15-415 Homework 8 Solutions

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Question 1

We can not deduce that $A \rightarrow C$. e.g. consider a table with tuples $(a1, b1, c1)$ and $(a1, b2, c2)$.

Question 2

Q2.1 Doesn't hold. Tuples that violate: $(T2, T3)$.

Q2.2 Can't say as this is just an instance which can be modified later. SQL query (many different queries are possible):

```
SELECT R2.A, R2.C
(SELECT A, C, count(distinct B) as CB
FROM R
GROUP BY A, C) AS R2
WHERE R2.CB > 1
```

If the above query returns some values (A, C) then the FD doesn't hold.

Q2.3 Can't say. SQL query:

```
SELECT R2.A, R2.B
(SELECT A, B, count(distinct (C, D)) as CCD
FROM R
GROUP BY A, B) AS R2
WHERE R2.CCD > 1
```

Q2.4 Doesn't hold. Tuples that violate: $(T2, T9)$ and $(T4, T6)$.

Question 3

Q3.1 It has 3 different candidate keys.

Q3.2 (A, B, D), (A, C, D) and (A, F, D).

Basically you have to compute the closure of all subsets of attributes testing which ones are minimal keys. You can take some shortcuts however. Note every key must contain A and D since neither of these attributes appear on the right side of any FDs. Working incrementally starting with (A, D) yields the three keys above.

Q3.3 $\{A\}^+ = \{A\}$.

Q3.4 $\{A, C\}^+ = \{A, B, C\}$.

Q3.5 No, R is not in 3NF. $F \rightarrow CE$ violates the definition.

Q3.6 All of the given FDs violate BCNF conditions.

Q3.7 $ACD \rightarrow E$.

An easy way of getting this is by recalling that (A, C, D) was a candidate key in R i.e. $ACD \rightarrow ABCDEF$. Simply project this FD on S to get the above FD.

Q3.8 None. All FDs are non-trivial and as all the candidate keys have 3 attributes, none of the left-hand side of the FDs can be a superkey.

Q3.9 (a) Yes, R2 and R3 joining attribute is F which is a superkey of R2. Then the subsequent joining attribute is AC which is the superkey in R1.

(b) No, $BD \rightarrow F$ spans two tables.

(c) Yes, AC is a superkey in R1, F in R2 and ADF in R3.

Question 4

Q4.1 Proof is:

$$AB \rightarrow BB \quad \text{(Augmentation of F1 with B)}$$
$$AB \rightarrow B \quad \text{(BB = B)}$$
$$AB \rightarrow C \quad \text{(Transitivity with F4 and above)}$$

Q4.2 Proof is:

$$AC \rightarrow BC \quad \text{(Augmentation of F1 with C)}$$
$$AC \rightarrow B \quad \text{(Decomposition of above)}$$

Q4.3 The minimal cover for S is $\{A \rightarrow B, B \rightarrow C\}$, because you can generate the other FDs using these two.

Question 5

Q5.1 The relation R1 is in 3NF. $AB \rightarrow C$ is OK because AB is a superkey and $C \rightarrow B$ is OK because B is part of the superkey AB ($C \rightarrow D$ is not applicable because D is not in R1).

Q5.2 Clearly R2 is in BCNF. $C \rightarrow D$ is the only applicable FD and BCNF is satisfied because C is a candidate key for R2.

Q5.3 There are only 3 *both* lossless and BCNF decompositions:

R1(C, B, D) and R2(A, C)

R1(A, B, C) and R2(C, D)

R1(C, B), R2(C, D) and R3(A, C)