Reminders

- Weight: 5% of the homework grade.
- Out of 100 points.
- Lead TA: B. Aditya Prakash

Notes:

- Rough time-estimate: 2~4 hours (about 30-60mins per question).
- You are encouraged to type your answers. Illegible handwriting may get no points, at the discretion of the grader.
- Whenever you are making an assumption, please state it clearly.

Question 1 [5 points]

You are given a table R(A, B, C) with the following FDs: \( AB \rightarrow C \) and \( B \rightarrow C \). Can we deduce from these that \( A \rightarrow C \) holds? If yes, give a short proof. If no, give a counter-example with 3 tuples or less.

Question 2 [20 points, 5 points each]

Table 1 shows an instance of relation R(A, B, C, D, E). In future, the instance can change by insertion, deletion, and update operations but the schema will remain the same.

Do the following four functional dependencies hold? Choose between (A) “always holds”; (B) “does not hold”; (C) “cannot say”.

(A) If it always holds, give the proof
(B) If it does not hold, give tuple-id(s) which violate the dependency
(C) If you cannot say, then give a SQL query that checks whether the given FD holds. Assume that no record has NULL values.

\[ \begin{align*}
Q2.1 & \quad A \rightarrow CD \\
Q2.2 & \quad AC \rightarrow B \\
Q2.3 & \quad AB \rightarrow CD \\
Q2.4 & \quad BD \rightarrow CE
\end{align*} \]
<table>
<thead>
<tr>
<th>tuple-id</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>4</td>
<td>7</td>
<td>22</td>
<td>48</td>
<td>1</td>
</tr>
<tr>
<td>T2</td>
<td>5</td>
<td>6</td>
<td>22</td>
<td>49</td>
<td>7</td>
</tr>
<tr>
<td>T3</td>
<td>5</td>
<td>7</td>
<td>24</td>
<td>53</td>
<td>9</td>
</tr>
<tr>
<td>T4</td>
<td>3</td>
<td>2</td>
<td>10</td>
<td>23</td>
<td>8</td>
</tr>
<tr>
<td>T5</td>
<td>6</td>
<td>0</td>
<td>12</td>
<td>30</td>
<td>3</td>
</tr>
<tr>
<td>T6</td>
<td>3</td>
<td>2</td>
<td>10</td>
<td>23</td>
<td>0</td>
</tr>
<tr>
<td>T7</td>
<td>2</td>
<td>3</td>
<td>10</td>
<td>22</td>
<td>9</td>
</tr>
<tr>
<td>T8</td>
<td>5</td>
<td>1</td>
<td>12</td>
<td>29</td>
<td>1</td>
</tr>
<tr>
<td>T9</td>
<td>5</td>
<td>6</td>
<td>22</td>
<td>49</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 1: Instance of R

**Question 3 [40 points]**

Consider the relation schema R(A, B, C, D, E, F) with functional dependencies \( AC \rightarrow B \), \( BD \rightarrow F \) and \( F \rightarrow CE \).

Q3.1 How many candidate keys does R have? [2 points]

Q3.2 List all the candidate keys of R. If a candidate key is composite then use parenthesis e.g. \( (A, B) \). [3 points]

Q3.3 Find the attribute closure of \( \{A\}^+ \). [5 points]

Q3.4 Find the attribute closure of \( \{A, C\}^+ \). [5 points]

Q3.5 Is R in 3NF? If yes, justify. If no, specify at least one FD which violates the definition? [5 points]

Q3.6 Which FD(s) (if any) of R violates BCNF? [5 points]

Q3.7 Suppose we project R onto S(A, C, D, E). Give one non-trivial FD that holds in S. [5 points]

Q3.8 Out of the 6 subsets of five out of the six attributes (A, B, C, D, E, F), how many and which ones are in BCNF w.r.t. to the given FD’s? [5 points]

Q3.9 Consider the decomposition of R into R1(A, B, C), R2(C, E, F) and R3(A, D, F). Give YES/NO answers for the following:

(a) Is this decomposition lossless? [2 points]

(b) Is this decomposition dependency preserving? [2 points]

(c) Is this decomposition in BCNF (i.e. are R1, R2 and R3 all in BCNF)? [1 point]

**Question 4 [20 points]**

Consider the following set \( S \) of functional dependencies:

\[
A \rightarrow B \quad (F1) \\
AB \rightarrow C \quad (F2) \\
AC \rightarrow B \quad (F3) \\
B \rightarrow C \quad (F4)
\]
Q4.1 Given F1 and F4, prove rigorously that F2 holds. [8 points]
Q4.2 Again, given F1 and F4 prove rigorously that F3 holds. [8 points]
Q4.3 Give a minimal cover for the set S. [4 points]
  
  Hint: Keep in mind the proofs above

Question 5 [15 points]

You are given the functional dependencies set \( S \equiv \{AB \rightarrow C, C \rightarrow B, C \rightarrow D\} \).

Q5.1 Given S, is the relation R1(A, B, C) in 3NF? If yes, justify. If no, specify at least one FD which violates the definition? [5 points]
Q5.2 Given S, give the strongest normal form (BCNF, 3NF, 2NF, 1NF) obeyed by the relation R2(C, D). [5 points]
Q5.3 Decompose the relation R(A, B, C, D) into a collection of BCNF relations. Of course make sure that the decomposition is lossless. Specifically:
  
  – How many different decompositions exist? [1 point]
  – List all such decompositions. [4 points]