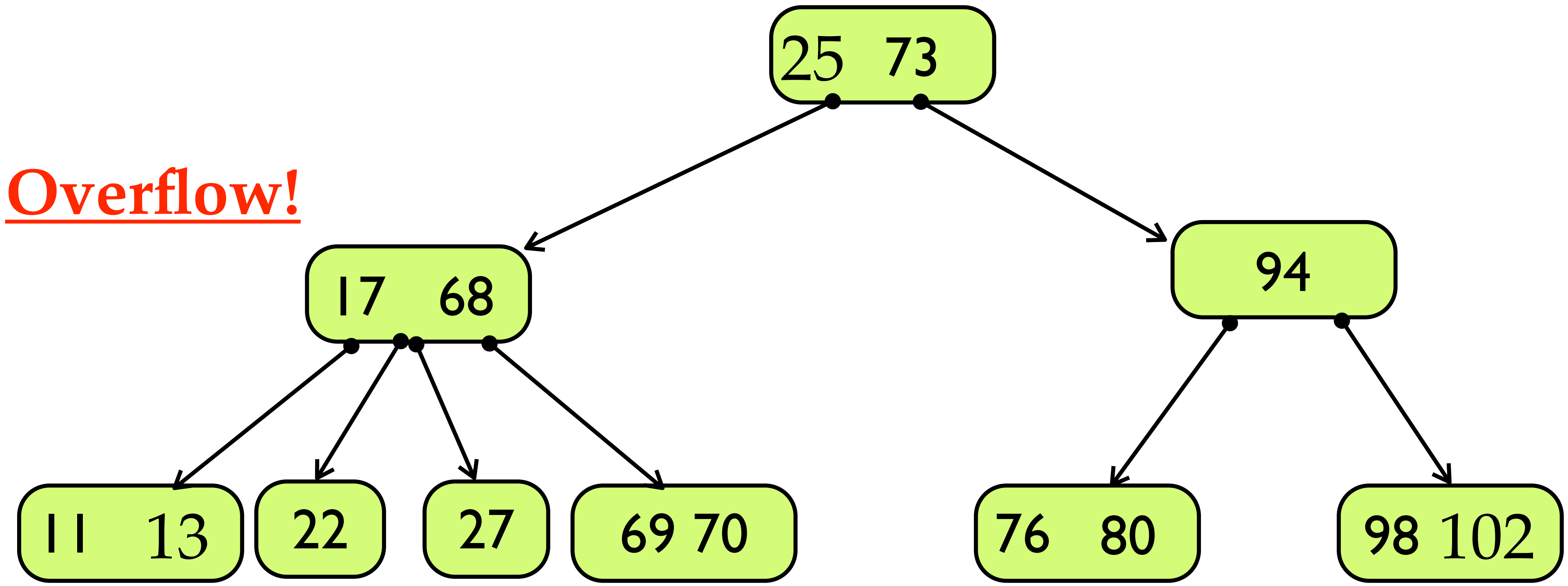

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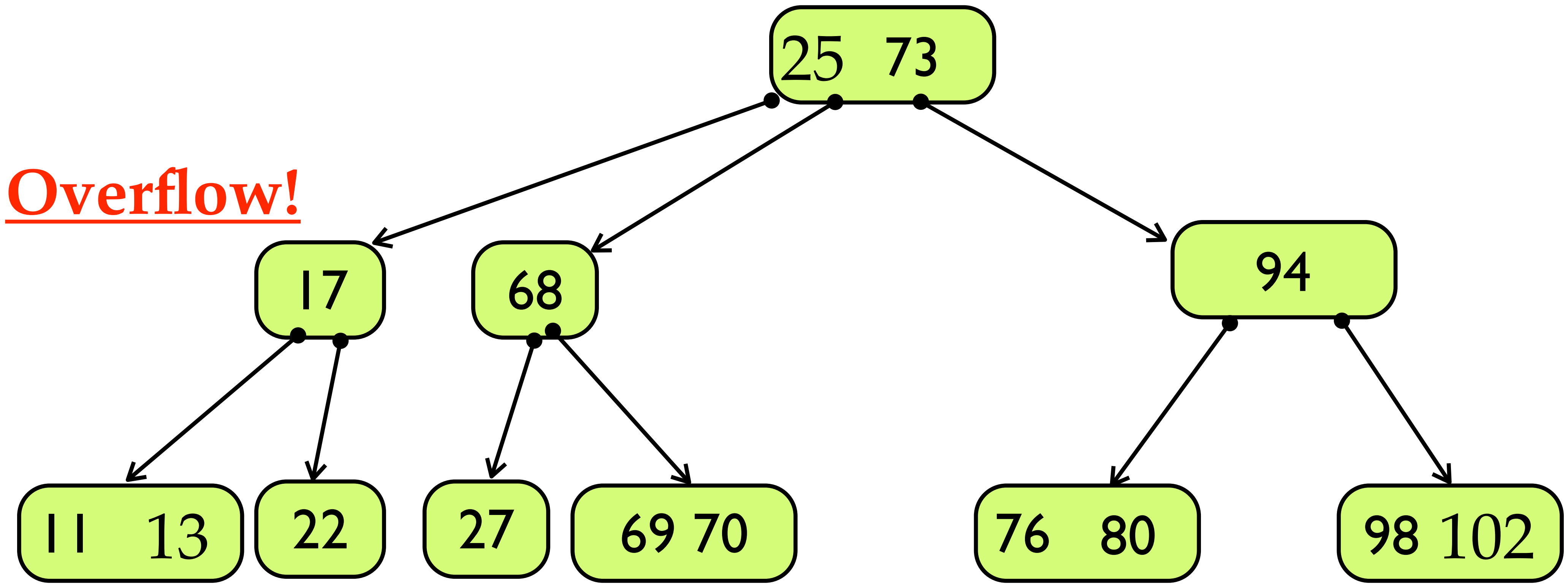

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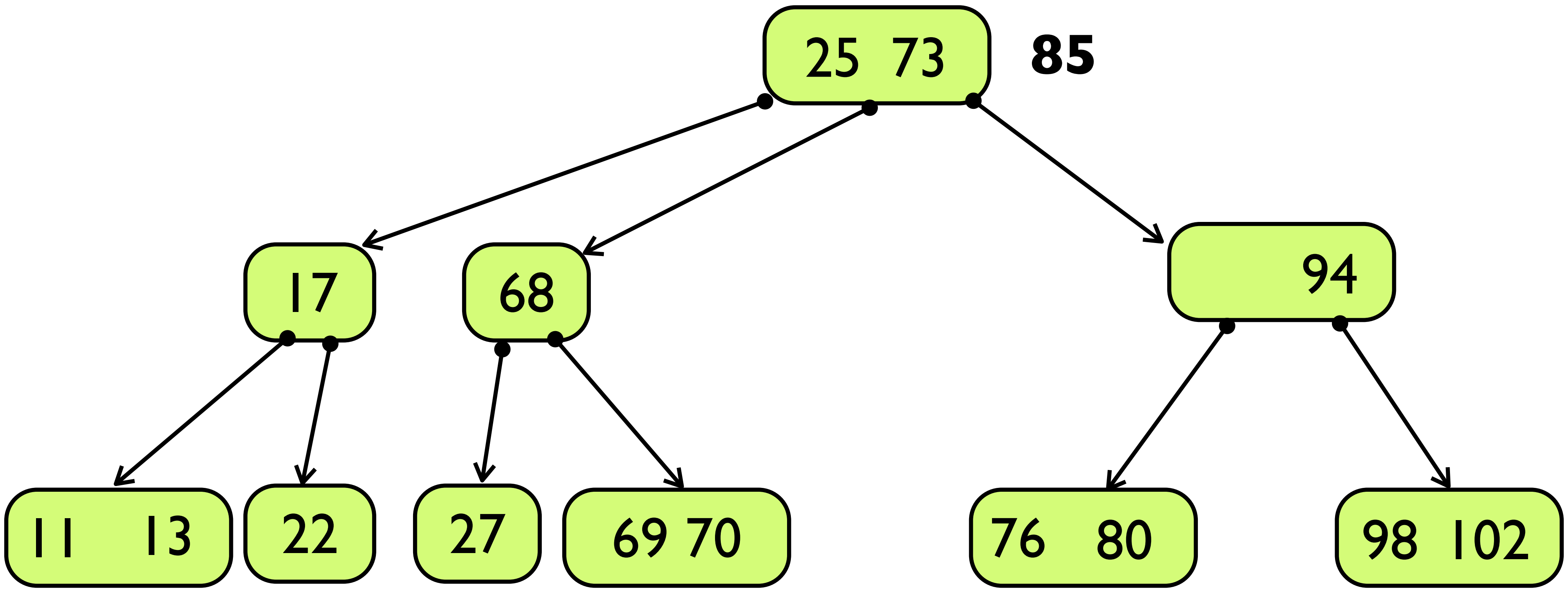
2,3 Tree Insertion – Another Splitting Example



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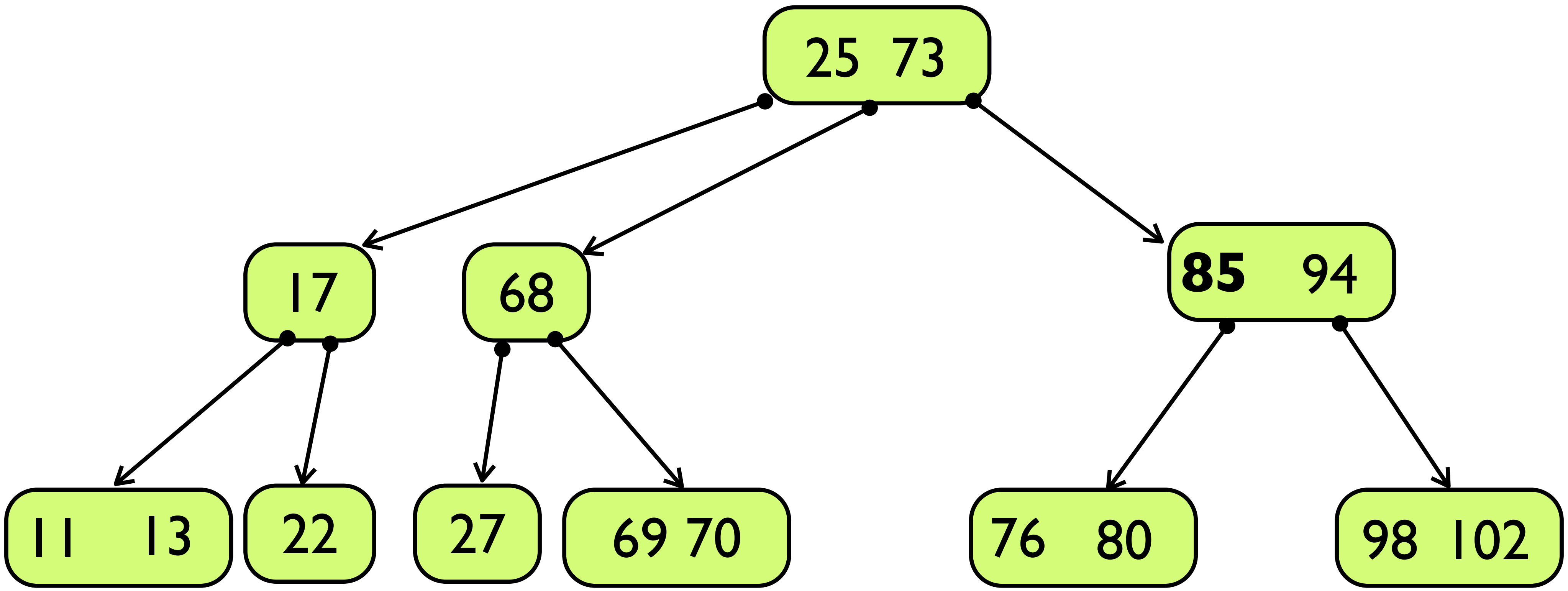

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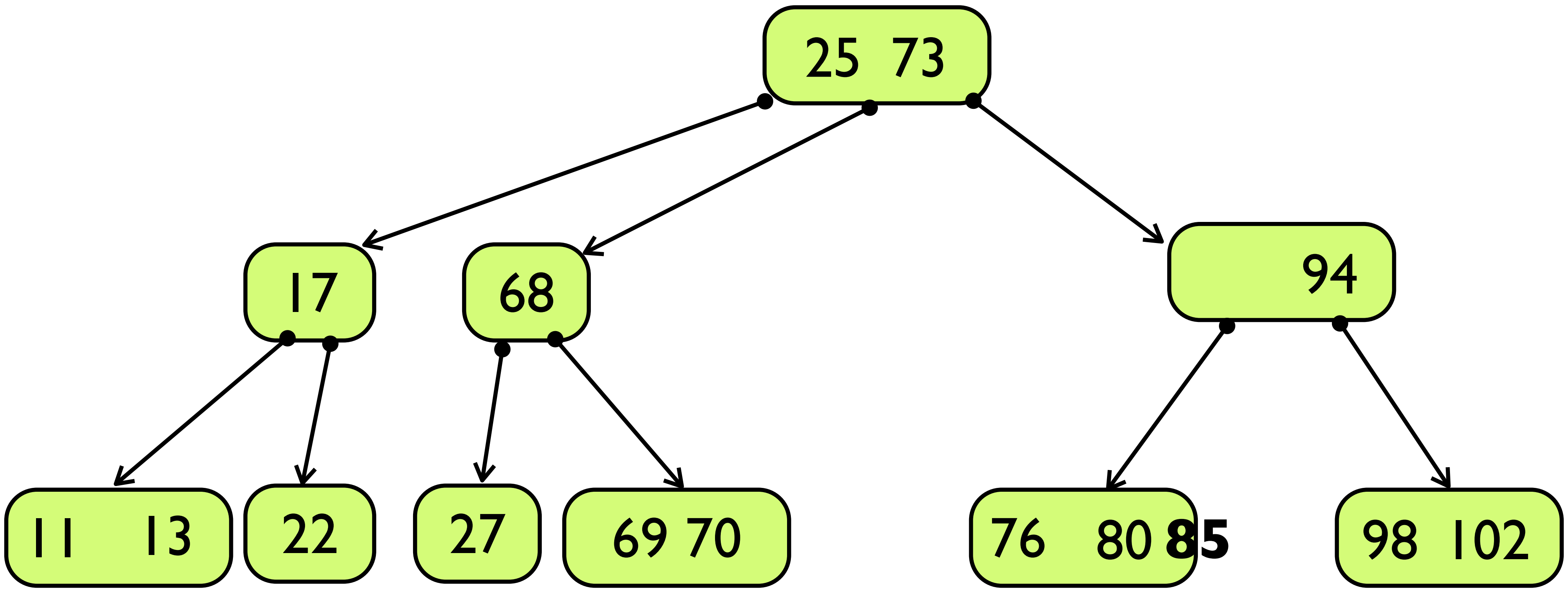

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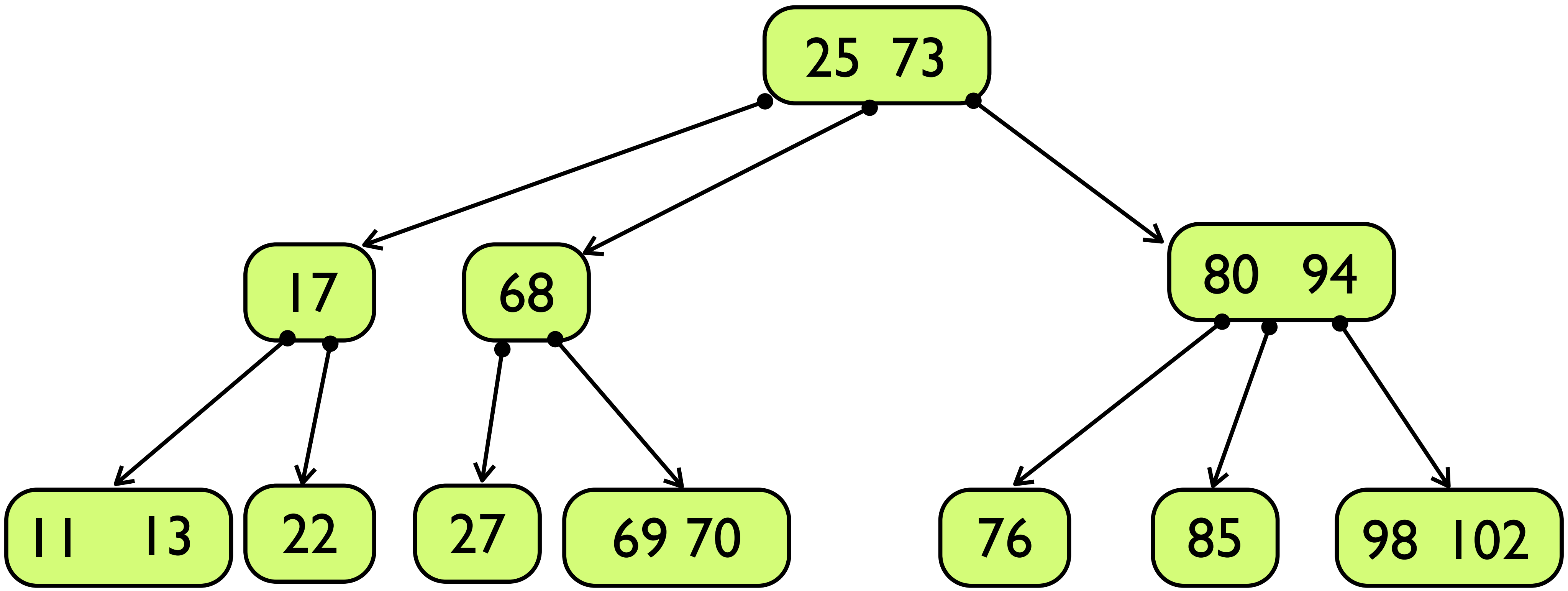

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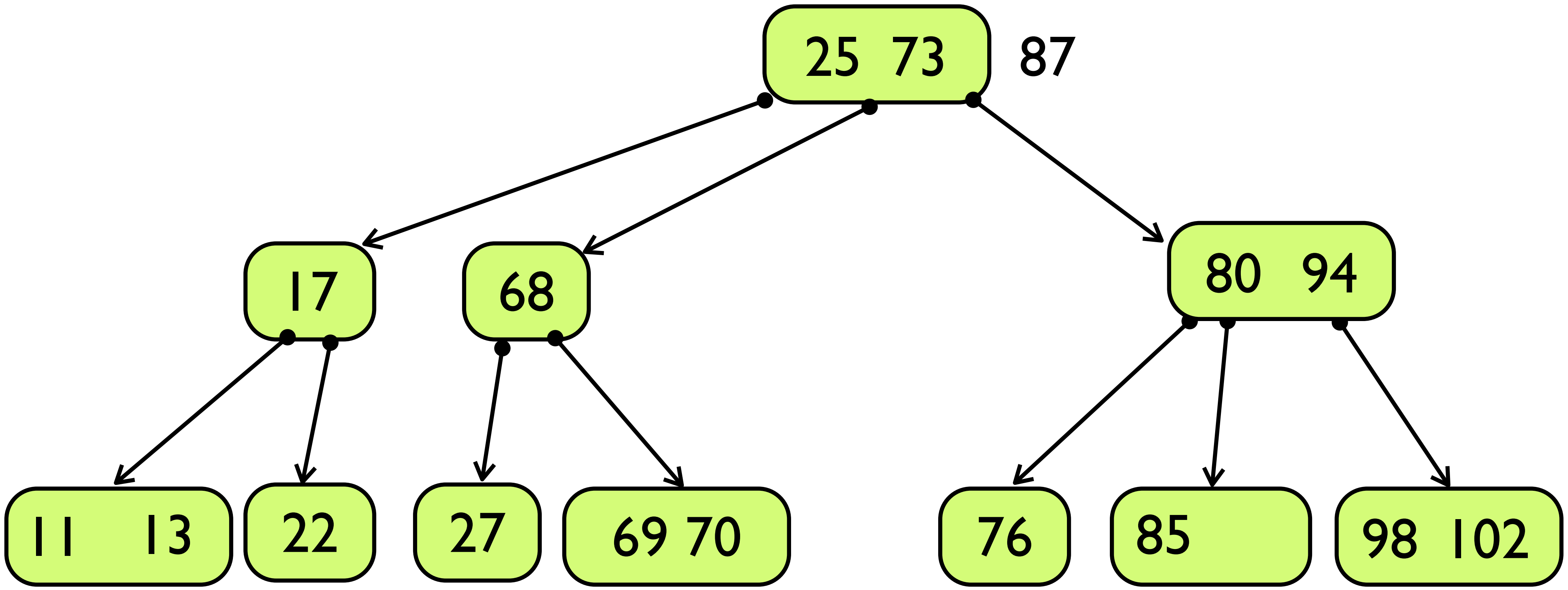

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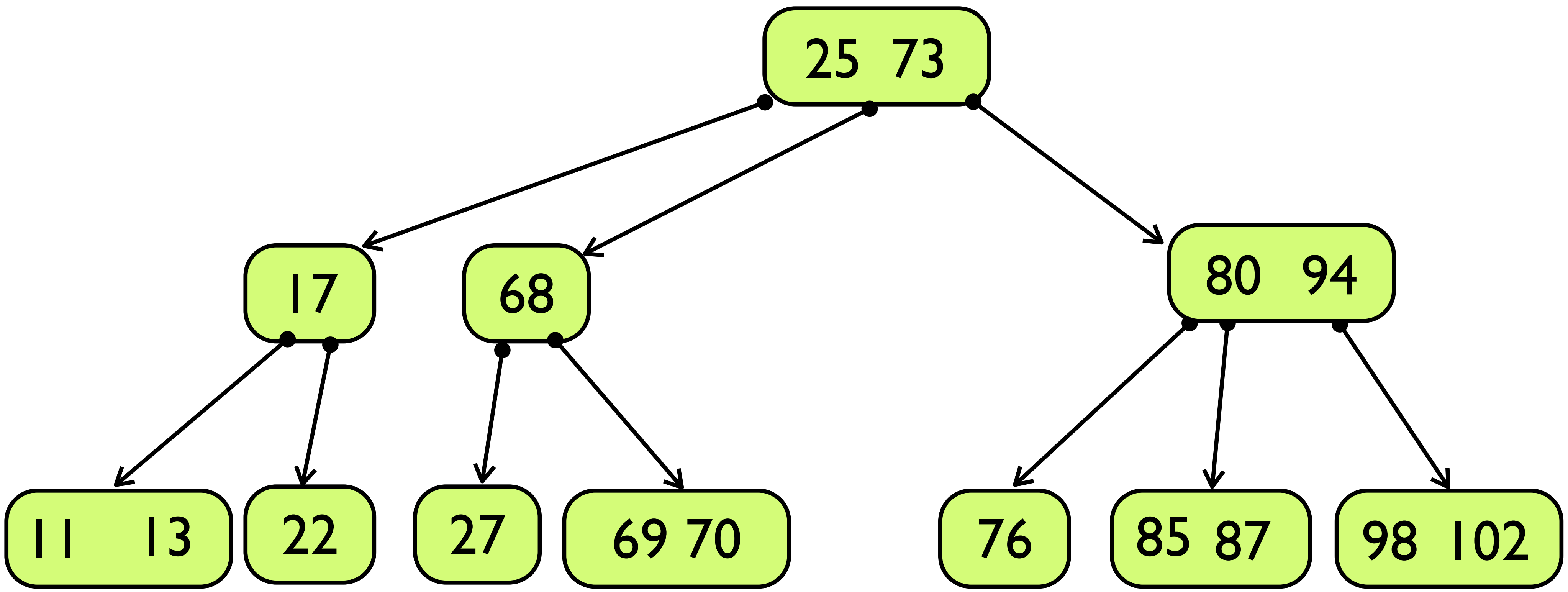

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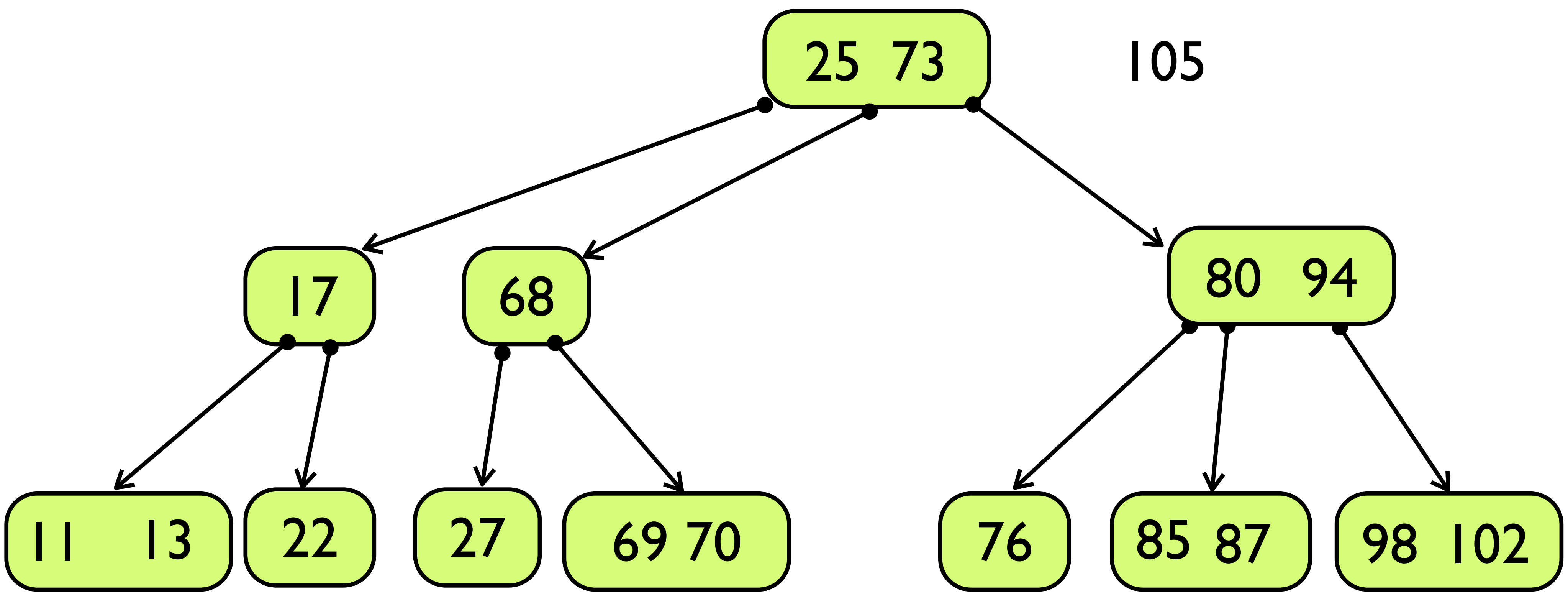

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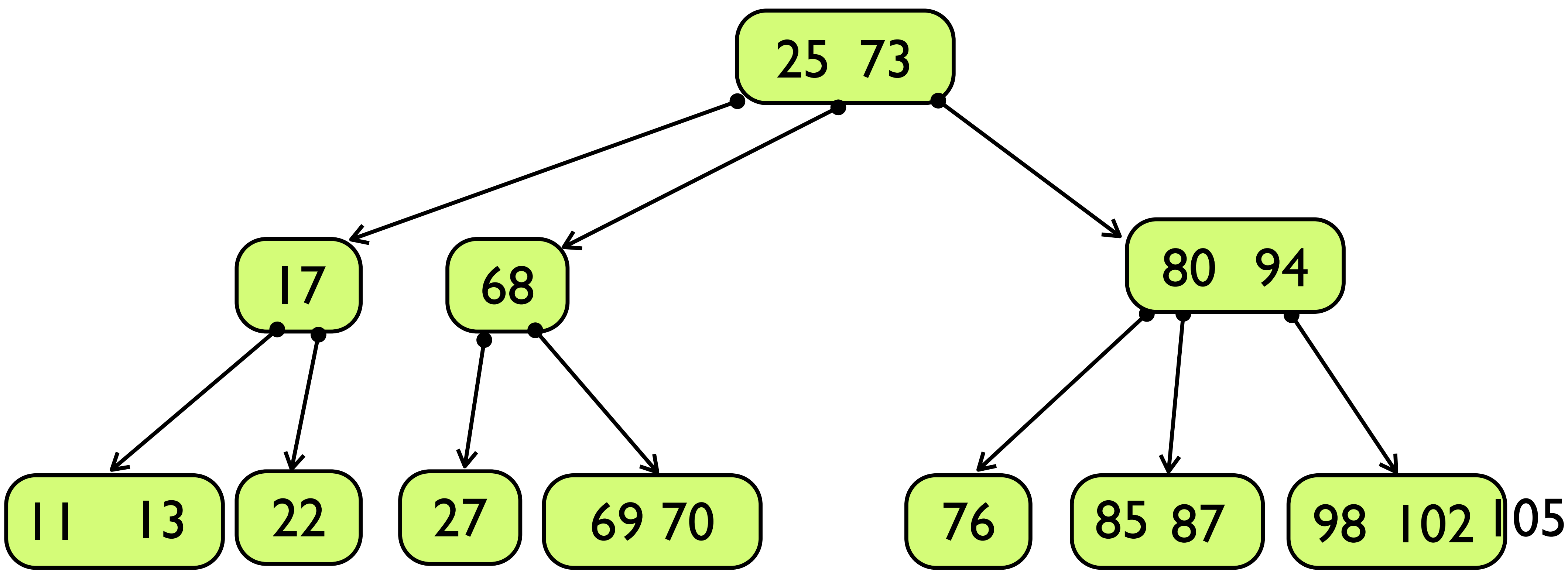

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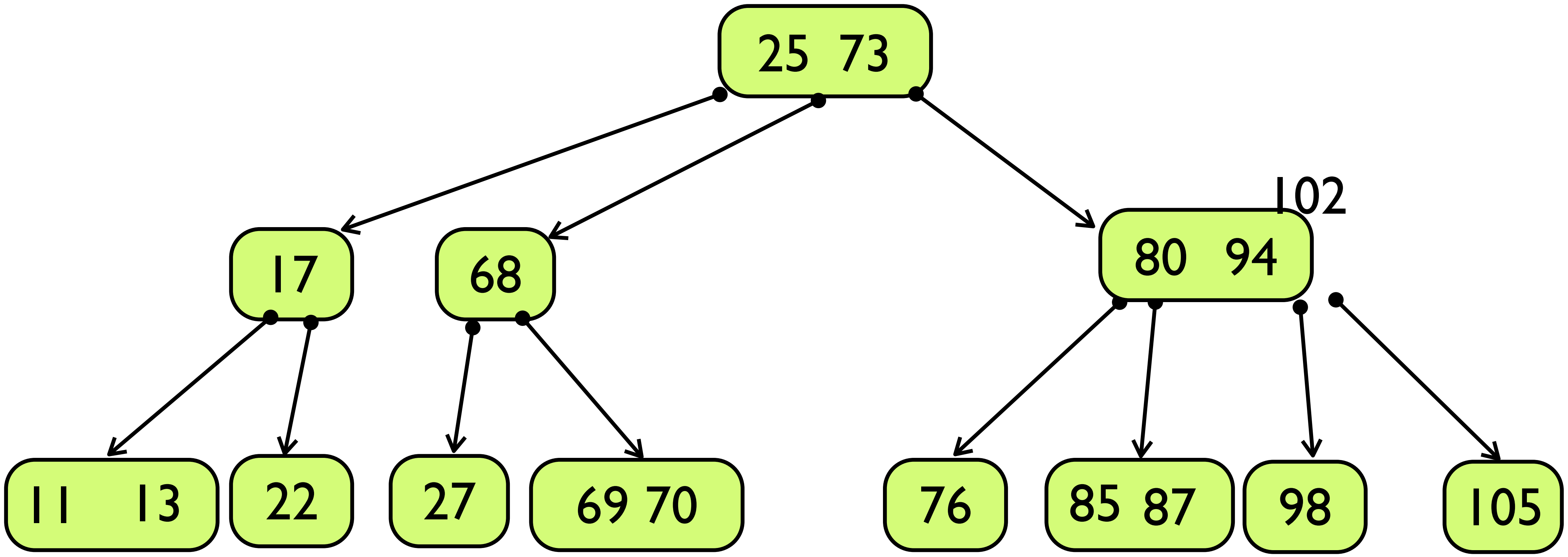

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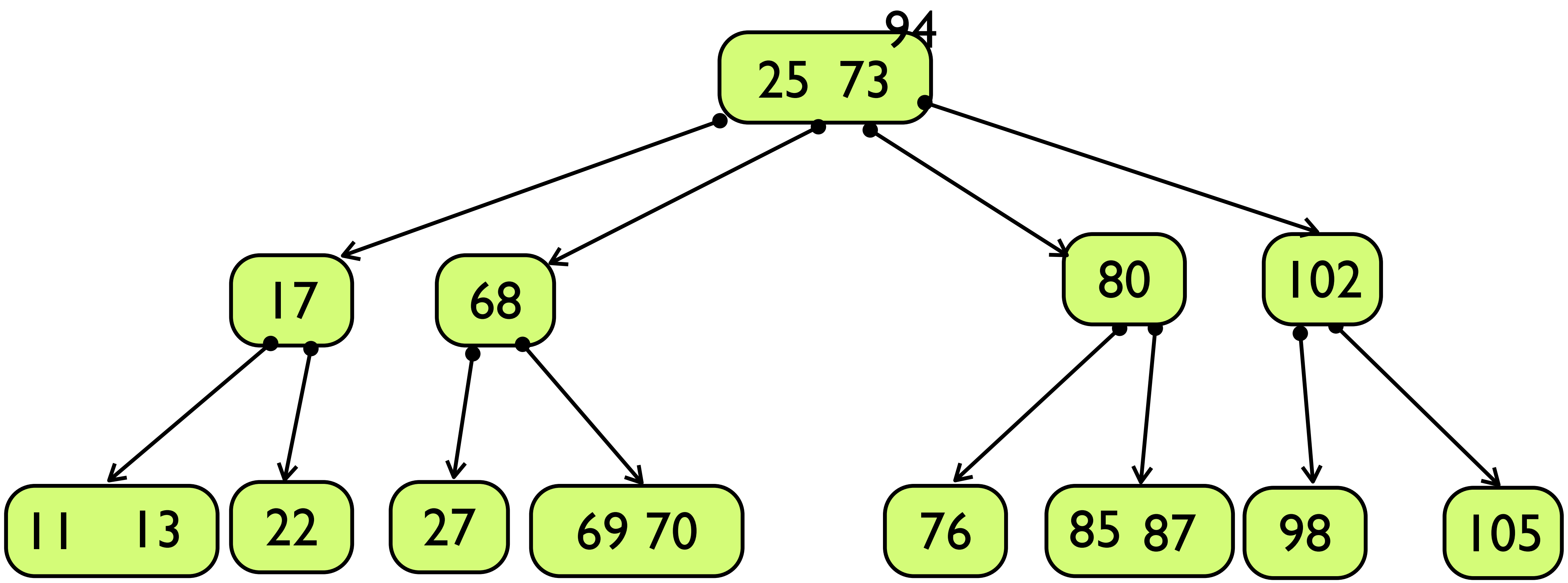

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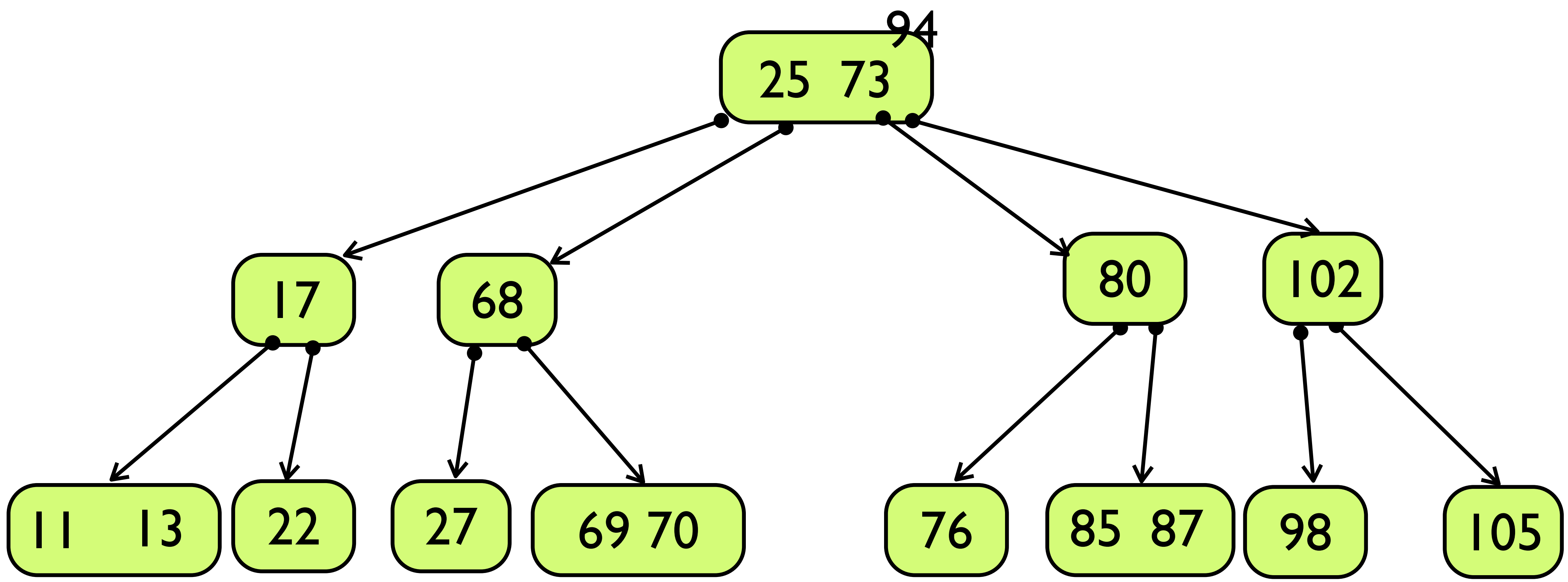

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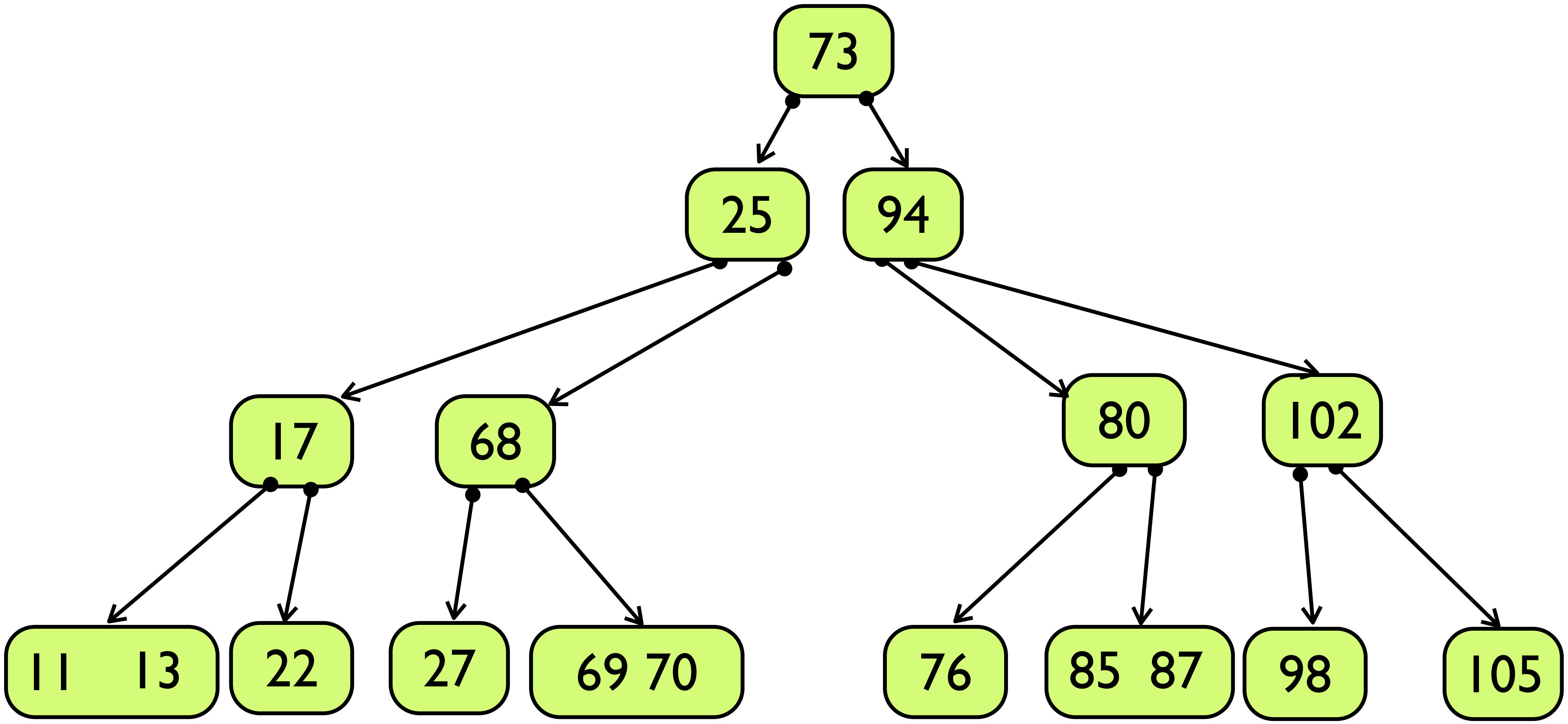
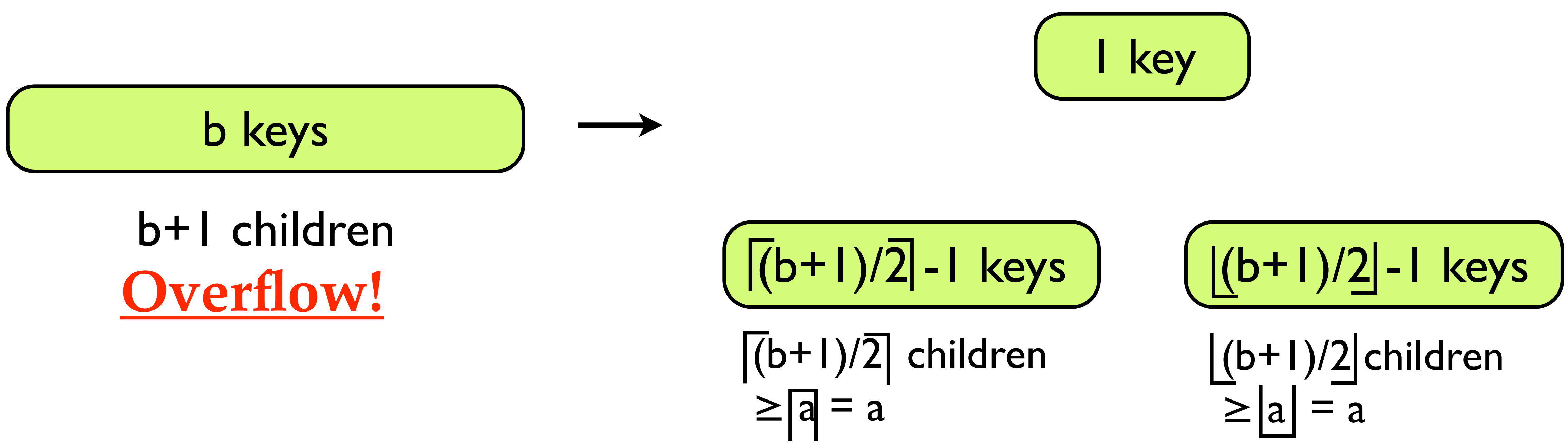
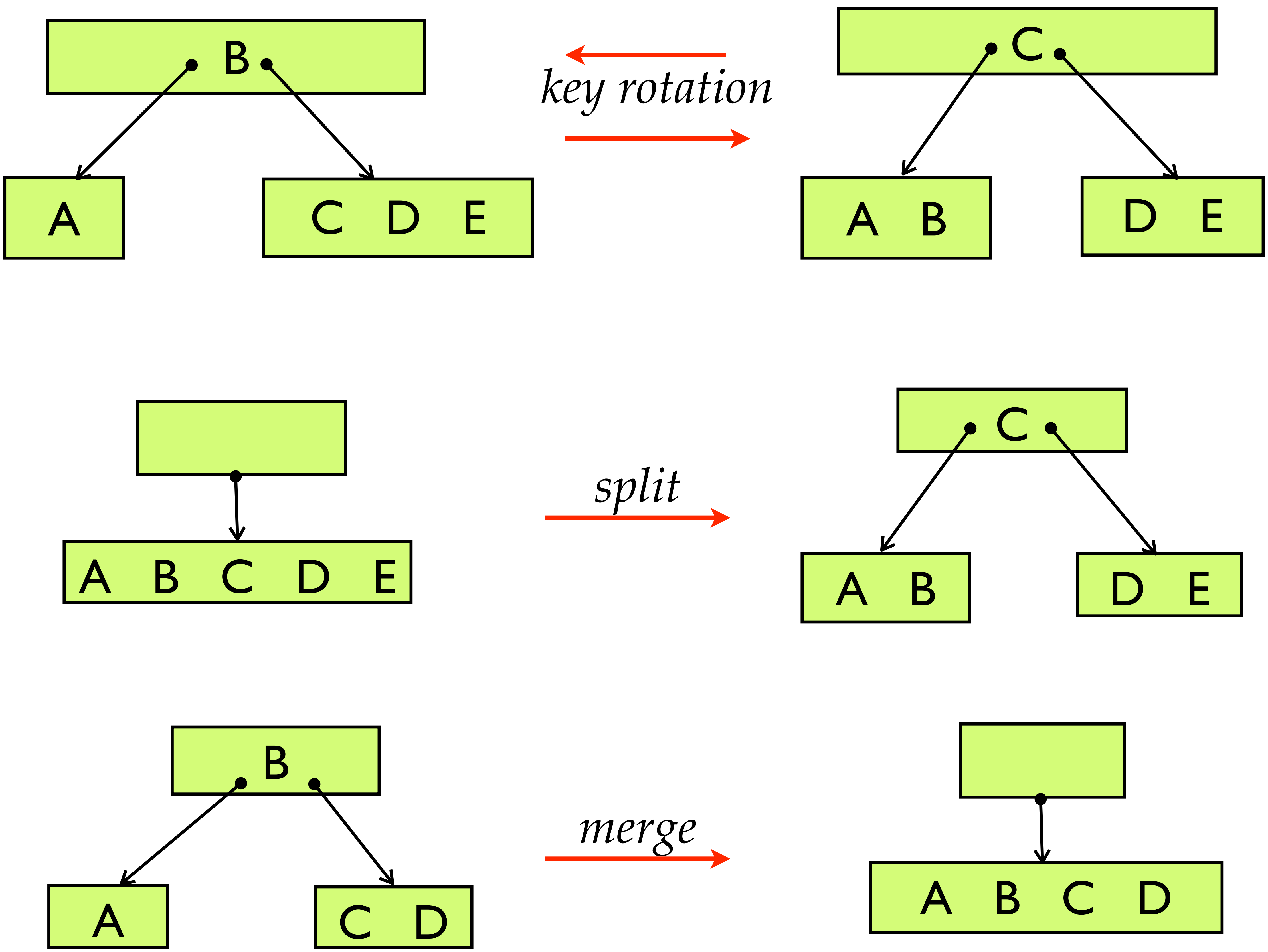


From 2,3-Trees to a,b-trees

- An a,b-tree is a generalization of a 2,3-tree, where each node (except the root) has between a and b children.
- Root can have between 2 and b children.
- We require that:
  - $a \geq 2$ (can't allow internal nodes to have 1 child)
  - $b \geq 2a - 1$ (need enough children to make split work)



a,b Insertions & Deletions



Deletion Details

- Try to borrow a key from a sibling if you have one that has an extra (≥ 1 more than the minimum)
- Otherwise, you have a sibling with exactly the minimum number a-1 of keys.
- Since you are underflowing, you must have one less than the minimum number of keys $= a$-2.
- Therefore, merging with your sibling produces a node with a-1 + a-2 = 2a-3 keys.
- This is one less than the maximum (2a-2 keys), so we have room to bring down the key that split us from our sibling.

B-trees

- A <u>*B-tree of order b*</u> is an a,b-tree with $b = 2a\text{-}1$
  - In other words, we choose the largest allowed a.

Each node (page) is at least 50% full.

- Want to have large b if bringing a node into memory is slow (say reading a disc block), but scanning the node once in memory is fast.

- b is usually chosen to match characteristics of the device.

- Ex. B-tree of order 1023 has $a = 512$.
  - If this B-tree stores n = 10 million records, its height no more than $O(\log_a n) \approx 2.58$. So only around 3 blocks need to be read from disk.

What if b is very large?

- Need to be able to find which subtree to traverse.

- Could linearly search through keys – technically constant time if b is a constant, but may be time consuming.

- Solution: Store a balanced tree (AVL or splay) at each node so that you can search for keys efficiently.