New Directions in Automated Mechanism Design

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Mechanism design

Make decisions based on the preferences (or other information) of one or more agents (as in social choice)

Focus on strategic (game-theoretic) agents with privately held information; have to be incentivized to reveal it truthfully

Popular approach in design of auctions, matching mechanisms, …
Sealed-bid auctions (on a single item)

Bidder $i$ determines how much the item is worth to her ($v_i$)
Writes a bid ($v'_i$) on a piece of paper
How would you bid?  How much would I make?
**First price:** Highest bid wins, pays bid
**Second price:** Highest bid wins, pays next-highest bid
**First price with reserve:** Highest bid wins iff it exceeds $r$, pays bid
**Second price with reserve:** Highest bid wins iff it exceeds $r$, pays next highest bid or $r$ (whichever is higher)
Revelation Principle

Anything you can achieve, you can also achieve with a truthful (AKA incentive compatible) mechanism.
Revelation Principle

Anything you can achieve, you can also achieve with a truthful (AKA incentive compatible) mechanism.
Automated mechanism design input

**Instance** is given by
Set of possible *outcomes*
Set of *agents*
  For each agent
  set of possible *types*
  *probability distribution* over these types

**Objective function**
Gives a value for each outcome for each combination of agents’ types
  E.g., social welfare, revenue

**Restrictions** on the mechanism
Are *payments* allowed?
Is randomization over outcomes allowed?
What versions of *incentive compatibility (IC) & individual rationality (IR)* are used?
How hard is designing an optimal deterministic mechanism (without reporting costs)?

[C. & Sandholm UAI’02, ICEC’03, EC’04]

<table>
<thead>
<tr>
<th>NP-complete (even with 1 reporting agent):</th>
<th>Solvable in polynomial time (for any constant number of agents):</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Maximizing social welfare (no payments)</td>
<td>1. Maximizing social welfare (not regarding the payments) (VCG)</td>
</tr>
<tr>
<td>2. Designer’s own utility over outcomes (no payments)</td>
<td></td>
</tr>
<tr>
<td>3. General (linear) objective that doesn’t regard payments</td>
<td></td>
</tr>
<tr>
<td>4. Expected revenue</td>
<td></td>
</tr>
</tbody>
</table>

1 and 3 hold even with no IR constraints
Positive results (randomized mechanisms)

[C. & Sandholm UAI’02, ICEC’03, EC’04]

• Use linear programming

• Variables:
  \( p(o \mid \theta_1, \ldots, \theta_n) \) = probability that outcome \( o \) is chosen given types \( \theta_1, \ldots, \theta_n \)
  (maybe) \( \pi_i(\theta_1, \ldots, \theta_n) \) = \( i \)'s payment given types \( \theta_1, \ldots, \theta_n \)

• Strategy-proofness constraints: for all \( i, \theta_1, \ldots, \theta_n, \theta_i' \):
  \[
  \sum_o p(o \mid \theta_1, \ldots, \theta_n)u_i(\theta_i, o) + \pi_i(\theta_1, \ldots, \theta_n) \geq \\
  \sum_o p(o \mid \theta_1, \ldots, \theta_i', \ldots, \theta_n)u_i(\theta_i, o) + \pi_i(\theta_1, \ldots, \theta_i', \ldots, \theta_n)
  \]

• Individual-rationality constraints: for all \( i, \theta_1, \ldots, \theta_n \):
  \[
  \sum_o p(o \mid \theta_1, \ldots, \theta_n)u_i(\theta_i, o) + \pi_i(\theta_1, \ldots, \theta_n) \geq 0
  \]

• Objective (e.g., sum of utilities)
  \[
  \sum_{\theta_1, \ldots, \theta_n} p(\theta_1, \ldots, \theta_n) \sum_i (\sum_o p(o \mid \theta_1, \ldots, \theta_n)u_i(\theta_i, o) + \pi_i(\theta_1, \ldots, \theta_n))
  \]

• Also works for BNE incentive compatibility, ex-interim individual rationality notions, other objectives, etc.

• For deterministic mechanisms, can still use mixed integer programming: require probabilities in \( \{0, 1\} \)
  – Remember typically designing the optimal deterministic mechanism is NP-hard
A simple example

One item for sale (free disposal)
2 agents, IID valuations: uniform over \{1, 2\}
Maximize expected revenue under ex-interim IR, Bayes-Nash equilibrium
How much can we get?
(What is optimal expected welfare?)

<table>
<thead>
<tr>
<th>Agent 1’s valuation</th>
<th>Agent 2’s valuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Status: OPTIMAL
Objective: \( \text{obj} = 1.5 \) (MAXimum)

[nonzero variables:]
- \( p_{t\_1\_1\_o3} 1 \) (probability of disposal for (1, 1))
- \( p_{t\_2\_1\_o1} 1 \) (probability 1 gets the item for (2, 1))
- \( p_{t\_1\_2\_o2} 1 \) (probability 2 gets the item for (1, 2))
- \( p_{t\_2\_2\_o2} 1 \) (probability 2 gets the item for (2, 2))
- \( \pi_{2\_2\_1} 2 \) (1’s payment for (2, 2))
- \( \pi_{2\_2\_2} 4 \) (2’s payment for (2, 2))

A slightly different distribution

One item for sale (free disposal)
2 agents, valuations drawn as on right
Maximize expected revenue under ex-interim IR, Bayes-Nash equilibrium
How much can we get?
(What is optimal expected welfare?)

<table>
<thead>
<tr>
<th>Agent 1’s valuation</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.251</td>
<td>0.250</td>
</tr>
<tr>
<td>Agent 2’s valuation</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>0.250</td>
<td>0.249</td>
</tr>
</tbody>
</table>

You’d better be really sure about your distribution!

Status: OPTIMAL
Objective: obj = 1.749 (MAXimum)

[some of the nonzero payment variables:]

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{1\ 1\ 2}$</td>
<td>62501</td>
<td></td>
</tr>
<tr>
<td>$\pi_{2\ 1\ 2}$</td>
<td>-62750</td>
<td></td>
</tr>
<tr>
<td>$\pi_{2\ 1\ 1}$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$\pi_{1\ 2\ 2}$</td>
<td>3.992</td>
<td></td>
</tr>
</tbody>
</table>
A nearby distribution without correlation

One item for sale (free disposal)
2 agents, valuations IID: 1 w/ .501, 2 w/ .499
Maximize expected revenue under ex-interim IR, Bayes-Nash equilibrium
How much can we get?
(What is optimal expected welfare?)

status:  OPTIMAL
Objective:  obj = 1.499 (MAXimum)
Cremer-McLean [1985]

For every agent, consider the following matrix $\Gamma$ of conditional probabilities, where $\Theta$ is the set of types for the agent and $\Omega$ is the set of signals (joint types for other agents, or something else observable to the auctioneer)

$$\Gamma = \begin{bmatrix} 
\pi(1|1) & \cdots & \pi(|\Omega||1) \\
\vdots & \ddots & \vdots \\
\pi(1||\Theta|) & \cdots & \pi(|\Omega|||\Theta||)
\end{bmatrix}$$

If $\Gamma$ has rank $|\Theta|$ for every agent then the auctioneer can allocate efficiently and extract the full surplus as revenue (!!)
Standard setup in mechanism design

(1) Designer has beliefs about agent’s type (e.g., preferences)

(2) Designer announces mechanism (typically mapping from reported types to outcomes)

(3) Agent strategically acts in mechanism (typically type report), however she likes at no cost

(4) Mechanism functions as specified
The mechanism may have more information about the specific agent!

application

online marketplaces
selling insurance
university admissions
webpage ranking

information

actions taken online
driving record
courses taken
links to page
Attempt 1 at fixing this

(0) Agent acts in the world (naively?)

(1) Designer obtains beliefs about agent’s type (e.g., preferences)

30%: \( v = 10 \)
70%: \( v = 20 \)

(2) Designer announces mechanism (typically mapping from reported types to outcomes)

(3) Agent \textit{strategically} acts in mechanism (typically type report), \textit{however she likes at no cost}

(4) Mechanism functions as specified
**Attempt 2: Sophisticated agent**

1. Designer has **prior** beliefs about agent’s type (e.g., preferences)

2. Designer announces mechanism (typically mapping from reported types to outcomes)

3. Agent **strategically** takes *possibly costly* actions

4. Mechanism functions as specified

*Show me pictures of cats

$v = 20$ → ![Gift Box]
Machine learning view

See also later work by Hardt, Megiddo, Papadimitriou, Wootters [2015/2016]
From Ancient Times...

Jacob and Esau

Trojan Horse
... to Modern Times

I don't always rig diesel cars to pass emissions tests

But when I do, it's a Volkswagen

MEANWHILE AT VW'S EMISSIONS TEST CENTER

THAT'S ANOTHER PASS

YOU USED SO MUCH OIL

YOU PASSED VW'S EMISSION TEST

STILL CLEANER THAN A NEW VOLKSWAGEN

TROUBLE...BREATHING...

I SENSE A TDI IS NEAR

BEIJING BEFORE VOLKSWAGEN

BEIJING AFTER VOLKSWAGEN

STILL LOWER EMISSIONS THAN A VOLKSWAGEN
<table>
<thead>
<tr>
<th>Types ( t \in T )</th>
<th>Actions ( a \in A )</th>
<th>Choice Function ( F : T \rightarrow A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>fresh</td>
<td>accept</td>
<td>fresh ( \rightarrow ) accept</td>
</tr>
<tr>
<td>ok</td>
<td>reject</td>
<td>ok ( \rightarrow ) accept</td>
</tr>
<tr>
<td>rotten</td>
<td></td>
<td>rotten ( \rightarrow ) reject</td>
</tr>
</tbody>
</table>

**Classifications \( t \in \hat{T} \):** fresh, ok, rotten
... continued

**Effort Function** \( E : T \times \hat{T} \rightarrow \mathbb{R} \):

<table>
<thead>
<tr>
<th></th>
<th>fresh</th>
<th>( \hat{ok} )</th>
<th>rotten</th>
</tr>
</thead>
<tbody>
<tr>
<td>fresh</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \hat{ok} )</td>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>rotten</td>
<td>30</td>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

**Valuation Function** \( V : T \times A \rightarrow \mathbb{R} \):

\[
V(\cdot, \text{accept}) = 20, \quad V(\cdot, \text{reject}) = 0
\]

**Mechanism** \( M : \hat{T} \rightarrow A \)

First Try: \( M = \text{fresh} \rightarrow \text{accept}, \ \hat{ok} \rightarrow \text{accept}, \ \text{rotten} \rightarrow \text{reject} \)
... continued

**Effort Function** \( E : T \times \hat{T} \to \mathbb{R} \):

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<td>0</td>
</tr>
<tr>
<td>ok</td>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>rotten</td>
<td>30</td>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

**Valuation Function** \( V : T \times A \to \mathbb{R} \):

\[
V(\cdot, \text{accept}) = 20, \quad V(\cdot, \text{reject}) = 0
\]

**Mechanism** \( M : \hat{T} \to A \)

First Try: \( M = \text{fresh} \to \text{accept}, \ \text{ok} \to \text{accept}, \ \text{rotten} \to \text{reject} \)

Better: \( M^* = \text{fresh} \to \text{accept}, \ \text{ok} \to \text{reject}, \ \text{rotten} \to \text{reject} \).
### Comparison With Other Models

<table>
<thead>
<tr>
<th>fresh</th>
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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
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<td>0</td>
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<td>0</td>
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Standard Mechanism Design

<table>
<thead>
<tr>
<th>fresh</th>
<th>ok</th>
<th>rotten</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>∞</td>
<td>0</td>
</tr>
<tr>
<td>∞</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
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Mechanism Design with Partial Verification

<table>
<thead>
<tr>
<th>fresh</th>
<th>ok</th>
<th>rotten</th>
</tr>
</thead>
<tbody>
<tr>
<td>∞</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.2</td>
<td>5</td>
<td>-∞</td>
</tr>
<tr>
<td>-3</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Mechanism Design with Signaling Costs

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Green and Laffont. *Partially verifiable information and mechanism design.* RES 1986

Auletta, Penna, Persiano, Ventre. *Alternatives to truthfulness are hard to recognize.* AAMAS 2011
Question

Given:

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<td>reject</td>
<td>ok $\rightarrow$ accept</td>
</tr>
<tr>
<td>rotten</td>
<td></td>
<td>rotten $\rightarrow$ reject</td>
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Classifications ($i \in \hat{T}$): fresh, ok, rotten

Effort Function ($E : T \times \hat{T} \rightarrow \mathbb{R}$):

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<td>0</td>
</tr>
<tr>
<td>30</td>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

Valuation Function ($V : T \times A \rightarrow \mathbb{R}$):

$V(\cdot, accept) = 20$, $V(\cdot, reject) = 0$

Then:

**Mechanism** $M : \hat{T} \rightarrow A$

Does there exist a mechanism which implements the choice function?

NP-complete!

Auletta, Penna, Persiano, Ventre. Alternatives to truthfulness are hard to recognize. AAMAS 2011
Non-bolded results are from:
Auletta, Penna, Persiano, Ventre. Alternatives to truthfulness are hard to recognize. AAMAS 2011

Hardness results fundamentally rely on revelation principle failing – conditions under which revelation principle still holds in Green & Laffont ’86 and Yu ’11 (partial verification), and Kephart & C. EC’16 (costly signaling).
When Samples Are Strategically Selected

Bob, Professor of Rocket Science

A NEW POSTDOC APPLICANT.

HMM.

SHE HAS 15 PAPERS AND I ONLY WANT TO READ 3.

ICML 2019, with

Hanrui Zhang
(Duke)

Yu Cheng
(Duke → IAS → UIC)
Academic hiring...

Charlie, Bob's student

GIVE ME 3 PAPERS BY ALICE THAT I NEED TO READ.

SURE.

CHARLIE IS EXCITED ABOUT HIRING ALICE

www.phdcomics.com
Academic hiring...

I need to choose the best 3 papers to convince Bob, so that he will hire Alice.

Charlie will definitely pick the best 3 papers by Alice, and I need to calibrate for that.
The general problem

A distribution (Alice) over paper qualities $\theta \in \{g, b\}$ arrives, which can be either a good one ($\theta = g$) or a bad one ($\theta = b$).
The general problem

The principal (Bob) announces a policy, according to which he decides, based on the report of the agent (Charlie), whether to accept $\theta$ (hire Alice)

I WILL HIRE ALICE IF YOU GIVE ME 3 GOOD PAPERS, OR 2 EXCELLENT PAPERS.

AND I WANT ALICE TO BE FIRST AUTHOR ON AT LEAST 2 OF THEM.
The general problem

The agent (Charlie) has access to $n(=15)$ iid samples (papers) from $\theta$ (Alice), from which he chooses $m(=3)$ as his report.
The general problem

The agent (Charlie) sends his report to the principal, aiming to convince the principal (Bob) to accept $\theta$ (Alice)

Charlie found 3 papers by Alice meeting Bob's criteria.

He is sure Bob will hire Alice upon seeing these 3 papers.
The general problem

The principal (Bob) observes the report of the agent (Charlie), and makes the decision according to the policy announced.

I read the 3 papers you sent me.

One is not so good, but the other two are incredible.

It looks like Alice is doing good work, so let's hire her.
Questions

How does strategic selection affect the principal’s policy?
Is it easier or harder to classify based on strategic samples, compared to when the principal has access to iid samples?
Should the principal ever have a diversity requirement (e.g., at least 1 mathematical paper and at least 1 experimental paper), or only go by total quality according to a single metric?
Agent’s problem:
• “How do I distinguish myself from other types?”
• “How many samples do I need for that?”

Principal’s problem:
• “How do I tell ML-flexible agents from others?”
• “At what point in their career can I reliably do that?”
One **good** and one **bad** distribution

Pick a subset of the right-hand side (to accept) that maximizes *(green mass covered - black mass covered)*

If positive, can (eventually) distinguish; otherwise not. NP-hard in general.

This subset covers \(0.5 + 0.2 = 0.7\) good mass and \(0.4 + 0.3 = 0.7\) bad mass, so it doesn’t work. (What does?)
But if we know the strategy for the good distribution (revelation principle holds):

Solve as maximum flow/matching from left to right with capacities on vertices
Duality gives set of signals to accept (~Hall’s marriage theorem)

Can place good mass on the signals side because we know the strategy
Optimization: reduction to min cut

-3 3
-3 2
2 2

In sampling case, can check existence of edges with previous technique

Values are $P(\text{type}) \times \text{value(\text{type})}$

(edges between types have capacity $\infty$)

Can be generalized to more outcomes than accept/reject, if types have the same utility over them.
Conclusion

First part:
When considering correlation, small changes can have a huge effect
Automatically designing robust mechanisms addresses this
Combines well with learning (under some conditions)

<table>
<thead>
<tr>
<th>0.251</th>
<th>0.250</th>
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<tbody>
<tr>
<td>0.250</td>
<td>0.249</td>
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<table>
<thead>
<tr>
<th>0.251001</th>
<th>0.249999</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.249999</td>
<td>0.249001</td>
</tr>
</tbody>
</table>

Second part:
With costly or limited misreporting, revelation principle can fail
Causes computational hardness in general
Sometimes agents report based on their samples
Some efficient algorithms for the infinite limit case; sample bounds

Thank you for your attention!