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15-826: Multimedia Databases and Data Mining

Lecture #20: SVD - part III (more case studies)

C. Faloutsos

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Must-read Material

- MM Textbook Appendix D
- Graph Mining Textbook, chapter 15.
- Kleinberg, J. (1998). Authoritative sources in a hyperlinked environment. Proc. 9th ACM-SIAM Symposium on Discrete Algorithms.
- Brin, S. and L. Page (1998). Anatomy of a Large-Scale Hypertextual Web Search Engine. 7th Intl World Wide Web Conf.

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Must-read Material, cont' d

- Haveliwala, Taher H. (2003) Topic-Sensitive PageRank: A Context-Sensitive Ranking Algorithm for Web Search. Extended version of the WWW2002 paper.
- Chen, C. M. and N. Roussopoulos (May 1994). Adaptive Selectivity Estimation Using Query Feedback. Proc. of the ACM-SIGMOD , Minneapolis, MN.

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Outline

Goal: 'Find similar / interesting things'

- Intro to DB
- Indexing - similarity search
- Data Mining

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Indexing - Detailed outline

- primary key indexing
- secondary key / multi-key indexing
- spatial access methods
- fractals
- text
- Singular Value Decomposition (SVD)
- multimedia
- ...

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SVD - Detailed outline

- Motivation
- Definition - properties
- Interpretation
- Complexity
- Case studies
- SVD properties
- More case studies
- Conclusions

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SVD - detailed outline

- ...
- Case studies
- SVD properties
- more case studies
  - google/Kleinberg algorithms
  - query feedbacks
- Conclusions

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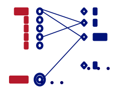
SVD - Other properties - summary

- can produce orthogonal basis (obvious) (who cares?)
- can solve over- and under-determined linear problems (see C(1) property)
- can compute 'fixed points' (= 'steady state prob. in Markov chains') (see C(4) property)

IMPORTANT!

Properties – sneak preview:

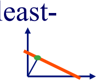
A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \mathbf{\Lambda}_{[r \times r]} \mathbf{V}^{\mathbf{T}}_{[r \times m]}$



B(5): $(\mathbf{A}^T \mathbf{A})^k \mathbf{v'} \sim (\text{constant}) \mathbf{v}_1$

C(1): $\mathbf{A}_{[n \times m]} \mathbf{x}_{[m \times 1]} = \mathbf{b}_{[n \times 1]}$

then, $\mathbf{x}_0 = \mathbf{V} \mathbf{\Lambda}^{(-1)} \mathbf{U}^T \mathbf{b}$: shortest, actual or least-squares solution

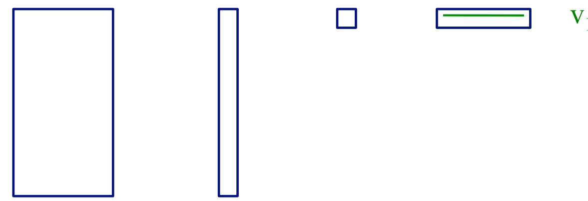


C(4): $\mathbf{A}^{\mathbf{T}} \mathbf{A} \mathbf{v}_1 = \lambda_1^2 \mathbf{v}_1$

IMPORTANT!

Properties – sneak preview:

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \mathbf{\Lambda}_{[r \times r]} \mathbf{V}^{\mathbf{T}}_{[r \times m]}$

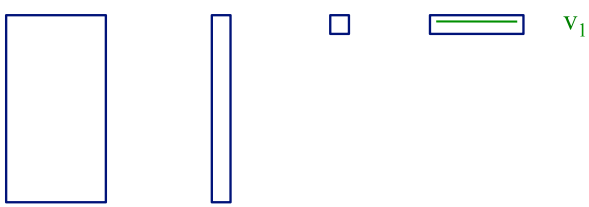


Document-term Doc-concept Concept-term

IMPORTANT!

Properties – sneak preview:

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \mathbf{\Lambda}_{[r \times r]} \mathbf{V}^{\mathbf{T}}_{[r \times m]}$

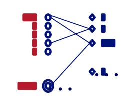


Libraries-books Libraries-concepts Concepts-books

IMPORTANT!

Properties – sneak preview:

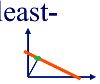
A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \mathbf{\Lambda}_{[r \times r]} \mathbf{V}^{\mathbf{T}}_{[r \times m]}$



B(5): $(\mathbf{A}^T \mathbf{A})^k \mathbf{v'} \sim (\text{constant}) \mathbf{v}_1$

C(1): $\mathbf{A}_{[n \times m]} \mathbf{x}_{[m \times 1]} = \mathbf{b}_{[n \times 1]}$

then, $\mathbf{x}_0 = \mathbf{V} \mathbf{\Lambda}^{(-1)} \mathbf{U}^T \mathbf{b}$: shortest, actual or least-squares solution



C(4): $\mathbf{A}^{\mathbf{T}} \mathbf{A} \mathbf{v}_1 = \lambda_1^2 \mathbf{v}_1$

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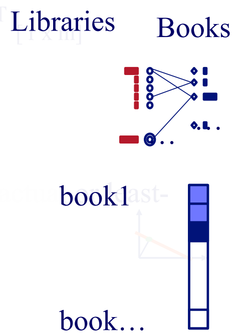
IMPORTANT!

Properties – sneak preview:

B(5): $(\mathbf{A}^T \mathbf{A})^k \mathbf{v}' \sim$ (constant) $\mathbf{v}_1$

Convergence



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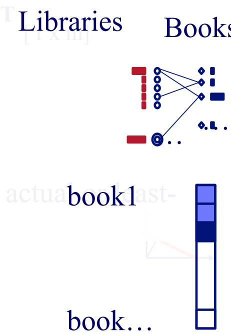
IMPORTANT!

Properties – sneak preview:

B(5): $(\mathbf{A}^T \mathbf{A})^k \mathbf{v}' \sim$ (constant) $\mathbf{v}_1$

Fixed point

C(4): $\mathbf{A}^T \mathbf{A}\, \mathbf{v}_1 = \lambda_1^2\, \mathbf{v}_1$



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IMPORTANT!

Properties – sneak preview:

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]}\, \mathbf{\Lambda}_{[r \times r]}\, \mathbf{V}^T_{[r \times m]}$

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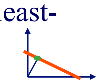
then, $\mathbf{x}_0 = \mathbf{V}\, \mathbf{\Lambda}^{(-1)}\, \mathbf{U}^T\, \mathbf{b}$: shortest, actual or least-squares solution



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SVD -outline of properties

- (A): obvious
- (B): less obvious
- (C): least obvious (and most powerful!)

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Properties - by defn.:

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \mathbf{\Lambda}_{[r \times r]} \mathbf{V}^T_{[r \times m]}$

A(1): $\mathbf{U}^T_{[r \times n]} \mathbf{U}_{[n \times r]} = \mathbf{I}_{[r \times r]}$ (identity matrix)

A(2): $\mathbf{V}^T_{[r \times n]} \mathbf{V}_{[n \times r]} = \mathbf{I}_{[r \times r]}$

A(3): $\mathbf{\Lambda}^k = \text{diag}(\lambda_1^k, \lambda_2^k, \ldots \lambda_r^k)$ (k: ANY real number)

A(4): $\mathbf{A}^T = \mathbf{V} \mathbf{\Lambda} \mathbf{U}^T$

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Less obvious properties

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \mathbf{\Lambda}_{[r \times r]} \mathbf{V}^T_{[r \times m]}$

B(1): $\mathbf{A}_{[n \times m]} (\mathbf{A}^T)_{[m \times n]} = ??$

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Less obvious properties

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \mathbf{\Lambda}_{[r \times r]} \mathbf{V}^T_{[r \times m]}$

B(1): $\mathbf{A}_{[n \times m]} (\mathbf{A}^T)_{[m \times n]} = \mathbf{U} \mathbf{\Lambda}^2 \mathbf{U}^T$

symmetric; Intuition?

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Less obvious properties

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \mathbf{\Lambda}_{[r \times r]} \mathbf{V}^T_{[r \times m]}$

B(1): $\mathbf{A}_{[n \times m]} (\mathbf{A}^T)_{[m \times n]} = \mathbf{U} \mathbf{\Lambda}^2 \mathbf{U}^T$

symmetric; Intuition?

'document-to-document' similarity matrix

B(2): symmetrically, for 'V'

$(\mathbf{A}^T)_{[m \times n]} \mathbf{A}_{[n \times m]} = \mathbf{V} \mathbf{\Lambda}^2 \mathbf{V}^T$

Intuition?

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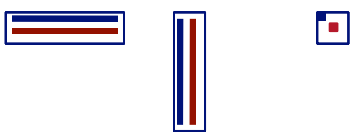
Reminder: 'column orthonormal'

- $\mathbf{V}^T$ $\mathbf{V}$ = $\mathbf{I}_{[r \times r]}$

v1

$\mathbf{v}_2$



$\mathbf{v}_1^T \times \mathbf{v}_1 = 1$

$\mathbf{v}_1^T \times \mathbf{v}_2 = 0$

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Less obvious properties

A: term-to-term similarity matrix

B(3): $((\mathbf{A}^T)_{[m \times n]} \mathbf{A}_{[n \times m]})^k = \mathbf{V} \mathbf{\Lambda}^{2k} \mathbf{V}^T$

and

B(4): $(\mathbf{A}^T \mathbf{A})^k \sim \mathbf{v}_1 \lambda_1^{2k} \mathbf{v}_1^T$ for k>>1

where

$\mathbf{v}_1$: [m x 1] first column (singular-vector) of **V**

λ_1: strongest singular value

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Proof of (B4)?

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Less obvious properties

B(4): $(\mathbf{A}^T \mathbf{A})^k \sim \mathbf{v}_1 \lambda_1^{2k} \mathbf{v}_1^T$ for k>>1

B(5): $(\mathbf{A}^T \mathbf{A})^k \mathbf{v}' \sim$ (constant) $\mathbf{v}_1$

ie., for (almost) any **v'**, it converges to a vector parallel to $\mathbf{v}_1$

Thus, useful to compute first singular vector/ value (as well as the next ones, too...)

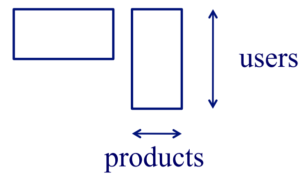
Proof of (B5)?

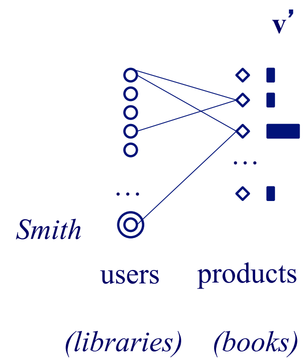
- B(5): $(\mathbf{A}^T \mathbf{A})^k \mathbf{v'} \sim$ (constant) $\mathbf{v}_1$
- == $(\mathbf{A}^T \mathbf{A}) \dots (\mathbf{A}^T \mathbf{A}) \mathbf{v'} \sim$ (constant) $\mathbf{v}_1$

k times

Property (B5)

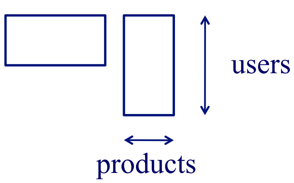
- Intuition:
  - $(\mathbf{A}^T \mathbf{A}) \mathbf{v'}$
  - $(\mathbf{A}^T \mathbf{A})^k \mathbf{v'}$

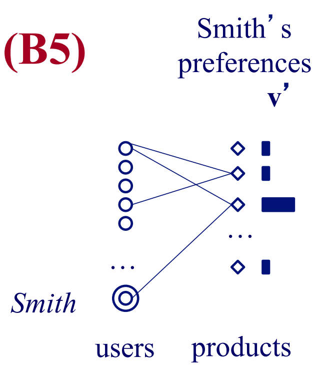




Property (B5)

- Intuition:
  - $(\mathbf{A}^T \mathbf{A}) \mathbf{v'}$
  - $(\mathbf{A}^T \mathbf{A})^k \mathbf{v'}$





Property (B5)

- Intuition:
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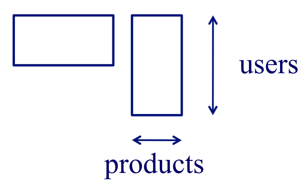
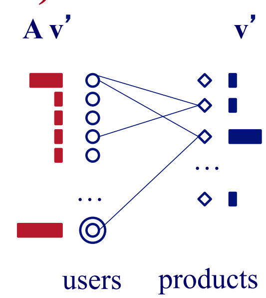
  - $(\mathbf{A}^T \mathbf{A})^k \mathbf{v'}$





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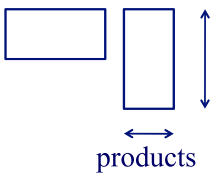
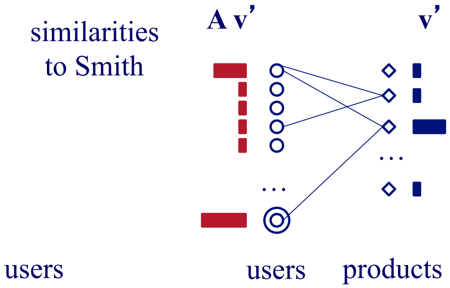
Property (B5)

- Intuition:
  - $(\mathbf{A}^T \mathbf{A})\ \mathbf{v'}$
  - $(\mathbf{A}^T \mathbf{A})^k\ \mathbf{v'}$





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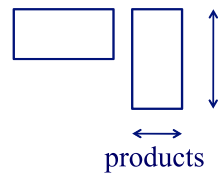
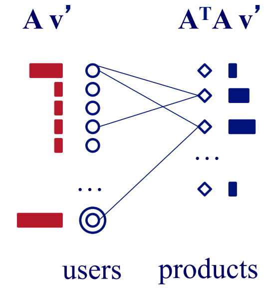
Property (B5)

- Intuition:
  - $(\mathbf{A}^T \mathbf{A})\ \mathbf{v'}$
  - $(\mathbf{A}^T \mathbf{A})^k\ \mathbf{v'}$





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Property (B5)

- Intuition:
  - $(\mathbf{A}^T \mathbf{A})\ \mathbf{v'}$ what Smith's 'friends' like
  - $(\mathbf{A}^T \mathbf{A})^k\ \mathbf{v'}$ what k-step-away-friends like

(ie., after *k* steps, we get what everybody likes, and Smith's initial opinions don't count)

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Less obvious properties - repeated:

A(0): $\mathbf{A}_{[n\ x\ m]} = \mathbf{U}_{[\ n\ x\ r\ ]}\ \mathbf{\Lambda}_{[\ r\ x\ r\ ]}\ \mathbf{V^T}_{[\ r\ x\ m]}$

B(1): $\mathbf{A}_{[n\ x\ m]}\ (\mathbf{A}^T)_{[m\ x\ n]} = \mathbf{U}\ \mathbf{\Lambda}^2\ \mathbf{U}^T$

B(2): $(\mathbf{A}^T)_{[m\ x\ n]}\ \mathbf{A}_{[n\ x\ m]} = \mathbf{V}\ \mathbf{\Lambda}^2\ \mathbf{V}^T$

B(3): $(\ (\mathbf{A}^T)_{[m\ x\ n]}\ \mathbf{A}_{[n\ x\ m]}\ )^k = \mathbf{V}\ \mathbf{\Lambda}^{2k}\ \mathbf{V}^T$

B(4): $(\mathbf{A}^T \mathbf{A})^k \sim v_1\ \lambda_1^{2k}\ v_1^T$

B(5): $(\mathbf{A}^T \mathbf{A})^k\ \mathbf{v'} \sim (\text{constant})\ \mathbf{v}_1$

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Least obvious properties

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \mathbf{\Lambda}_{[r \times r]} \mathbf{V}^{T}_{[r \times m]}$

C(1): $\mathbf{A}_{[n \times m]} \mathbf{x}_{[m \times 1]} = \mathbf{b}_{[n \times 1]}$

let $\mathbf{x}_0 = \mathbf{V} \mathbf{\Lambda}^{(-1)} \mathbf{U}^T \mathbf{b}$

- if under-specified, $\mathbf{x}_0$ gives 'shortest' solution
- if over-specified, it gives the 'solution' with the smallest least squares error

(see Num. Recipes, p. 62)

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Least obvious properties

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \mathbf{\Lambda}_{[r \times r]} \mathbf{V}^{T}_{[r \times m]}$

C(1): $\mathbf{A}_{[n \times m]} \mathbf{x}_{[m \times 1]} = \mathbf{b}_{[n \times 1]}$

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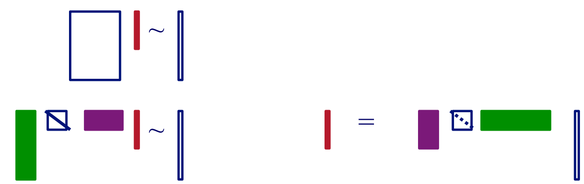

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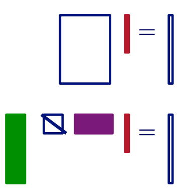
Slowly:



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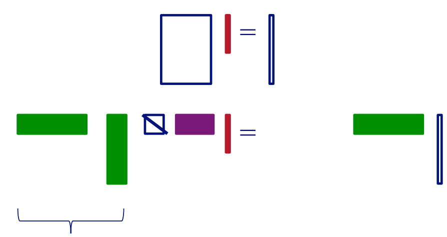
Slowly:



Identity
U: column-orthonormal

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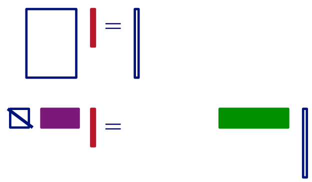
Slowly:



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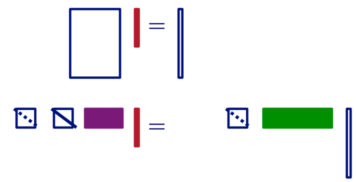
Slowly:



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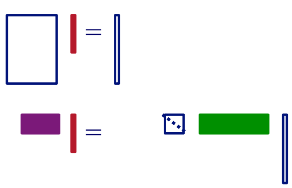
Slowly:



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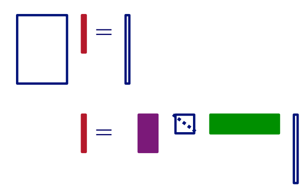
Slowly:



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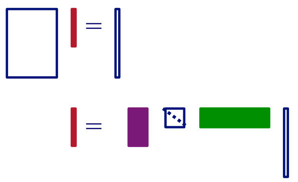
Slowly:



$\mathbf{x}$ $\mathbf{V}$ $\mathbf{\Lambda}^{-1}$ $\mathbf{U}^T$ $\mathbf{b}$

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Slowly:

Important: **DROP** small values of $\mathbf{\Lambda}$ (say, $< 10^{-6} * \lambda_1$)

$\mathbf{x}$ $\mathbf{V}$ $\mathbf{\Lambda}^{-1}$ $\mathbf{U}^T$ $\mathbf{b}$

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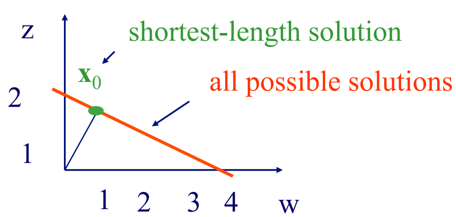
Least obvious properties

Illustration: under-specified, eg

[1 2] [w z] ^{T} = 4 (ie, 1 w + 2 z = 4)

A=??

b=??



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Exercise

Verify formula:

$\mathbf{A}$ = [1 2] $\mathbf{b}$ = [4]

$\mathbf{A} = \mathbf{U}\,\mathbf{\Lambda}\,\mathbf{V}^T$

$\mathbf{U}$ = ??

$\mathbf{\Lambda}$ = ??

$\mathbf{V}$= ??

$\mathbf{x_0} = \mathbf{V}\,\mathbf{\Lambda}^{(-1)}\,\mathbf{U}^T\,\mathbf{b}$

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Exercise

Verify formula:

$\mathbf{A}$ = [1 2] $\mathbf{b}$ = [4]

$\mathbf{A} = \mathbf{U}\ \mathbf{\Lambda}\ \mathbf{V}^T$

$\mathbf{U}$ = [1]

$\mathbf{\Lambda}$ = [sqrt(5)]

$\mathbf{V}$= [1/sqrt(5) 2/sqrt(5)] ^{T}

$\mathbf{x}_0 = \mathbf{V}\ \mathbf{\Lambda}^{(-1)}\ \mathbf{U}^T\ \mathbf{b}$

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Exercise

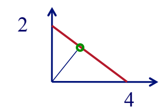
Verify formula:

$\mathbf{A}$ = [1 2] $\mathbf{b}$ = [4]

$\mathbf{A} = \mathbf{U}\ \mathbf{\Lambda}\ \mathbf{V}^T$

$\mathbf{U}$ = [1]

$\mathbf{\Lambda}$ = [sqrt(5)]



$\mathbf{V}$= [1/sqrt(5) 2/sqrt(5)] ^{T}

$\mathbf{x}_0 = \mathbf{V}\ \mathbf{\Lambda}^{(-1)}\ \mathbf{U}^T\ \mathbf{b}$ = [1/5 2/5] ^{T} [4]

= [4/5 8/5] ^{T} : w= 4/5, z = 8/5

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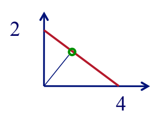
Exercise

Verify formula:



Show that w= 4/5, z = 8/5 is

(a) A solution to 1*w + 2*z = 4 and

(b) Minimal (wrt Euclidean norm)

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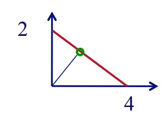
Exercise

Verify formula:



Show that w= 4/5, z = 8/5 is

(a) A solution to 1*w + 2*z = 4 and

A: easy

(b) Minimal (wrt Euclidean norm)

A: [4/5 8/5] is perpenticular to [2 -1]

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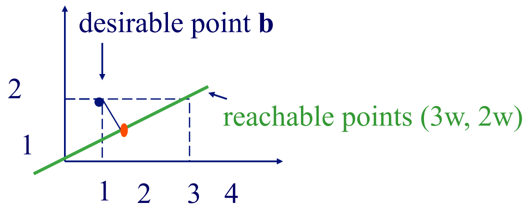
Least obvious properties – cont'd

Illustration: over-specified, eg

$[3\ 2]^T\ [w] = [1\ 2]^T$ (ie, 3 w = 1; 2 w = 2)

A=??
b=??



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CMU SCS

Exercise

Verify formula:

$\mathbf{A} = [3\ 2]^T$ $\mathbf{b} = [1\ 2]^T$

$\mathbf{A} = \mathbf{U}\ \mathbf{\Lambda}\ \mathbf{V}^T$

$\mathbf{U}$ = ??

$\mathbf{\Lambda}$ = ??

$\mathbf{V}$ = ??

$\mathbf{x_0} = \mathbf{V}\ \mathbf{\Lambda}^{(-1)}\ \mathbf{U}^T\ \mathbf{b}$

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CMU SCS

Exercise

Verify formula:

$\mathbf{A} = [3\ 2]^T$ $\mathbf{b} = [1\ 2]^T$

$\mathbf{A} = \mathbf{U}\ \mathbf{\Lambda}\ \mathbf{V}^T$

$\mathbf{U} = [\ 3/\mathrm{sqrt}(13)\quad 2/\mathrm{sqrt}(13)\ ]^T$

$\mathbf{\Lambda} = [\ \mathrm{sqrt}(13)\ ]$

$\mathbf{V} = [\ 1\ ]$

$\mathbf{x_0} = \mathbf{V}\ \mathbf{\Lambda}^{(-1)}\ \mathbf{U}^T\ \mathbf{b} = [\ 7/13\ ]$

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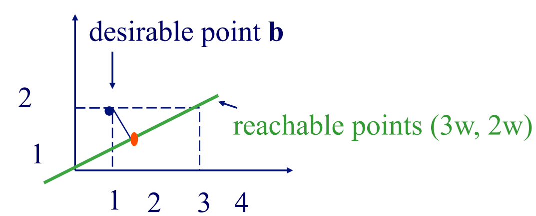
Exercise

Verify formula:

$[3\ 2]^T\ [7/13] = [1\ 2]^T$

$[21/13\ \ 14/13\ ]^T$ -> 'red point'



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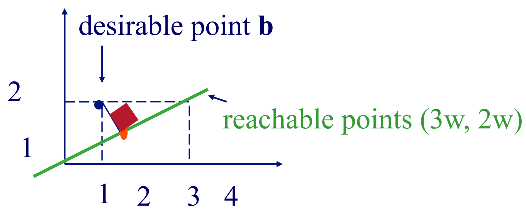
CMU SCS

Exercise

Verify formula:

$[3\ 2]^T\ [7/13] = [1\ 2]^T$

$[21/13\ \ 14/13\ ]^T$ -> 'red point' - perpenticular?



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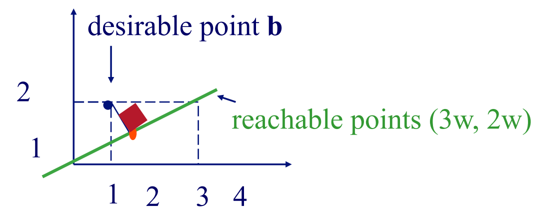
CMU SCS

Exercise

Verify formula:

A: [3 2] . ([1 2] – [21/13 14/13]) =

[3 2] . [-8/13 12/13] = [3 2] . [-2 3] = 0



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Least obvious properties - cont'd

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]}\ \mathbf{\Lambda}_{[r \times r]}\ \mathbf{V}^{\mathbf{T}}_{[r \times m]}$

C(2): $\mathbf{A}_{[n \times m]}\ \mathbf{v_1}_{[m \times 1]} = \lambda_1\ \mathbf{u_1}_{[n \times 1]}$

where $\mathbf{v_1}$, $\mathbf{u_1}$ the first (column) vectors of **V**, **U.** ($\mathbf{v_1}$ == right-singular-vector)

C(3): symmetrically: $\mathbf{u_1}^T \mathbf{A} = \lambda_1 \mathbf{v_1}^T$

$\mathbf{u_1}$ == left-singular-vector

Therefore:

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Least obvious properties - cont'd

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]}\ \mathbf{\Lambda}_{[r \times r]}\ \mathbf{V}^{\mathbf{T}}_{[r \times m]}$

C(4): $\mathbf{A}^T \mathbf{A}\ \mathbf{v_1} = \lambda_1^2\ \mathbf{v_1}$

(**fixed point** - the dfn of eigenvector for a symmetric matrix)

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Least obvious properties - altogether

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \mathbf{\Lambda}_{[r \times r]} \mathbf{V}^T_{[r \times m]}$

C(1): $\mathbf{A}_{[n \times m]} \mathbf{x}_{[m \times 1]} = \mathbf{b}_{[n \times 1]}$
then, $\mathbf{x}_0 = \mathbf{V} \mathbf{\Lambda}^{(-1)} \mathbf{U}^T \mathbf{b}$: shortest, actual or least-squares solution

C(2): $\mathbf{A}_{[n \times m]} \mathbf{v}_{1\,[m \times 1]} = \lambda_1 \mathbf{u}_{1\,[n \times 1]}$

C(3): $\mathbf{u}_1^T \mathbf{A} = \lambda_1 \mathbf{v}_1^T$

C(4): $\mathbf{A}^T \mathbf{A} \mathbf{v}_1 = \lambda_1^2 \mathbf{v}_1$

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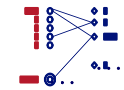
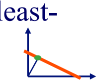
Properties - conclusions

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \mathbf{\Lambda}_{[r \times r]} \mathbf{V}^T_{[r \times m]}$



B(5): $(\mathbf{A}^T \mathbf{A})^k \mathbf{v}' \sim (\text{constant})\, \mathbf{v}_1$

C(1): $\mathbf{A}_{[n \times m]} \mathbf{x}_{[m \times 1]} = \mathbf{b}_{[n \times 1]}$
then, $\mathbf{x}_0 = \mathbf{V} \mathbf{\Lambda}^{(-1)} \mathbf{U}^T \mathbf{b}$: shortest, actual or least-squares solution



C(4): $\mathbf{A}^T \mathbf{A} \mathbf{v}_1 = \lambda_1^2 \mathbf{v}_1$

CMU SCS

SVD - detailed outline

- ...
- Case studies
- SVD properties
- more case studies
  - Kleinberg/google algorithms
  - query feedbacks
- Conclusions

CMU SCS

Kleinberg's algo (HITS)

Kleinberg, Jon (1998). *Authoritative sources in a hyperlinked environment.* Proc. 9th ACM-SIAM Symposium on Discrete Algorithms.

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Kleinberg's algorithm

- Problem dfn: given the web and a query
- find the most 'authoritative' web pages for this query

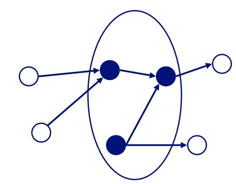
Step 0: find all pages containing the query terms

Step 1: expand by one move forward and backward

CMU SCS

Kleinberg's algorithm

- Step 1: expand by one move forward and backward



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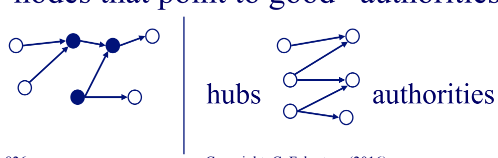
Kleinberg's algorithm

- on the resulting graph, give high score (= 'authorities') to nodes that many important nodes point to
- give high importance score ('hubs') to nodes that point to good 'authorities')



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Kleinberg's algorithm

- on the resulting graph, give high score (= 'authorities') to nodes that many important nodes point to
- give high importance score ('hubs') to nodes that point to good 'authorities')

Kleinberg's algorithm

observations

- recursive definition!
- each node (say, 'i'-th node) has both an authoritativeness score a_i and a hubness score h_i

Kleinberg's algorithm

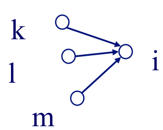
Let E be the set of edges and **A** be the adjacency matrix:

the (i,j) is 1 if the edge from i to j exists

Let h and a be [n x 1] vectors with the 'hubness' and 'authoritativiness' scores.

Then:

Kleinberg's algorithm



Then:

$$a_i = h_k + h_l + h_m$$

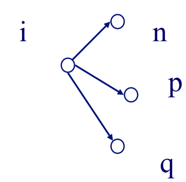
that is

a_i = Sum (h_j) over all j that (j,i) edge exists

or

$\mathbf{a} = \mathbf{A}^T \mathbf{h}$

Kleinberg's algorithm



symmetrically, for the 'hubness':

$$h_i = a_n + a_p + a_q$$

that is

h_i = Sum (q_j) over all j that (i,j) edge exists

or

$\mathbf{h} = \mathbf{A}\ \mathbf{a}$

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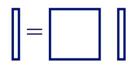
Kleinberg's algorithm

In conclusion, we want vectors **h** and **a** such that:

$$\mathbf{h} = \mathbf{A}\ \mathbf{a}$$

$$\mathbf{a} = \mathbf{A}^T\ \mathbf{h}$$



Recall properties:

C(2): $\mathbf{A}_{[n \times m]}\ \mathbf{v_1}_{[m \times 1]} = \lambda_1\ \mathbf{u_1}_{[n \times 1]}$

C(3): $\mathbf{u_1}^T \mathbf{A} = \lambda_1\ \mathbf{v_1}^T$

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Kleinberg's algorithm

In short, the solutions to

$$\mathbf{h} = \mathbf{A}\ \mathbf{a}$$

$$\mathbf{a} = \mathbf{A}^T\ \mathbf{h}$$

are the <u>left- and right- singular-vectors</u> of the adjacency matrix **A.**

Starting from random **a'** and iterating, we'll eventually converge

(Q: to which of all the singular-vectors? why?)

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Kleinberg's algorithm

(Q: to which of all the singular-vectors? why?)

A: to the ones of the strongest singular-value, because of property B(5):

B(5): $(\mathbf{A}^T \mathbf{A})^k\ \mathbf{v'} \sim \text{(constant)}\ \mathbf{v}_1$

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Kleinberg's algorithm - results

Eg., for the query 'java':

0.328 www.gamelan.com

0.251 java.sun.com

0.190 www.digitalfocus.com ("the java developer")

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Kleinberg's algorithm - discussion

- 'authority' score can be used to find 'similar pages' (how?)
- closely related to 'citation analysis', social networs / 'small world' phenomena

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SVD - detailed outline

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- Conclusions

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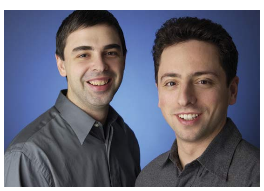
PageRank (google)



Larry Page   Sergey Brin

•Brin, Sergey and Lawrence Page (1998). *Anatomy of a Large-Scale Hypertextual Web Search Engine*. 7th Intl World Wide Web Conf.

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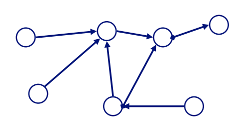
Problem: PageRank

Given a directed graph, find its most interesting/central node



A node is important, if it is connected with important nodes (recursive, but OK!)

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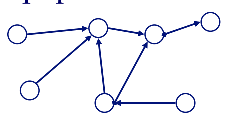
Problem: PageRank - solution

Given a directed graph, find its most interesting/central node

Proposed solution: Random walk; spot most 'popular' node (-> steady state prob. (ssp))



A node has high ssp, if it is connected with high ssp nodes (recursive, but OK!)

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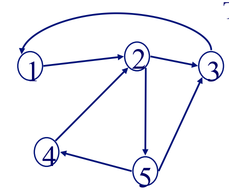
(Simplified) PageRank algorithm

- Let **A** be the adjacency matrix;
- let **B** be the transition matrix: transpose, column-normalized - then



From / To

B

		1		
1			1	
	1/2			1/2
				1/2
	1/2			

p
p1
p2
p3
p4
p5

=

p
p1
p2
p3
p4
p5

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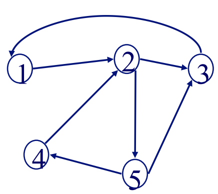
(Simplified) PageRank algorithm

- **B p = p**



B **p** = **p**

		1		
1			1	
	1/2			1/2
				1/2
	1/2			

p
p1
p2
p3
p4
p5

=

p
p1
p2
p3
p4
p5

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(Simplified) PageRank algorithm

- **B p** = 1 * **p**
- thus, **p** is the **eigenvector** that corresponds to the highest eigenvalue (=1, since the matrix is column-normalized)
- Why does such a **p** exist?
  - **p** exists if **B** is nxn, nonnegative, irreducible [Perron–Frobenius theorem]

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(Simplified) PageRank algorithm

- **B p** = 1 * **p**
- thus, **p** is the **eigenvector(*)** that corresponds to the highest eigenvalue (=1, since the matrix is column-normalized)
- Why does such a **p** exist?
  - **p** exists if **B** is nxn, nonnegative, irreducible [Perron–Frobenius theorem]

All<-> all

(*) dfn: a few foils later

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(Simplified) PageRank algorithm

- In short: imagine a particle randomly moving along the edges
- compute its steady-state probabilities (ssp)

Full version of algo: with occasional random jumps

Why? To make the matrix irreducible

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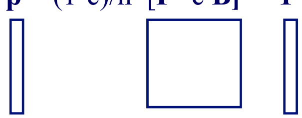
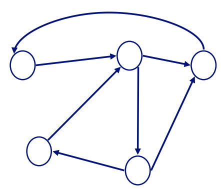
Full Algorithm

- With probability *1-c*, fly-out to a random node
- Then, we have

p = c **B p** + (1-c)/n **1** =>

p = (1-c)/n [**I** - c **B**] $^{-1}$ **1**





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Full Algorithm

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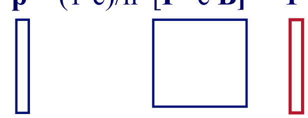
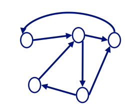


$$\begin{bmatrix} 1 \\ 1 \\ \cdots \\ 1 \end{bmatrix}$$



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Alternative notation – eigenvector viewpoint

M Modified transition matrix

$\mathbf{M} = c\,\mathbf{B} + (1-c)/n\ \mathbf{1}\ \mathbf{1}^T$

CLIQUE

Then

$\mathbf{p} = \mathbf{M}\,\mathbf{p}$

That is: the steady state probabilities =

PageRank scores form the *first eigenvector* of the 'modified transition matrix'

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Parenthesis: intuition behind eigenvectors

- Definition
- 2 properties
- intuition

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Formal definition

If **A** is a (n x n) square matrix

(λ , **x**) is an **eigenvalue/eigenvector** pair of **A** if

$\mathbf{A}\,\mathbf{x} = \lambda\,\mathbf{x}$

CLOSELY related to singular values:

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Property #1: Eigen- vs singular-values

if

$\mathbf{B}_{[n\ x\ m]} = \mathbf{U}_{[n\ x\ r]}\ \mathbf{\Lambda}_{[\ r\ x\ r]}\ (\mathbf{V}_{[m\ x\ r]})^T$

then $\mathbf{A} = (\mathbf{B}^T\mathbf{B})$ is symmetric and

C(4): $\mathbf{B}^T\,\mathbf{B}\ \mathbf{v}_i = \lambda_i^2\ \mathbf{v}_i$

ie, $\mathbf{v}_1$, $\mathbf{v}_2$, ...: eigenvectors of $\mathbf{A} = (\mathbf{B}^T\mathbf{B})$

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Property #2

- If $\mathbf{A}_{[nxn]}$ is a real, symmetric matrix
- Then it has *n* real eigenvalues

(if **A** is not symmetric, some eigenvalues may be complex)

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Property #3

- If $\mathbf{A}_{[nxn]}$ is a real, symmetric matrix
- Then it has *n* real eigenvalues
- And they agree with its *n* singular values, except possibly for the sign

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Parenthesis: intuition behind eigenvectors

- Definition
- 2 properties
- **intuition**

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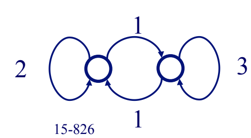
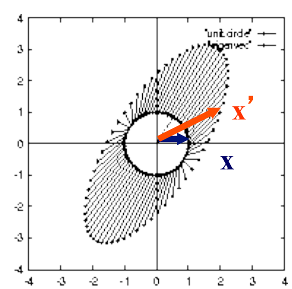
Intuition

- **A** as vector transformation

$$\underbrace{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}_{\mathbf{x}'} = \underbrace{\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\mathbf{x}}$$





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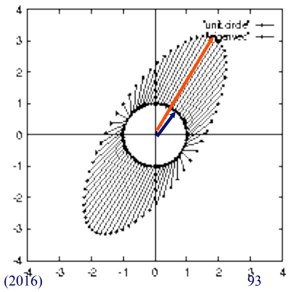
Intuition

- By defn., eigenvectors remain parallel to themselves ('**fixed points**')

$$\underset{\lambda_1}{3.62} * \underset{\mathbf{v_1}}{\begin{bmatrix} 0.52 \\ 0.85 \end{bmatrix}} = \underset{\mathbf{A}}{\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}} \underset{\mathbf{v_1}}{\begin{bmatrix} 0.52 \\ 0.85 \end{bmatrix}}$$



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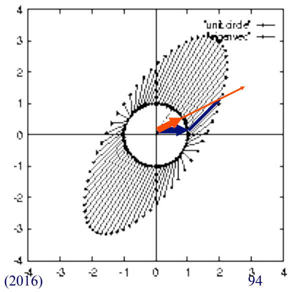
Convergence

- Usually, fast:



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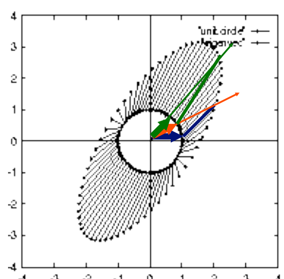
Convergence

- Usually, fast:



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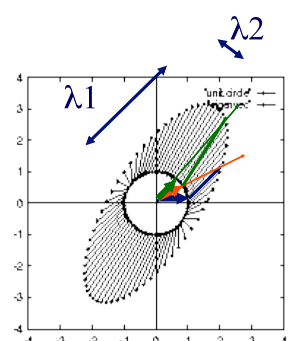
Convergence

- Usually, fast:
- depends on ratio
  λ1 : λ2



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Closing the parenthesis wrt intuition behind eigenvectors

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Kleinberg/PageRank - conclusions

SVD helps in graph analysis:

hub/authority scores: strongest left- and right-singular-vectors of the adjacency matrix

random walk on a graph: steady state probabilities are given by the strongest eigenvector of the transition matrix

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SVD - detailed outline

- ...
- Case studies
- SVD properties
- more case studies
  - google/Kleinberg algorithms
  - query feedbacks
- Conclusions

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Query feedbacks

[Chen & Roussopoulos, sigmod 94]

Sample problem:

estimate selectivities (e.g., '*how many movies were made between 1940 and 1945*?'

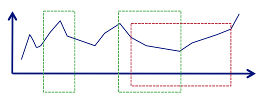
for query optimization,

LEARNING from the query results so far!!

Query feedbacks

- Given: past queries and their results
  - #movies(1925,1935) = 52
  - #movies(1948, 1990) = 123
  - …
  - And a new query, say #movies(1979,1980)?
- Give your best estimate

#movies

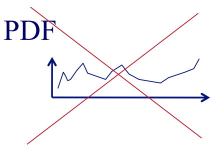


year

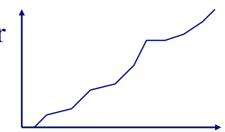
Query feedbacks

Idea #1: consider a function for the CDF (cummulative distr. function), eg., 6-th degree polynomial (or splines, or anything else)

PDF



count, so far



year

Query feedbacks

For example

F(x) = # movies made until year 'x'

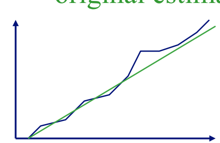
$= a_1 + a_2 * x + a_3 * x^2 + \ldots a_7 * x^6$

Query feedbacks

GREAT idea #2: adapt your model, as you see the actual counts of the actual queries

original estimate

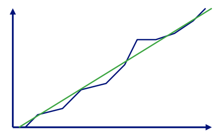
count, so far



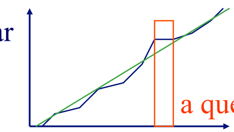
actual

year

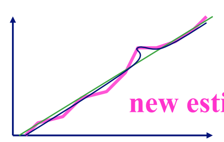
Query feedbacks



Query feedbacks



Query feedbacks



Query feedbacks

Eventually, the problem becomes:

- estimate the parameters a_1, ... a_7 of the model
- to minimize the least squares errors from the real answers so far.

Formally:

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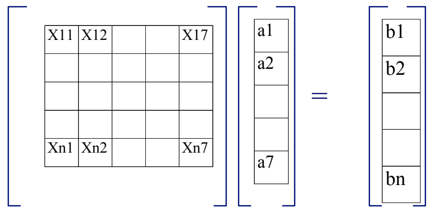
Query feedbacks

Formally, with *n* queries and 6-th degree polynomials:



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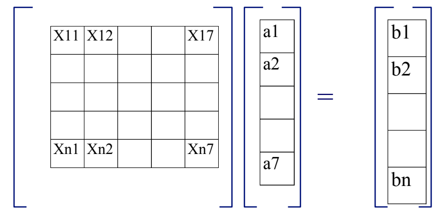
where $x_{i,j}$ such that Sum ($x_{i,j}$ * a_i) = our estimate for the # of movies and b_j: the actual



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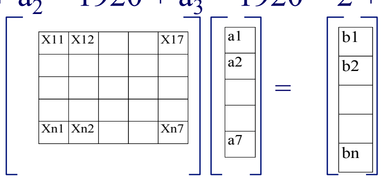
For example, for query '*find the count of movies during (1920-1932)*' :

$a_1 + a_2 * 1932 + a_3 * 1932^{**}2 + \ldots$

-

$(a_1 + a_2 * 1920 + a_3 * 1920^{**}2 + \ldots)$



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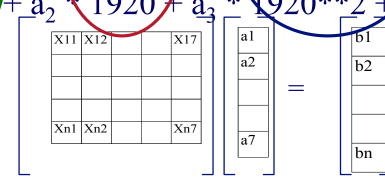
Query feedbacks

And thus X11 = 0; X12 = 1932-1920, etc

$a_1 + a_2 * 1932 + a_3 * 1932^{**}2 + \ldots$

-

$(a_1 + a_2 * 1920 + a_3 * 1920^{**}2 + \ldots)$



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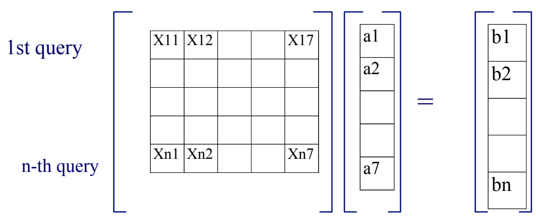
Query feedbacks

In matrix form:

X **a** = **b**



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Query feedbacks

In matrix form:

$$\mathbf{X}\,\mathbf{a} = \mathbf{b}$$

and the least-squares estimate for **a** is

$$\mathbf{a} = \mathbf{V}\,\mathbf{\Lambda}^{(-1)}\,\mathbf{U}^T\,\mathbf{b}$$

according to property C(1)

(let $\mathbf{X} = \mathbf{U}\,\mathbf{\Lambda}\,\mathbf{V}^T$)

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Query feedbacks - enhancements

The solution

$$\mathbf{a} = \mathbf{V}\,\mathbf{\Lambda}^{(-1)}\,\mathbf{U}^T\,\mathbf{b}$$

works, but needs expensive SVD each time a new query arrives

GREAT Idea #3: Use 'Recursive Least Squares', to adapt **a** incrementally.

Details: in paper - intuition:

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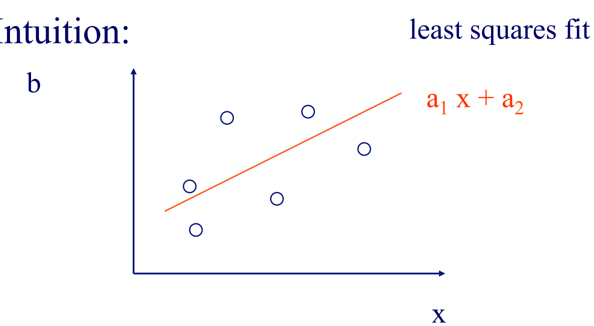
Query feedbacks - enhancements

Intuition:



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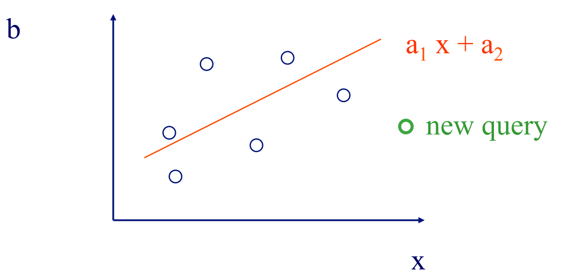
Query feedbacks - enhancements

Intuition:



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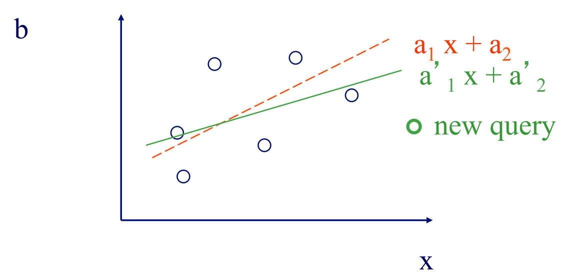
Query feedbacks - enhancements

Intuition:



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Query feedbacks - enhancements

the new coefficients can be quickly computed from the old ones, plus statistics in a (7x7) matrix

(no need to know the details, although the RLS is a brilliant method)

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Query feedbacks - enhancements

GREAT idea #4: 'forgetting' factor - we can even down-play the weight of older queries, since the data distribution might have changed.

(comes for 'free' with RLS...)

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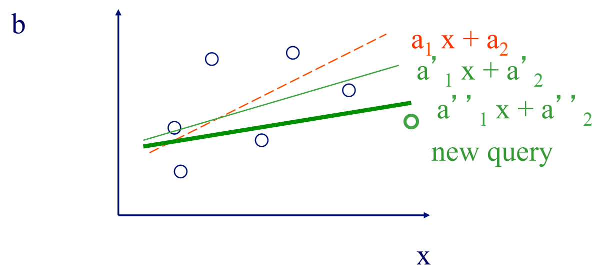
Query feedbacks - enhancements

Intuition: least squares fit



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Query feedbacks - conclusions

SVD helps find the Least Squares solution, to adapt to query feedbacks

(RLS = Recursive Least Squares is a great method to incrementally update least-squares fits)

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SVD - detailed outline

- ...
- Case studies
- SVD properties
- more case studies
  - google/Kleinberg algorithms
  - query feedbacks
- Conclusions

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Conclusions

- SVD: a **valuable** tool
- given a document-term matrix, it finds 'concepts' (LSI)
- ... and can reduce dimensionality (KL)
- ... and can find rules (PCA; RatioRules)

Conclusions cont'd

- ... and can find fixed-points or steady-state probabilities (google/ Kleinberg/ Markov Chains)
- ... and can solve optimally over- and under-constraint linear systems (least squares / query feedbacks)

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