Question 1

We can not deduce that $A \rightarrow C$. e.g. consider a table with tuples $(a_1, b_1, c_1)$ and $(a_1, b_2, c_2)$.

Question 2

Q2.1 Doesn’t hold. Tuples that violate: $(T_2, T_3)$.

Q2.2 Can’t say as this is just an instance which can be modified later. SQL query (many different queries are possible):

```sql
SELECT R2.A, R2.C
(SELECT A, C, count(distinct B) as CB FROM R
GROUP BY A, C) AS R2
WHERE R2.CB > 1
```

If the above query returns some values $(A, C)$ then the FD doesn’t hold.

Q2.3 Can’t say. SQL query:

```sql
SELECT R2.A, R2.B
(SELECT A, B, count(distinct (C, D)) as CCD FROM R
GROUP BY A, B) AS R2
WHERE R2.CCD > 1
```

Q2.4 Doesn’t hold. Tuples that violate: $(T_2, T_9)$ and $(T_4, T_6)$.

Question 3

Q3.1 It has 3 different candidate keys.

Q3.2 $(A, B, D)$, $(A, C, D)$ and $(A, F, D)$.

Basically you have to compute the closure of all subsets of attributes testing which ones are minimal keys. You can take some shortcuts however. Note every key must contain $A$ and $D$ since neither of these attributes appear on the right side of any FDs. Working incrementally starting with $(A, D)$ yields the three keys above.

Q3.3 \(\{A\}^+ = \{A\}\). 

Q3.4 \(\{A, C\}^+ = \{A, B, C\}\).
Q3.5 No, R is not in 3NF. \( F \rightarrow CE \) violates the definition.

Q3.6 All of the given FDs violate BCNF conditions.

Q3.7 \( ACD \rightarrow E \).

An easy way of getting this is by recalling that \((A, C, D)\) was a candidate key in R i.e. \( ACD \rightarrow ABCDEF \). Simply project this FD on S to get the above FD.

Q3.8 None. All FDs are non-trivial and as all the candidate keys have 3 attributes, none of the left-hand side of the FDs can be a superkey.

Q3.9 (a) Yes, R2 and R3 joining attribute is F which is a superkey of R2. Then the subsequent joining attribute is AC which is the superkey in R1.

(b) No, \( BD \rightarrow F \) spans two tables.

(c) Yes, AC is a superkey in R1, F in R2 and ADF in R3.

**Question 4**

Q4.1 Proof is:

\[
AB \rightarrow BB \quad \text{(Augmentation of F1 with B)}
\]

\[
AB \rightarrow B \quad \text{(BB = B)}
\]

\[
AB \rightarrow C \quad \text{(Transitivity with F4 and above)}
\]

Q4.2 Proof is:

\[
AC \rightarrow BC \quad \text{(Augmentation of F1 with C)}
\]

\[
AC \rightarrow B \quad \text{(Decomposition of above)}
\]

Q4.3 The minimal cover for S is \( \{ A \rightarrow B, B \rightarrow C \} \), because you can generate the other FDs using these two.

**Question 5**

Q5.1 The relation R1 is in 3NF. \( AB \rightarrow C \) is OK because AB is a superkey and \( C \rightarrow B \) is OK because B is part of the superkey AB \((C \rightarrow D \) is not applicable because D is not in R1).

Q5.2 Clearly R2 is in BCNF. \( C \rightarrow D \) is the only applicable FD and BCNF is satisfied because C is a candidate key for R2.

Q5.3 There are only 3 both lossless and BCNF decompositions:

- \( R1(C, B, D) \) and \( R2(A, C) \)
- \( R1(A, B, C) \) and \( R2(C, D) \)
- \( R1(C, B), R2(C, D) \) and \( R3(A, C) \)