

15-826: Multimedia Databases and Data Mining

Lecture #28: Graph mining - patterns
Christos Faloutsos

Must-read Material

- [Graph mining textbook] Deepayan Chakrabarti and Christos Faloutsos [*Graph Mining: Laws, Tools and Case Studies*](#), Morgan Claypool, 2012
 - Part I (patterns)

Must-read Material

- Michalis Faloutsos, Petros Faloutsos and Christos Faloutsos, On Power-Law Relationships of the Internet Topology, SIGCOMM 1999.
- R. Albert, H. Jeong, and A.-L. Barabasi, Diameter of the World Wide Web Nature, 401, 130-131 (1999).
- Reka Albert and Albert-Laszlo Barabasi Statistical mechanics of complex networks, Reviews of Modern Physics, 74, 47 (2002).
- Jure Leskovec, Jon Kleinberg, Christos Faloutsos Graphs over Time: Densification Laws, Shrinking Diameters and Possible Explanations, KDD 2005, Chicago, IL, USA

Main outline



- Introduction
- Indexing
- Mining
 - Graphs – patterns
 - Graphs – generators and tools
 - Association rules
 - ...

Outline



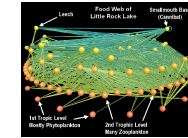
- ➔ • Introduction – Motivation
- Problem#1: Patterns in graphs
- Problem#2: Scalability
- Conclusions

15-826

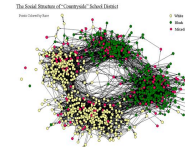
(c) C. Faloutsos, 2017

5

Graphs - why should we care?



Food Web
[Martinez '91]



Friendship Network
[Moody '01]



Internet Map
[lumeta.com]

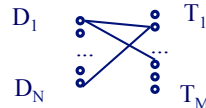
15-826

(c) C. Faloutsos, 2017

6

Graphs - why should we care?

- IR: bi-partite graphs (doc-terms)



- web: hyper-text graph

- ... and more:

15-826

(c) C. Faloutsos, 2017

7

Graphs - why should we care?

- 'viral' marketing
- web-log ('blog') news propagation
- computer network security: email/IP traffic and anomaly detection
-

15-826

(c) C. Faloutsos, 2017

8

Outline



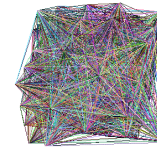
- Introduction – Motivation
- ➔ • Problem#1: Patterns in graphs
 - Static graphs
 - Weighted graphs
 - Time evolving graphs
- Problem#2: Scalability
- Conclusions

15-826

(c) C. Faloutsos, 2017

9

Problem #1 - network and graph mining



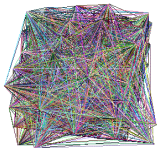
- What does the Internet look like?
- What does FaceBook look like?
- What is 'normal' / 'abnormal' ?
- which patterns/laws hold?

15-826

(c) C. Faloutsos, 2017

10

Problem #1 - network and graph mining



- What does the Internet look like?
- What does FaceBook look like?
- What is 'normal' / 'abnormal' ?
- which patterns/laws hold?
 - To spot **anomalies** (rarities), we have to discover **patterns**

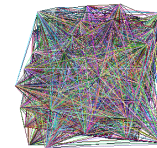


15-826

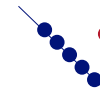
(c) C. Faloutsos, 2017

11

Problem #1 - network and graph mining



- What does the Internet look like?
- What does FaceBook look like?
- What is 'normal' / 'abnormal' ?
- which patterns/laws hold?
 - To spot **anomalies** (rarities), we have to discover **patterns**
 - **Large** datasets reveal patterns/anomalies that may be invisible otherwise...



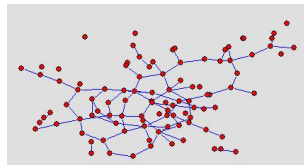
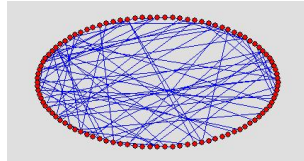
15-826

(c) C. Faloutsos, 2017

12

Are real graphs random?

- random (Erdos-Renyi) graph – 100 nodes, avg degree = 2
- before layout
- after layout
- No obvious patterns



(generated with: pajek

<http://vlado.fmf.uni-lj.si/pub/networks/pajek/>)

15-826

(c) C. Faloutsos, 2017

13

Graph mining

- Are real graphs random?

15-826

(c) C. Faloutsos, 2017

14

Laws and patterns

- Are real graphs random?
- A: NO!!
 - Diameter (‘6 degrees’ , ‘Kevin Bacon’)
 - in- and out- degree distributions
 - other (surprising) patterns
- So, let’ s look at the data



15-826

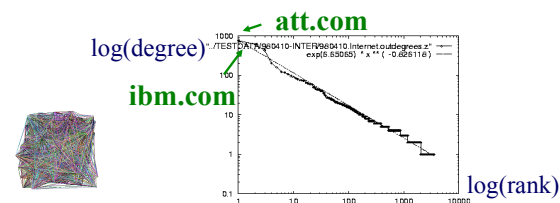
(c) C. Faloutsos, 2017

15

Solution# S.1

- Power law in the degree distribution [SIGCOMM99]

internet domains



15-826

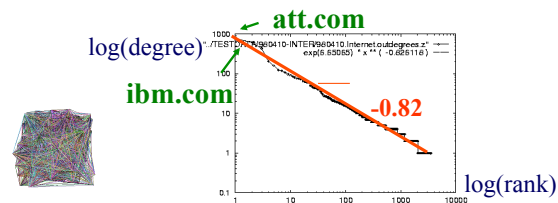
(c) C. Faloutsos, 2017

16

Solution# S.1

- Power law in the degree distribution [SIGCOMM99]

internet domains



15-826

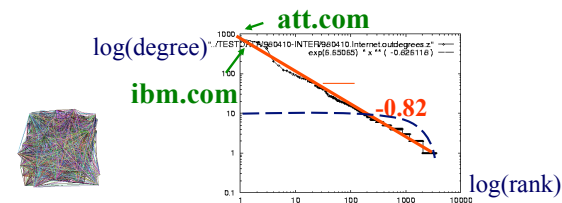
(c) C. Faloutsos, 2017

17

Solution# S.1

- Q: So what?

internet domains



15-826

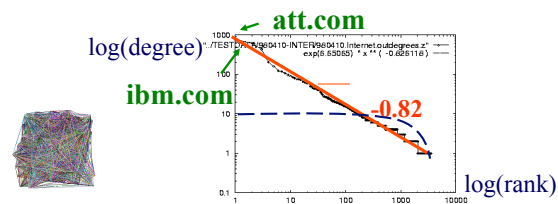
(c) C. Faloutsos, 2017

18

Solution# S.1

- Q: So what?
- A1: # of two-step-away pairs: $\text{= friends of friends (F.O.F.)}$

internet domains



15-826

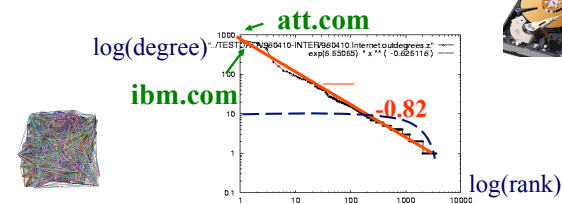
(c) C. Faloutsos, 2017

19

Solution# S.1

- Q: So what?
- A1: # of two-step-away pairs: $100^2 * N = 10 \text{ Trillion}$

internet domains



15-826

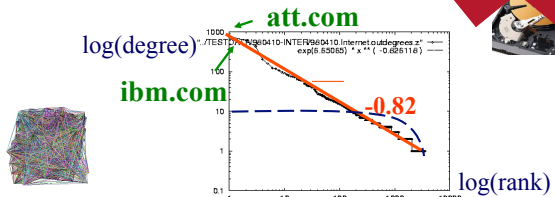
(c) C. Faloutsos, 2017

20

Carnegie Mellon

Solution# S.1

- Q: So what? = friends of friends (F.O.F.)
- A1: # of two-step-away pairs: 100^2 Trillion internet domains



15-826 (c) C. Faloutsos, 2017 21

Carnegie Mellon

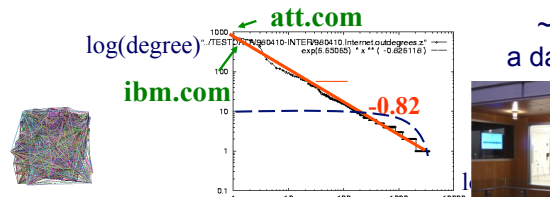
Solution# S.1

Gaussian trap

- Q: So what? = friends of friends (F.O.F.)
- A1: # of two-step-away pairs: $O(d_{\max}^2) \sim 10M^2$

\downarrow

$\sim 0.8PB \rightarrow$
a data center(!)



15-826 (c) C. Faloutsos, 2017 DCO @ CMU 22

Carnegie Mellon

Solution# S.1

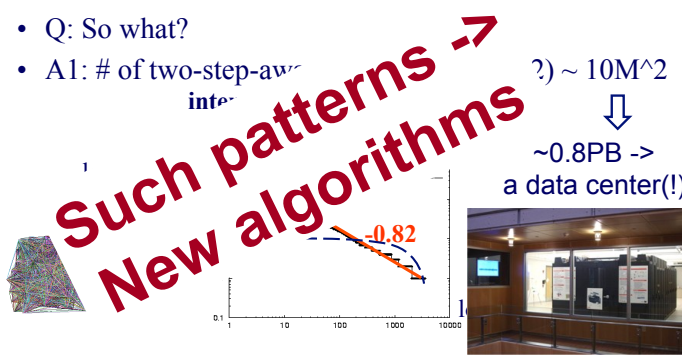
Gaussian trap

- Q: So what?
- A1: # of two-step-away pairs: $100^2 \sim 10M^2$

\downarrow

$\sim 0.8PB \rightarrow$
a data center(!)

**Such patterns \rightarrow
New algorithms**

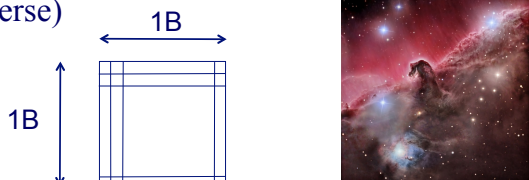


15-826 (c) C. Faloutsos, 2017 23

Carnegie Mellon

Observation – big-data:

- $O(N^2)$ algorithms are \sim intractable - $N=1B$
- N^2 seconds = 31B years ($>2x$ age of universe)



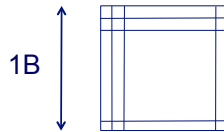
15-826 (c) C. Faloutsos, 2017 24

Carnegie Mellon

Observation – big-data:

- $O(N^2)$ algorithms are ~intractable - $N=1B$

- N^2 seconds = ~~31B~~^{31M} years
- 1,000 machines



15-826

(c) C. Faloutsos, 2017

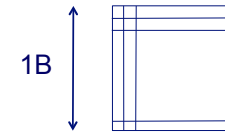
25

Carnegie Mellon

Observation – big-data:

- $O(N^2)$ algorithms are ~intractable - $N=1B$

- N^2 seconds = ~~31B~~^{31K} years
- 1M machines



Google Y!

15-826

(c) C. Faloutsos, 2017

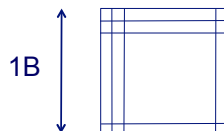
26

Carnegie Mellon

Observation – big-data:

- $O(N^2)$ algorithms are ~intractable - $N=1B$

- N^2 seconds = ~~31B~~³ years
- 10B machines ~ \$10Trillion



15-826

(c) C. Faloutsos, 2017

27

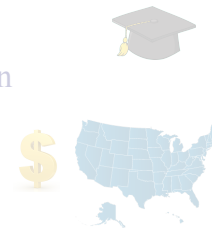
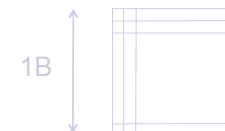
Carnegie Mellon

Observation – big-data:

- $O(N^2)$ algorithms are ~intractable - $N=1B$

And parallelism might not help

- N^2 seconds = ~~31B~~³ years
- 10B machines ~ \$10Trillion



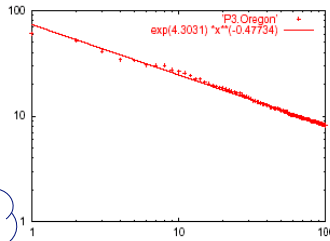
15-826

(c) C. Faloutsos, 2017

28

Solution# S.2: Eigen Exponent E

Eigenvalue



Exponent = slope

$$E = -0.48$$

May 2001

$$A x = \lambda x$$

Rank of decreasing eigenvalue

- A2: power law in the eigenvalues of the adjacency matrix

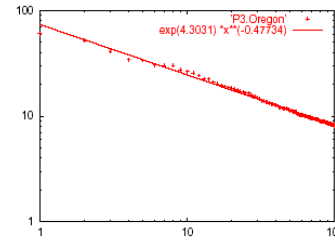
15-826

(c) C. Faloutsos, 2017

29

Solution# S.2: Eigen Exponent E

Eigenvalue



Exponent = slope

$$E = -0.48$$

May 2001

Rank of decreasing eigenvalue

- [Mihail, Papadimitriou '02]: slope is $\frac{1}{2}$ of rank exponent

15-826

(c) C. Faloutsos, 2017

30

But:

How about graphs from other domains?

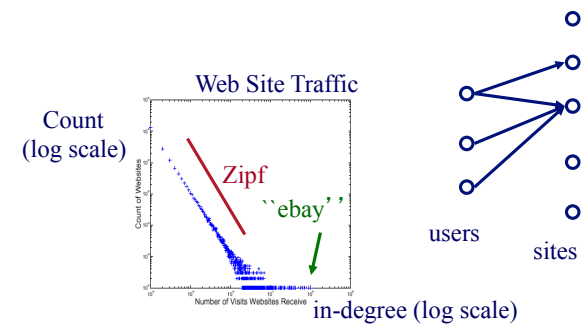
15-826

(c) C. Faloutsos, 2017

31

More power laws:

- web hit counts [w/ A. Montgomery]

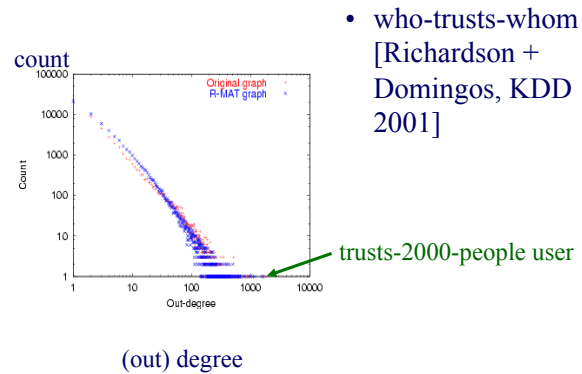


15-826

(c) C. Faloutsos, 2017

32

epinions.com



15-826

(c) C. Faloutsos, 2017

33

And numerous more

- # of sexual contacts
- Income [Pareto] – '80-20 distribution'
- Duration of downloads [Bestavros+]
- Duration of UNIX jobs ('mice and elephants')
- Size of files of a user
- ...
- 'Black swans'

15-826

(c) C. Faloutsos, 2017

34

Outline

- Introduction – Motivation
- Problem#1: Patterns in graphs
 - Static graphs
 - degree, diameter, eigen,
 - Triangles
 - Weighted graphs
 - Time evolving graphs



15-826

(c) C. Faloutsos, 2017

35

Solution# S.3: Triangle 'Laws'



- Real social networks have a lot of triangles

15-826

(c) C. Faloutsos, 2017

36

Solution# S.3: Triangle 'Laws'



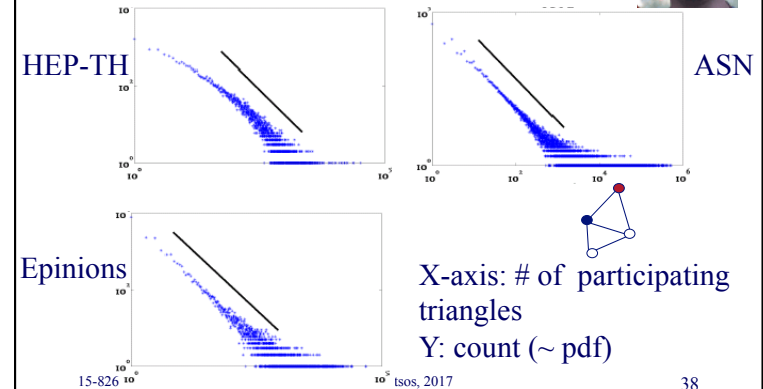
- Real social networks have a lot of triangles
 - Friends of friends are friends
- Any patterns?

15-826

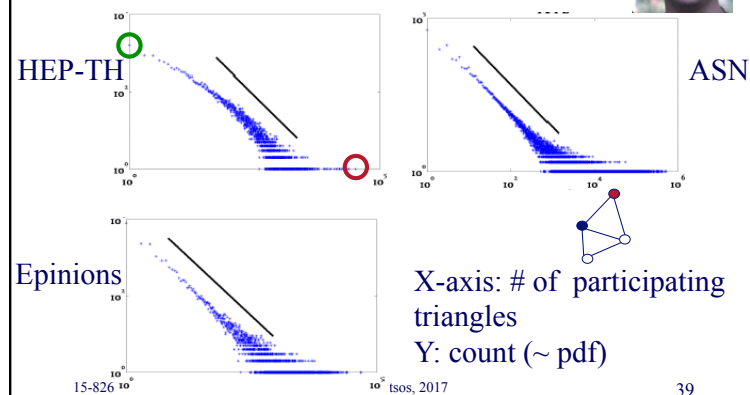
(c) C. Faloutsos, 2017

37

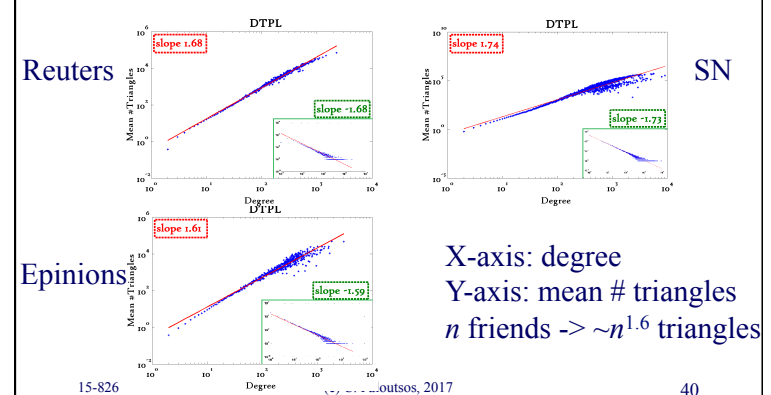
Triangle Law: #S.3 [Tsourakakis ICDM 2008]



Triangle Law: #S.3 [Tsourakakis ICDM 2008]



Triangle Law: #S.4 [Tsourakakis ICDM 2008]



Triangle Law: Computations

[Tsourakakis ICDM 2008]

But: triangles are expensive to compute
(3-way join; several approx. algos)
Q: Can we do that quickly?

15-826

(c) C. Faloutsos, 2017

41

Triangle Law: Computations

[Tsourakakis ICDM 2008]

But: triangles are expensive to compute
(3-way join; several approx. algos)
Q: Can we do that quickly?

A: Yes!

#triangles = $\frac{1}{6} \sum (\lambda_i^3)$
(and, because of skewness (S2),
we only need the top few eigenvalues!)

15-826

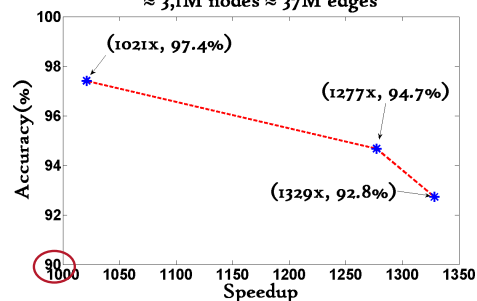
(c) C. Faloutsos, 2017

42

Triangle Law: Computations

[Tsourakakis ICDM 2008]

Wikipedia graph 2006-Nov-04
≈ 3.1M nodes ≈ 37M edges



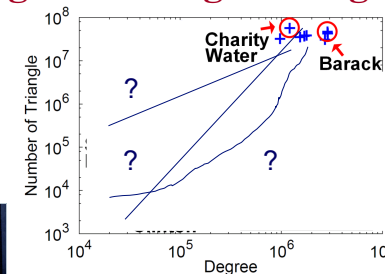
15-826

1000x+ speed-up, >90% accuracy

(c) C. Faloutsos, 2017

43

Triangle counting for large graphs?



Anomalous nodes in Twitter (~ 3 billion edges)

[U Kang, Brendan Meeder, +, PAKDD'11]

15-826

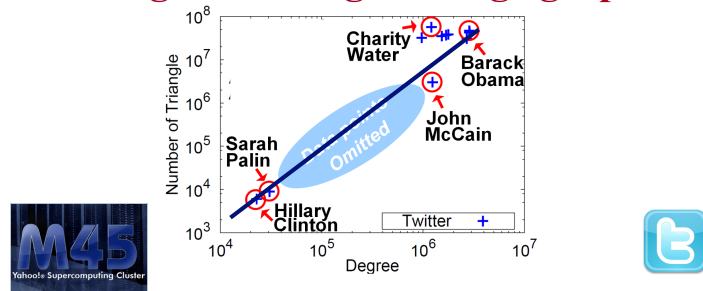


(c) C. Faloutsos, 2017

44

Carnegie Mellon

Triangle counting for large graphs?



Anomalous nodes in Twitter (~ 3 billion edges)
[U Kang, Brendan Meeder, +, PAKDD'11]

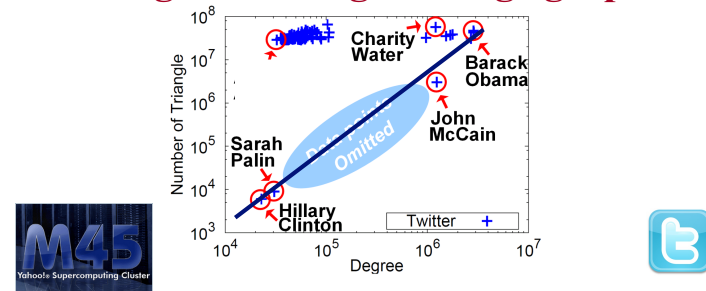
15-826

(c) C. Faloutsos, 2017

45

Carnegie Mellon

Triangle counting for large graphs?



Anomalous nodes in Twitter (~ 3 billion edges)
[U Kang, Brendan Meeder, +, PAKDD'11]

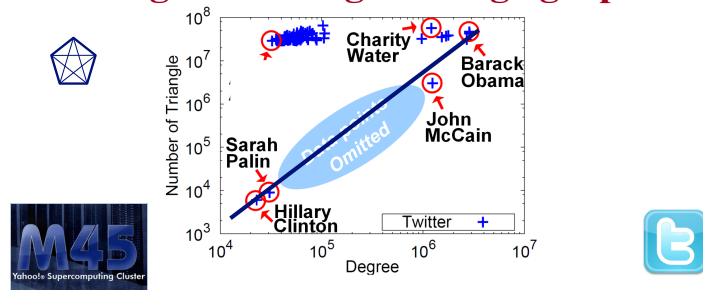
15-826

(c) C. Faloutsos, 2017

46

Carnegie Mellon

Triangle counting for large graphs?



Anomalous nodes in Twitter (~ 3 billion edges)
[U Kang, Brendan Meeder, +, PAKDD'11]

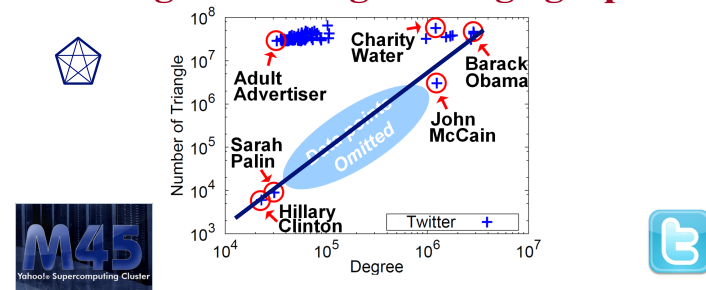
15-826

(c) C. Faloutsos, 2017

47

Carnegie Mellon

Triangle counting for large graphs?



Anomalous nodes in Twitter (~ 3 billion edges)
[U Kang, Brendan Meeder, +, PAKDD'11]

15-826

(c) C. Faloutsos, 2017

48

Any other 'laws' ?

Yes!

15-826

(c) C. Faloutsos, 2017

49

Any other 'laws' ?

Yes!

- Small diameter (\sim constant!) –
 - six degrees of separation / 'Kevin Bacon'
 - small worlds [Watts and Strogatz]

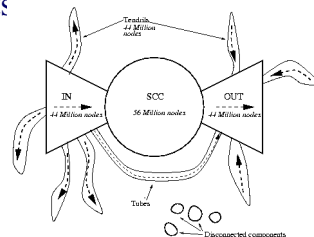
15-826

(c) C. Faloutsos, 2017

50

Any other 'laws' ?

- Bow-tie, for the web [Kumar+ '99]
- IN, SCC, OUT, 'tendrils'
- disconnected components



15-826

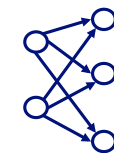
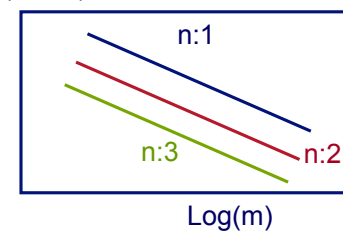
(c) C. Faloutsos, 2017

51

Any other 'laws' ?

- power-laws in communities (bi-partite cores) [Kumar+, '99]

Log(count)



2:3 core
(m:n core)

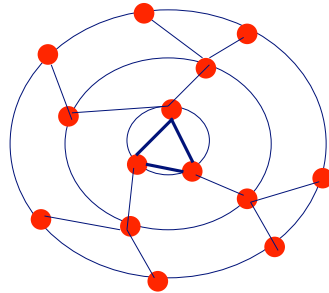
15-826

(c) C. Faloutsos, 2017

52

Any other 'laws' ?

- “Jellyfish” for Internet [Tauro+ '01]
- core: ~clique
- ~5 concentric layers
- many 1-degree nodes



15-826

(c) C. Faloutsos, 2017

53

EigenSpokes



B. Aditya Prakash, Mukund Seshadri, Ashwin Sridharan, Sridhar Machiraju and Christos Faloutsos: *EigenSpokes: Surprising Patterns and Scalable Community Chipping in Large Graphs*, PAKDD 2010, Hyderabad, India, 21-24 June 2010.

Useful for fraud detection!

15-826

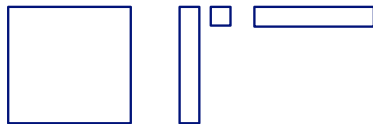
(c) C. Faloutsos, 2017

54

EigenSpokes

- Eigenvectors of adjacency matrix
 - equivalent to singular vectors (symmetric, undirected graph)

$$A = U\Sigma U^T$$



15-826

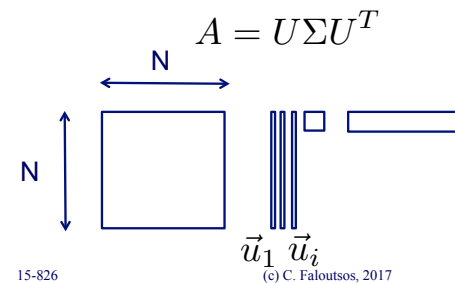
(c) C. Faloutsos, 2017

55

EigenSpokes



- Eigenvectors of adjacency matrix
 - equivalent to singular vectors (symmetric, undirected graph)



15-826

(c) C. Faloutsos, 2017

56

Carnegie Mellon

EigenSpokes

details

- Eigenvectors of adjacency matrix
 - equivalent to singular vectors (symmetric, undirected graph)

$$A = U\Sigma U^T$$

15-826 (c) C. Faloutsos, 2017 57

Carnegie Mellon

EigenSpokes

details

- Eigenvectors of adjacency matrix
 - equivalent to singular vectors (symmetric, undirected graph)

$$A = U\Sigma U^T$$

15-826 (c) C. Faloutsos, 2017 58

Carnegie Mellon

EigenSpokes

details

- Eigenvectors of adjacency matrix
 - equivalent to singular vectors (symmetric, undirected graph)

$$A = U\Sigma U^T$$

15-826 (c) C. Faloutsos, 2017 59

Carnegie Mellon

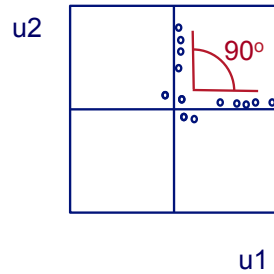
EigenSpokes

- EE plot:
- Scatter plot of scores of u1 vs u2
- One would expect
 - Many points @ origin
 - A few scattered ~randomly

15-826 (c) C. Faloutsos, 2017 60

EigenSpokes

- EE plot:
- Scatter plot of scores of u_1 vs u_2
- One would expect
 - Many points @ origin
 - A few scattered ~randomly



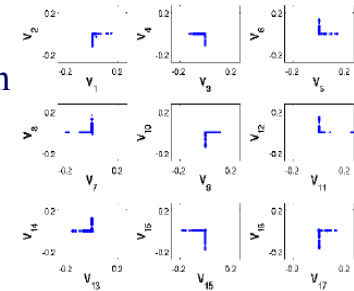
15-826

(c) C. Faloutsos, 2017

61

EigenSpokes - pervasiveness

- Present in mobile social graph
 - across time and space
- Patent citation graph



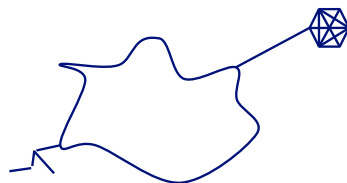
15-826

(c) C. Faloutsos, 2017

62

EigenSpokes - explanation

Near-cliques, or near-bipartite-cores, loosely connected



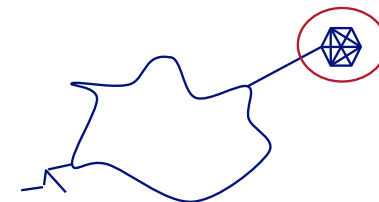
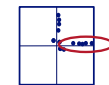
15-826

(c) C. Faloutsos, 2017

63

EigenSpokes - explanation

Near-cliques, or near-bipartite-cores, loosely connected



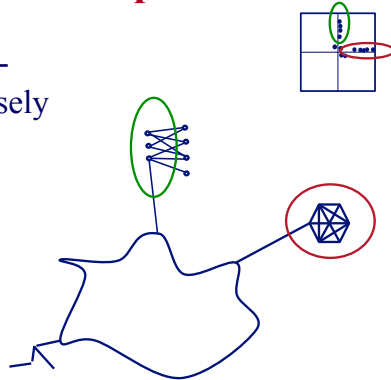
15-826

(c) C. Faloutsos, 2017

64

EigenSpokes - explanation

Near-cliques, or near-bipartite-cores, loosely connected



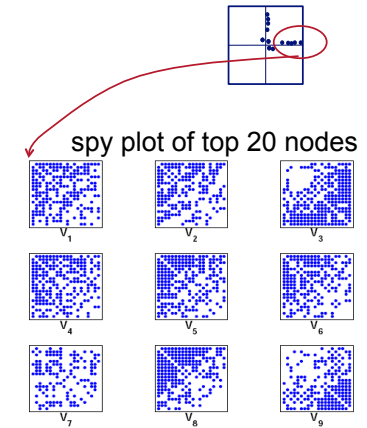
15-826

(c) C. Faloutsos, 2017

65

EigenSpokes - explanation

Near-cliques, or near-bipartite-cores, loosely connected



So what?

- Extract nodes with high scores
- high connectivity
- Good “communities”

15-826

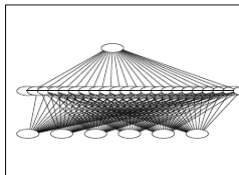
(c) C. Faloutsos, 2017

66

Bipartite Communities!

patents from same inventor(s)
‘cut-and-paste’ bibliography!

magnified bipartite community



Useful for fraud detection!

15-826

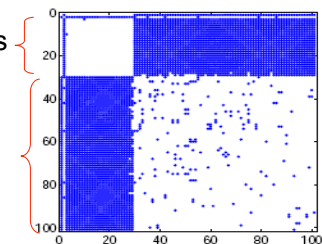
(c) C. Faloutsos, 2017

67

Bipartite Communities!

IP – port scanners

victims



Useful for fraud detection!


15-826

(c) C. Faloutsos, 2017

68

Carnegie Mellon

Outline





- Introduction – Motivation
- Problem#1: Patterns in graphs
 - Static graphs
 - degree, diameter, eigen,
 - Triangles
 - Weighted graphs
 - ➡ – Time evolving graphs
- Problem#2: Scalability
- Conclusions

15-826 (c) C. Faloutsos, 2017 69

Carnegie Mellon

Observations on weighted graphs?

- A: yes - even more 'laws' !

M. McGlohon, L. Akoglu, and C. Faloutsos
Weighted Graphs and Disconnected Components: Patterns and a Generator.
 SIG-KDD 2008

15-826 (c) C. Faloutsos, 2017 70

Carnegie Mellon

Observation W.1: Fortification

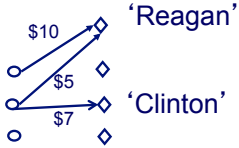
Q: How do the weights of nodes relate to degree?

15-826 (c) C. Faloutsos, 2017 71

Carnegie Mellon

Observation W.1: Fortification

More donors, more \$?

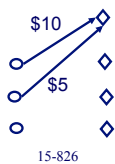


15-826 (c) C. Faloutsos, 2017 72

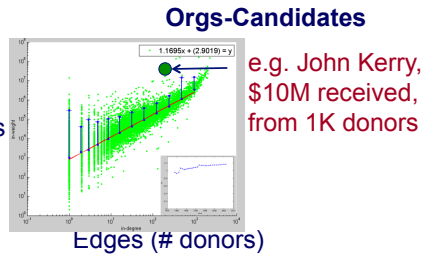
Observation W.1: fortification: Snapshot Power Law

- Weight: super-linear on in-degree
- exponent 'iw': $1.01 < iw < 1.26$

**More donors,
even more \$**



In-weights
(\$)



(c) C. Faloutsos, 2017

73

Outline



- Introduction – Motivation
- Problem#1: Patterns in graphs
 - Static graphs
 - Weighted graphs
 - ➔ – Time evolving graphs
- Problem#2: Scalability
- Conclusions

15-826

(c) C. Faloutsos, 2017

74

Problem: Time evolution

- with Jure Leskovec (CMU -> Stanford)
- and Jon Kleinberg (Cornell – sabb. @ CMU)



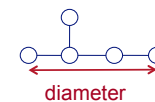
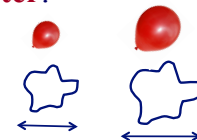
15-826

(c) C. Faloutsos, 2017

75

T.1 Evolution of the Diameter

- Prior work on Power Law graphs hints at **slowly growing diameter**:
 - [diameter $\sim O(N^{1/3})$]
 - diameter $\sim O(\log N)$
 - diameter $\sim O(\log \log N)$
- What is happening in real data?



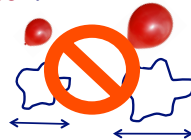
15-826

(c) C. Faloutsos, 2017

76

T.1 Evolution of the Diameter

- Prior work on Power Law graphs hints at **slowly growing diameter**:
 - [diameter $\sim O(N^{1/3})$]
 - diameter $\sim O(\log N)$
 - diameter $\sim O(\log \log N)$
- What is happening in real data?
- Diameter **shrinks** over time



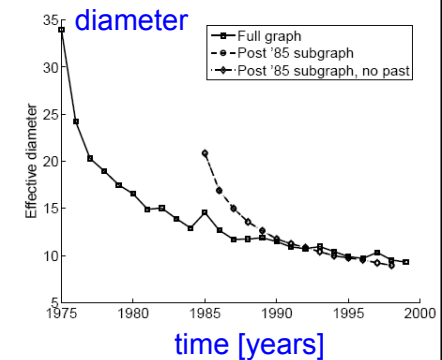
15-826

(c) C. Faloutsos, 2017

77

T.1 Diameter – “Patents”

- Patent citation network
- 25 years of data
- @1999
 - 2.9 M nodes
 - 16.5 M edges



15-826

(c) C. Faloutsos, 2017

78

T.2 Temporal Evolution of the Graphs

- $N(t)$... nodes at time t
- $E(t)$... edges at time t
- Suppose that

$$N(t+1) = 2 * N(t)$$
- Q: what is your guess for

$$E(t+1) = ? 2 * E(t)$$

15-826

(c) C. Faloutsos, 2017

79

T.2 Temporal Evolution of the Graphs

- $N(t)$... nodes at time t
- $E(t)$... edges at time t
- Suppose that

$$N(t+1) = 2 * N(t)$$
- Q: what is your guess for

$$E(t+1) = ? 2 * E(t)$$
- A: over-doubled!
 - But obeying the “Densification Power Law”

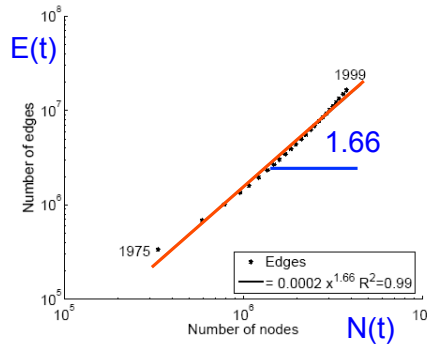
15-826

(c) C. Faloutsos, 2017

80

T.2 Densification – Patent Citations

- Citations among patents granted
- @1999
 - 2.9 M nodes
 - 16.5 M edges
- Each year is a datapoint



15-826

(c) C. Faloutsos, 2017

81

Outline



- Introduction – Motivation
- Problem#1: Patterns in graphs
 - Static graphs
 - Weighted graphs
 - ➔ – Time evolving graphs
- Problem#2: Scalability
- Conclusions

15-826

(c) C. Faloutsos, 2017

82

More on Time-evolving graphs

M. McGlohon, L. Akoglu, and C. Faloutsos
Weighted Graphs and Disconnected Components: Patterns and a Generator.
SIG-KDD 2008

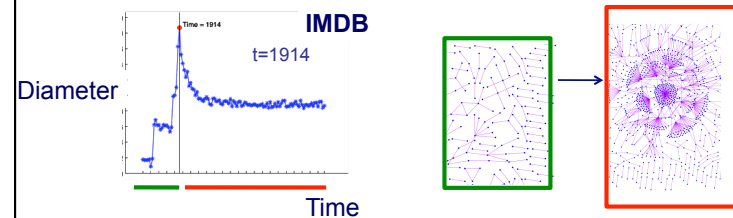
15-826

(c) C. Faloutsos, 2017

83

[Gelling Point]

- Most real graphs display a gelling point
- After gelling point, they exhibit typical behavior. This is marked by a spike in diameter.



15-826

(c) C. Faloutsos, 2017

84

Observation T.3: NLCC behavior

Q: How do NLCC's emerge and join with the GCC?

('NLCC' = non-largest conn. components)

- Do they continue to grow in size?
- or do they shrink?
- or stabilize?



15-826

(c) C. Faloutsos, 2017

85

Observation T.3: NLCC behavior

Q: How do NLCC's emerge and join with the GCC?

('NLCC' = non-largest conn. components)

- Do they continue to grow in size?
- or do they shrink?
- or stabilize?



15-826

(c) C. Faloutsos, 2017

86

Observation T.3: NLCC behavior

Q: How do NLCC's emerge and join with the GCC?

('NLCC' = non-largest conn. components)

- YES** – Do they continue to grow in size?
- YES** – or do they shrink?
- YES** – or stabilize?

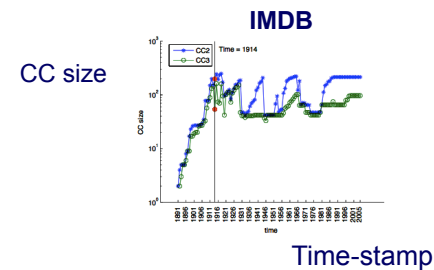
15-826

(c) C. Faloutsos, 2017

87

Observation T.3: NLCC behavior

- After the gelling point, the GCC takes off, but NLCC's remain ~constant (actually, **oscillate**).



15-826

(c) C. Faloutsos, 2017

88

Timing for Blogs

- with Mary McGlohon (CMU->Google)
 - Jure Leskovec (CMU->Stanford)
 - Natalie Glance (now at Google)
 - Mat Hurst (now at MSR)
- [SDM' 07]

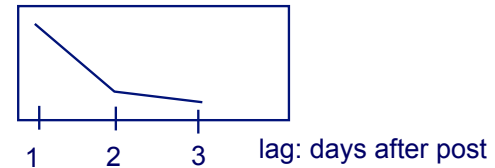
15-826

(c) C. Faloutsos, 2017

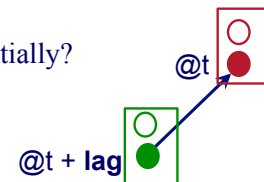
89

T.4 : popularity over time

in links



Post popularity drops-off – exponentially?

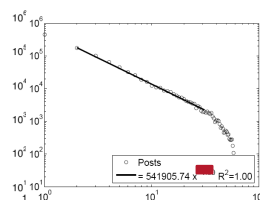


15-826

(c) C. Faloutsos, 2017

90

T.4 : popularity over time

in links
(log)days after post
(log)

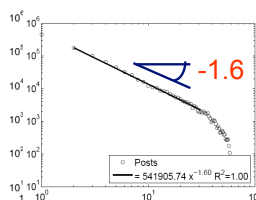
Post popularity drops-off – exponentially?
POWER LAW!
Exponent?

15-826

(c) C. Faloutsos, 2017

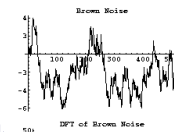
91

T.4 : popularity over time

in links
(log)days after post
(log)

Post popularity drops-off – exponentially?
POWER LAW!
Exponent? -1.6

- close to -1.5: Barabasi's stack model
- and like the zero-crossings of a random walk



15-826

(c) C. Faloutsos, 2017

92

-1.5 slope

J. G. Oliveira & A.-L. Barabási Human Dynamics: The Correspondence Patterns of Darwin and Einstein. *Nature* **437**, 1251 (2005). [[PDF](#)]

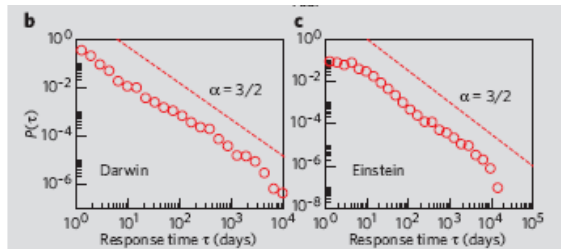


Figure 1 | The correspondence patterns of Darwin and Einstein.

93

T.5: duration of phonecalls

Surprising Patterns for the Call Duration Distribution of Mobile Phone Users

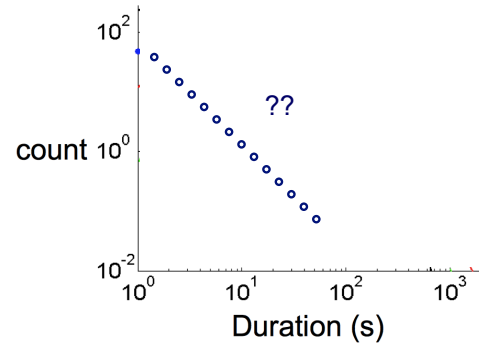


Pedro O. S. Vaz de Melo, Leman Akoglu, Christos Faloutsos, Antonio A. F. Loureiro
PKDD 2010

15-826

(c) C. Faloutsos, 2017

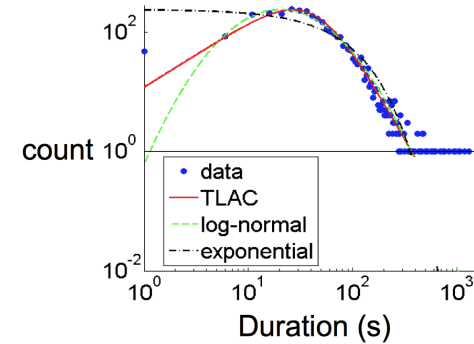
94

Probably, power law (?)

15-826

(c) C. Faloutsos, 2017

95

No Power Law!

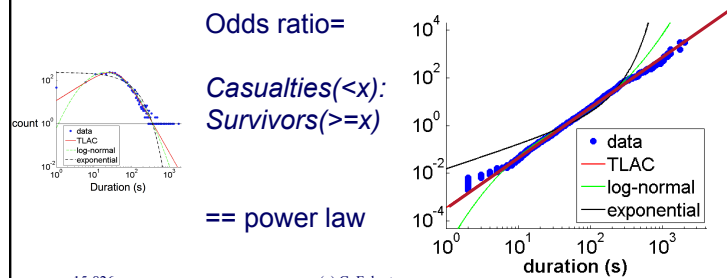
15-826

(c) C. Faloutsos, 2017

96

'TLaC: Lazy Contractor'

- The longer a task (phonecall) has taken,
- The even longer it will take



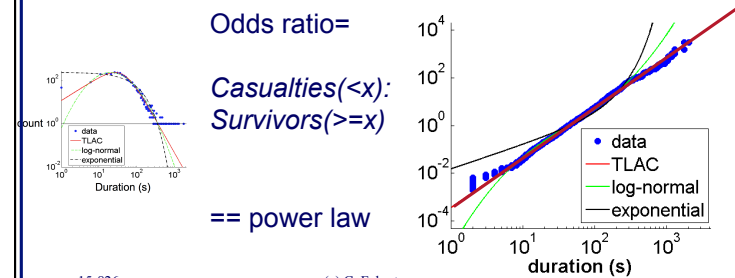
15-826

(c) C. Faloutsos, 2017

97

Log-logistic distribution

- $CDF(t)/(1 - CDF(t)) == OR(t)$
- For log-logistic: $\log[OR(t)] = \beta + \rho * \log(t)$



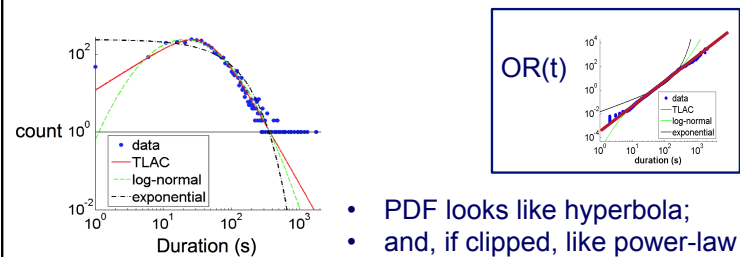
15-826

(c) C. Faloutsos, 2017

98

Log-logistic distribution

- $CDF(t)/(1 - CDF(t)) == OR(t)$
- For log-logistic: $\log[OR(t)] = \beta + \rho * \log(t)$



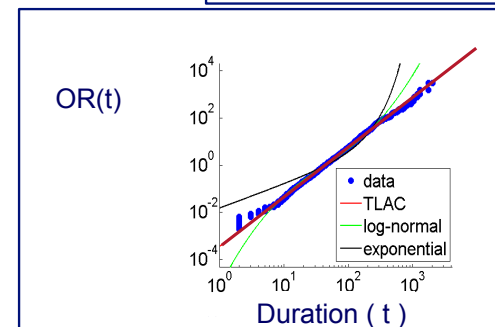
15-826

(c) C. Faloutsos, 2017

99

Log-logistic distribution

- $CDF(t)/(1 - CDF(t)) == OR(t)$
- For log-logistic: $\log[OR(t)] = \beta + \rho * \log(t)$



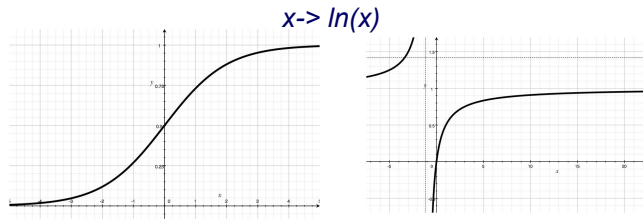
15-826

100

Carnegie Mellon

Log-logistic distribution

- Logistic distribution: CDF \rightarrow sigmoid
- LOG-Logistic distribution:



$$\text{CDF}(x) = 1/(1+\exp(-x))$$

15-826

(c) C. Faloutsos, 2017

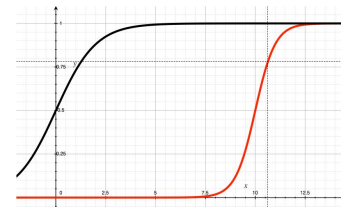
101

$$\text{CDF}(x) = 1/(1+1/x)$$

Carnegie Mellon

Log-logistic distribution

- Logistic distribution: CDF \rightarrow sigmoid
- LOG-Logistic distribution:



$$\text{CDF}(x) = 1/(1+\exp(-(x-m)/s)) \quad \text{CDF}(x) = 1/(1+\exp(-(\ln(x)-m)/s))$$

15-826

(c) C. Faloutsos, 2017

102

Carnegie Mellon

Data Description

- Data from a private mobile operator of a large city
 - 4 months of data
 - 3.1 million users
 - more than 1 billion phone records
- Over 96% of 'talkative' users obeyed a TLAC distribution ('talkative': >30 calls)

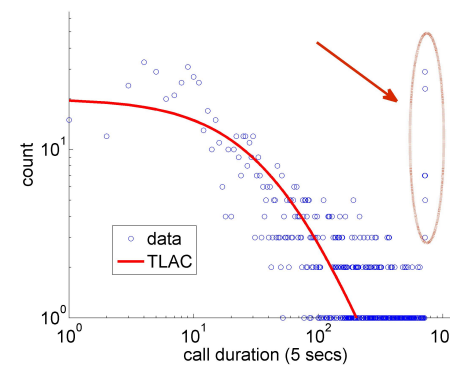
15-826

(c) C. Faloutsos, 2017

103

Carnegie Mellon

Outliers:



15-826

(c) C. Faloutsos, 2017

104

Outline



- Introduction – Motivation
- Problem#1: Patterns in graphs
- ➡ • Problem#2: Scalability -PEGASUS
- Conclusions

15-826

(c) C. Faloutsos, 2017

105

Scalability



- Google: > 450,000 processors in clusters of ~2000 processors each [Barroso, Dean, Hölzle, “Web Search for a Planet: The Google Cluster Architecture” IEEE Micro 2003]
- Yahoo: 5Pb of data [Fayyad, KDD’ 07]
- Problem: machine failures, on a daily basis
- How to parallelize data mining tasks, then?
- A: map/reduce – hadoop (open-source clone)
<http://hadoop.apache.org/>



15-826

(c) C. Faloutsos, 2017

106

Outline – Algorithms & results

	Centralized	Hadoop/ PEGASUS
Degree Distr.	old	old
Pagerank	old	old
➡ Diameter/ANF	old	HERE
Conn. Comp	old	HERE
Triangles	done	HERE
Visualization	started	

15-826

(c) C. Faloutsos, 2017

107

HADI for diameter estimation

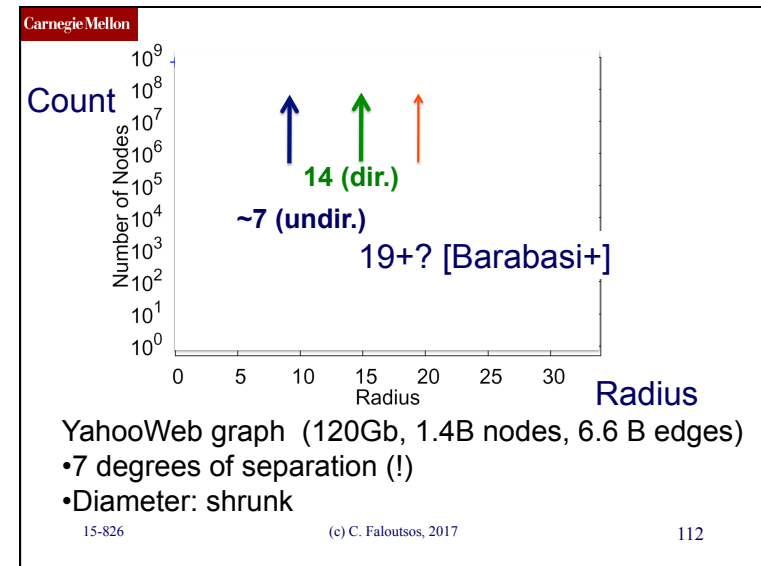
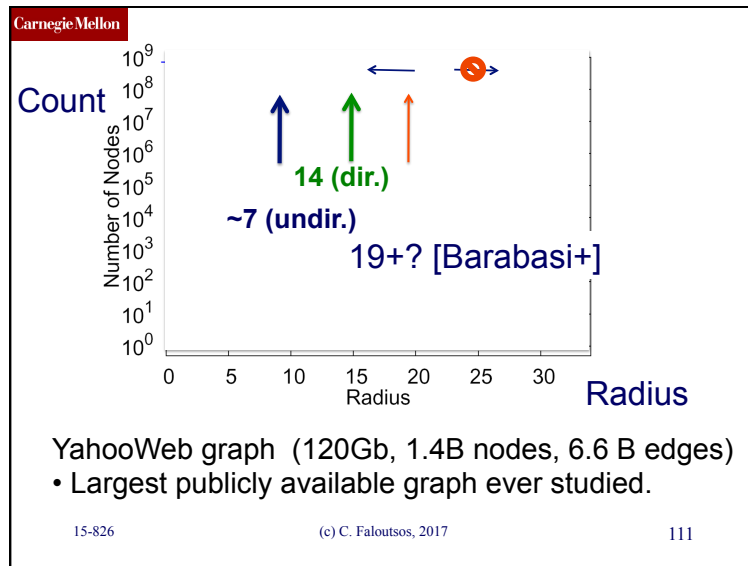
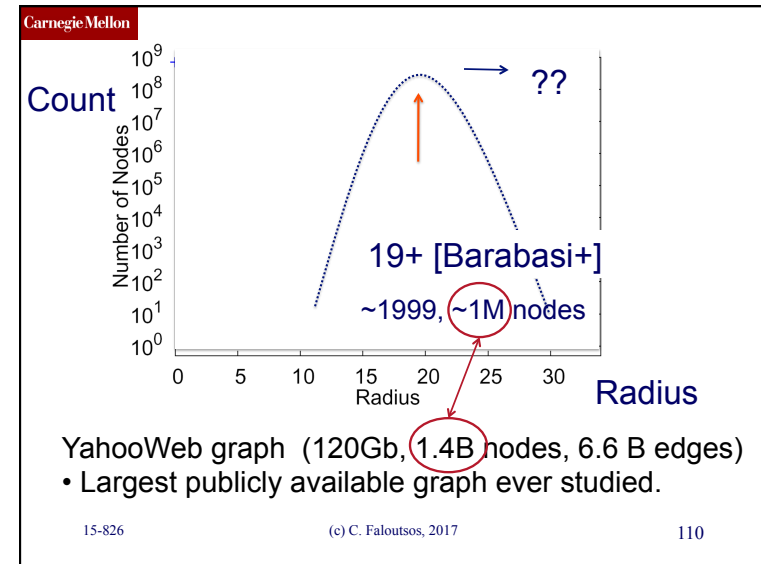
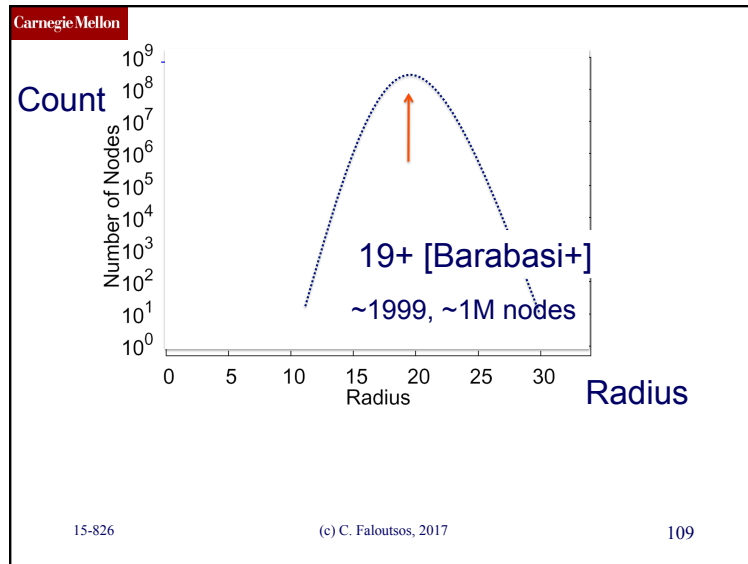


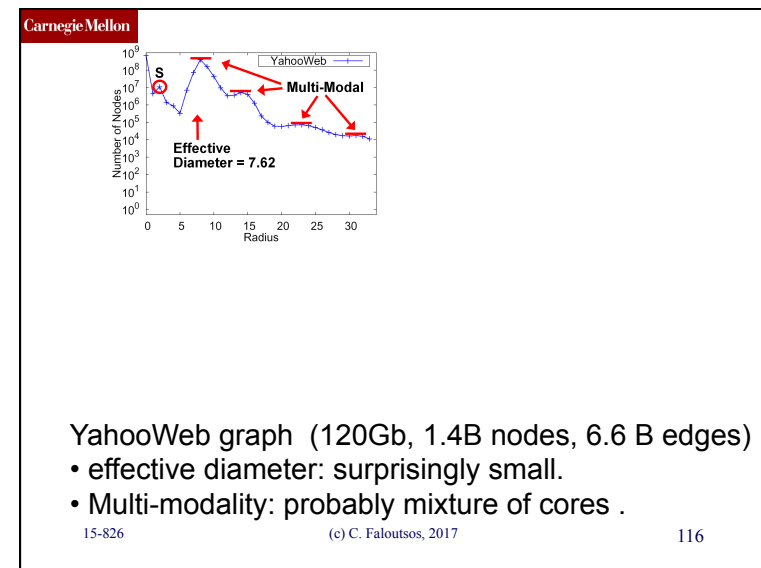
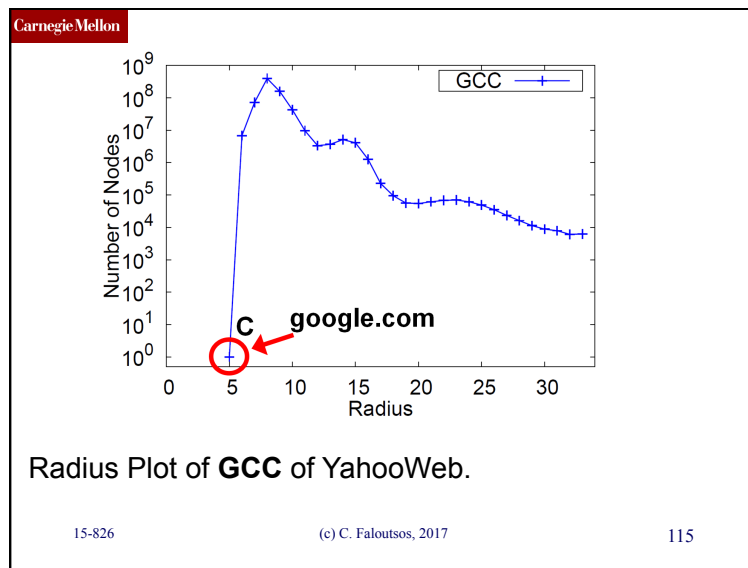
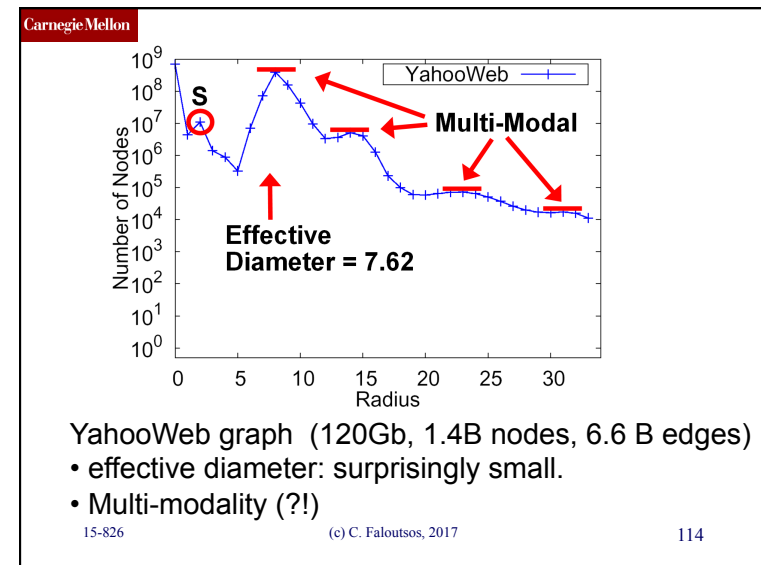
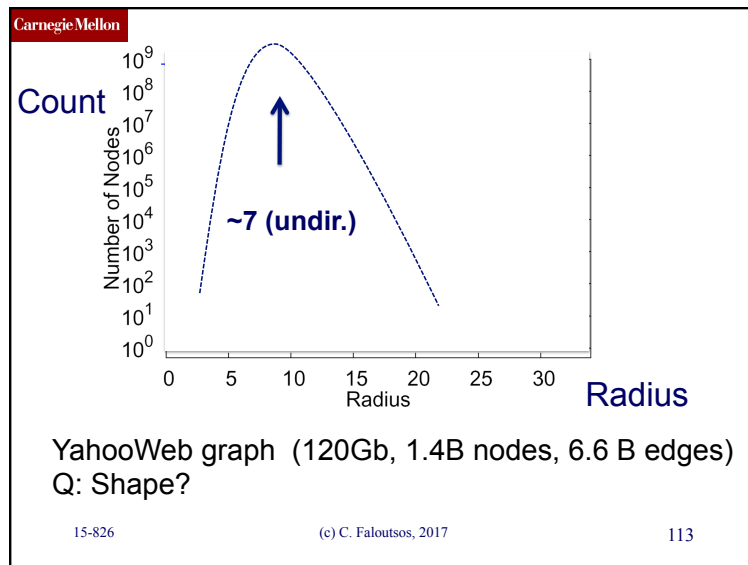
- *Radius Plots for Mining Tera-byte Scale Graphs* **U Kang**, Charalampos Tsourakakis, Ana Paula Appel, Christos Faloutsos, Jure Leskovec, SDM’10
- Naively: diameter needs $O(N^2)$ space and up to $O(N^3)$ time – **prohibitive** ($N \sim 1B$)
- Our HADI: linear on E ($\sim 10B$)
 - Near-linear scalability wrt # machines
 - Several optimizations \rightarrow 5x faster

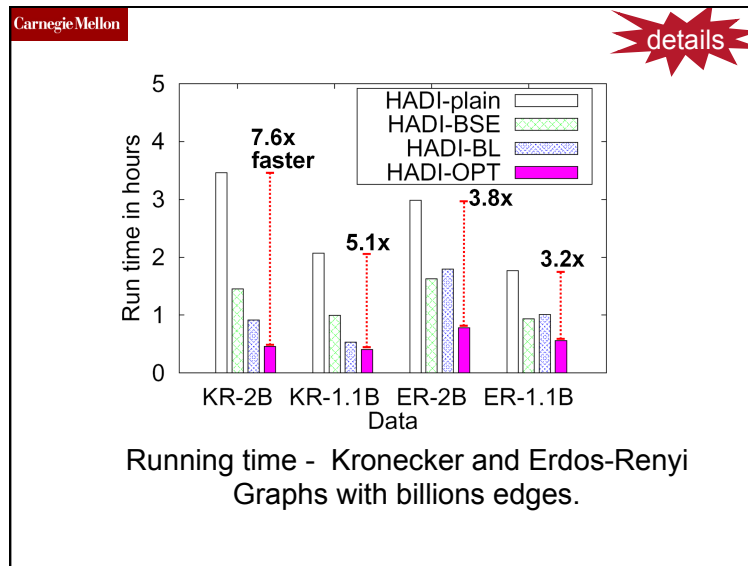
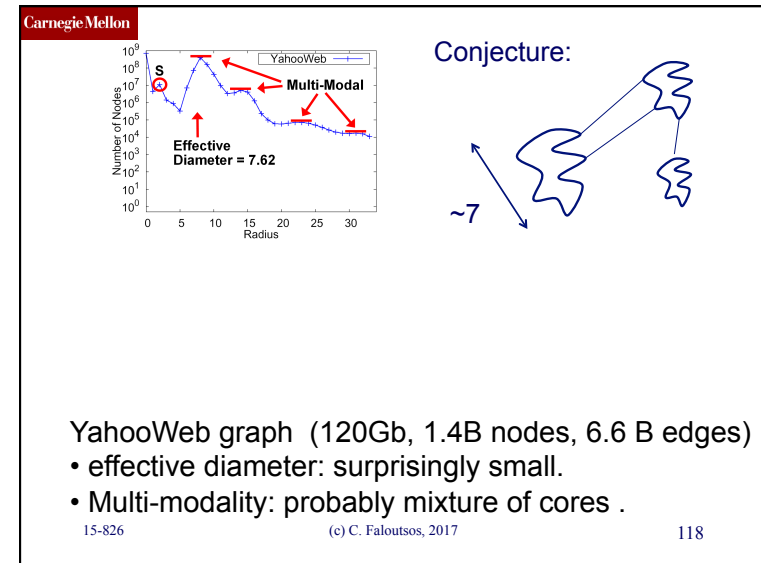
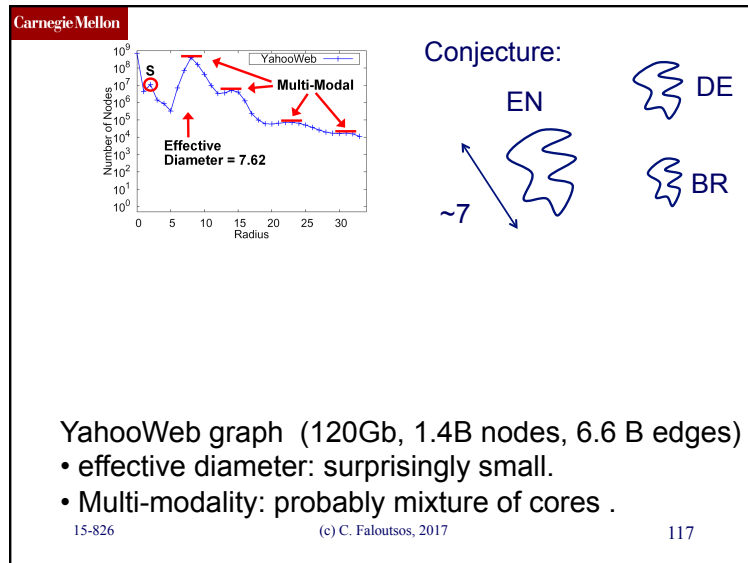
15-826

(c) C. Faloutsos, 2017

108







Carnegie Mellon

Outline – Algorithms & results

	Centralized	Hadoop/ PEGASUS
Degree Distr.	old	old
Pagerank	old	old
Diameter/ANF	old	HERE
Conn. Comp	old	HERE
Triangles		HERE
Visualization	started	

15-826 (c) C. Faloutsos, 2017 120

Generalized Iterated Matrix Vector Multiplication (GIMV)

[PEGASUS: A Peta-Scale Graph Mining System - Implementation and Observations.](#)

U Kang, Charalampos E. Tsourakakis, and Christos Faloutsos.

(ICDM) 2009, Miami, Florida, USA.
Best Application Paper (runner-up).

15-826

(c) C. Faloutsos, 2017

121

Generalized Iterated Matrix Vector Multiplication (GIMV)

details

- PageRank
- proximity (RWR)
- Diameter
- Connected components
- (eigenvectors,
- Belief Prop.
- ...)

Matrix – vector
Multiplication
(iterated)

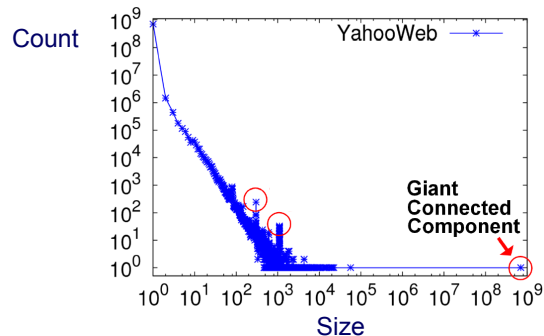
15-826

(c) C. Faloutsos, 2017

122

Example: GIM-V At Work

- Connected Components – 4 observations:



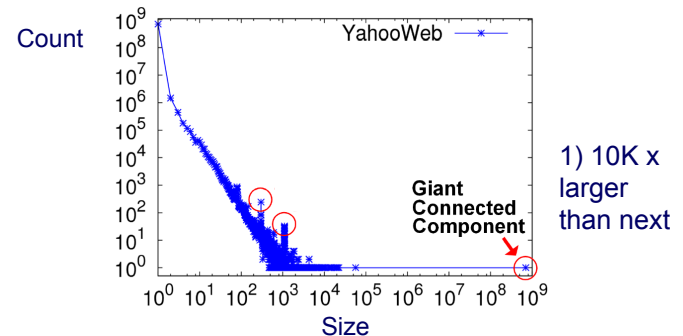
15-826

(c) C. Faloutsos, 2017

123

Example: GIM-V At Work

- Connected Components



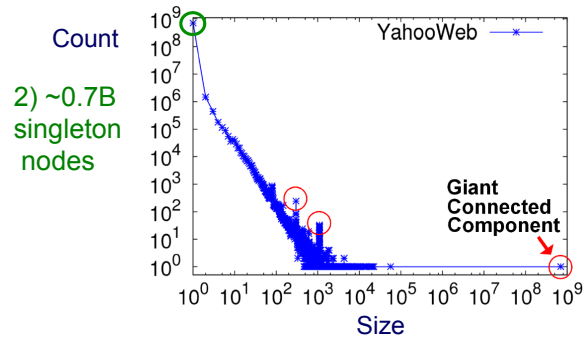
15-826

(c) C. Faloutsos, 2017

124

Example: GIM-V At Work

- Connected Components



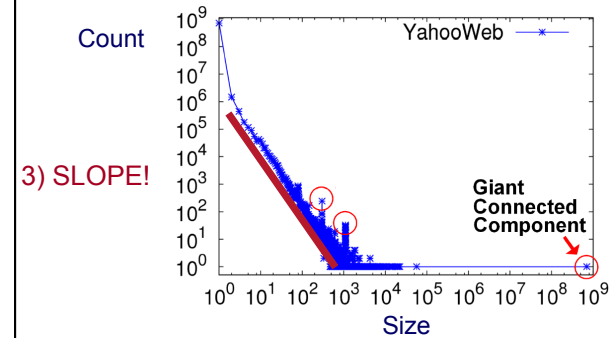
15-826

(c) C. Faloutsos, 2017

125

Example: GIM-V At Work

- Connected Components



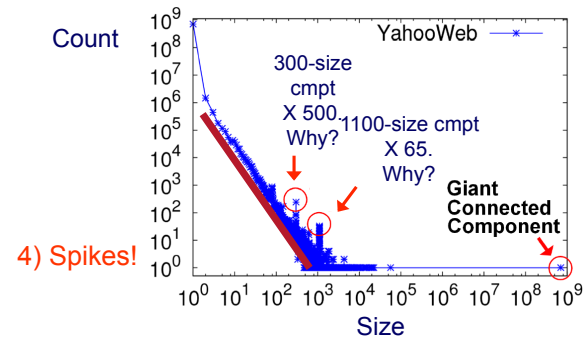
15-826

(c) C. Faloutsos, 2017

126

Example: GIM-V At Work

- Connected Components



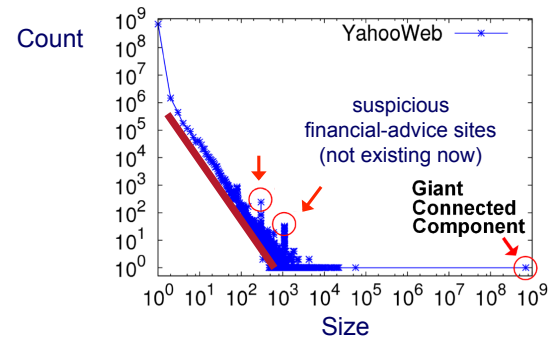
15-826

(c) C. Faloutsos, 2017

127

Example: GIM-V At Work

- Connected Components



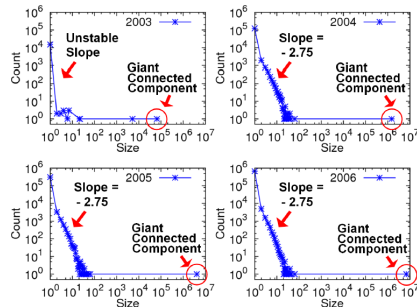
15-826

(c) C. Faloutsos, 2017

128

GIM-V At Work

- Connected Components over Time
- **LinkedIn: 7.5M nodes and 58M edges**



Stable tail slope
after the gelling point

15-826

(c) C. Faloutsos, 2017

129

Outline



- Introduction – Motivation
- Problem#1: Patterns in graphs
- DELETE
- Problem#2: Scalability
- ➔ Conclusions

15-826

(c) C. Faloutsos, 2017

130

OVERALL CONCLUSIONS – low level:

- Several new **patterns** (fortification, shrinking diameter, triangle-laws, conn. components, etc)
- Log-logistic distribution: ubiquitous
- New **tools**:
 - anomaly detection (OddBall), belief propagation, immunization
- **Scalability**: PEGASUS / hadoop

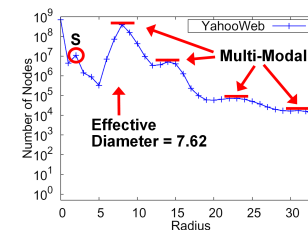
15-826

(c) C. Faloutsos, 2017

131

OVERALL CONCLUSIONS – high level

- **BIG DATA**: Large datasets reveal patterns/outliers that are invisible otherwise



15-826

(c) C. Faloutsos, 2017

132

References

- Leman Akoglu, Christos Faloutsos: *RTG: A Recursive Realistic Graph Generator Using Random Typing*. ECML/PKDD (1) 2009: 13-28
- Deepayan Chakrabarti, Christos Faloutsos: *Graph mining: Laws, generators, and algorithms*. ACM Comput. Surv. 38(1): (2006)

15-826

(c) C. Faloutsos, 2017

133

References

- Deepayan Chakrabarti, Yang Wang, Chenxi Wang, Jure Leskovec, Christos Faloutsos: *Epidemic thresholds in real networks*. ACM Trans. Inf. Syst. Secur. 10(4): (2008)

15-826

(c) C. Faloutsos, 2017

134

References

- Jure Leskovec, Jon Kleinberg and Christos Faloutsos: *Graphs over Time: Densification Laws, Shrinking Diameters and Possible Explanations*, KDD 2005 (Best Research paper award).
- Jure Leskovec, Deepayan Chakrabarti, Jon M. Kleinberg, Christos Faloutsos: *Realistic, Mathematically Tractable Graph Generation and Evolution, Using Kronecker Multiplication*. PKDD 2005: 133-145

15-826

(c) C. Faloutsos, 2017

135

References

- Jimeng Sun, Yinglian Xie, Hui Zhang, Christos Faloutsos. *Less is More: Compact Matrix Decomposition for Large Sparse Graphs*, SDM, Minneapolis, Minnesota, Apr 2007.
- Jimeng Sun, Spiros Papadimitriou, Philip S. Yu, and Christos Faloutsos, *GraphScope: Parameter-free Mining of Large Time-evolving Graphs* ACM SIGKDD Conference, San Jose, CA, August 2007

15-826

(c) C. Faloutsos, 2017

136

References

- Jimeng Sun, Dacheng Tao, Christos Faloutsos: *Beyond streams and graphs: dynamic tensor analysis*. KDD 2006: 374-383

15-826

(c) C. Faloutsos, 2017

137

References

- Hanghang Tong, Christos Faloutsos, and Jia-Yu Pan, *Fast Random Walk with Restart and Its Applications*, ICDM 2006, Hong Kong.
- Hanghang Tong, Christos Faloutsos, *Center-Piece Subgraphs: Problem Definition and Fast Solutions*, KDD 2006, Philadelphia, PA

15-826

(c) C. Faloutsos, 2017

138

References

- Hanghang Tong, Christos Faloutsos, Brian Gallagher, Tina Eliassi-Rad: Fast best-effort pattern matching in large attributed graphs. KDD 2007: 737-746

15-826

(c) C. Faloutsos, 2017

139

(Project info)

www.cs.cmu.edu/~pegasus



Chau,
Polo



Akoglu,
Leman

Koutra,
Danae



Kang, U

Prakash,
Aditya



McGlohon,
Mary



Tong,
Hanghang

15-826

(c) C. Faloutsos, 2017

140