


15-826: Multimedia Databases and Data Mining


Lecture #12: Fractals - case studies Part III
(quadrees, knn queries)
C. Faloutsos



Must-read Material

- Alberto Belussi and Christos Faloutsos, [Estimating the Selectivity of Spatial Queries Using the 'Correlation' Fractal Dimension](#) Proc. of VLDB, p. 299-310, 1995


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Optional Material

Optional, but very useful: Manfred Schroeder
*Fractals, Chaos, Power Laws: Minutes
from an Infinite Paradise* W.H. Freeman
and Company, 1991

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Outline



Goal: 'Find **similar / interesting** things'

- Intro to DB
- ➔ • Indexing - similarity search
- Data Mining

Optional = NOT in exam
(but useful as mental drill!)

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Optional

Indexing - Detailed outline

- primary key indexing
- secondary key / multi-key indexing
- spatial access methods
 - z-ordering
 - R-trees
 - misc
- ➔ fractals
 - intro
 - applications
- text

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Optional

Indexing - Detailed outline

- fractals
 - intro
 - applications
 - ✓ disk accesses for R-trees (range queries)
 - ✓ dimensionality reduction
 - ✓ dim. curse revisited
- ➔
 - quad-tree analysis [Gaede+]
 - nn queries [Belussi+]

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Optional

Fractals and Quadrees

- Problem: how many quadtree nodes will we need, to store a region in some level of approximation? [Gaede+96]

Faloutsos, C. and V. Gaede (Sept. 1996). Analysis of the z-ordering Method Using the Hausdorff Fractal Dimension. VLDB, Bombay, India.

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Optional

Fractals and Quadrees

- I.e.:

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Fractals and Quadrees

- I.e.:

of quadtree 'blocks' (= # gray nodes)

level of quadtree

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Fractals and Quadrees

- Datasets:

Franconia

Brain Atlas

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Fractals and Quadrees

- Hint:
 - assume that the boundary is self-similar, with a given fd
 - how will the quad-tree (oct-tree) look like?

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Fractals and Quadrees

○ white

● blue

● black

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Fractals and Quadrees

Let $p_g(i)$ the prob. to find a gray node at level i .
 If self-similar, what can we say for $p_g(i)$?

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Fractals and Quadrees

Let $p_g(i)$ the prob. to find a gray node at level i .
 If self-similar, what can we say for $p_g(i)$?

A: $p_g(i) = p_g = \text{constant}$

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Fractals and Quadrees

Assume only 'gray' and 'white' nodes (ie., no volume')
 Assume that p_g is given - how many gray nodes at level i ?

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Fractals and Quadrees

Assume only 'gray' and 'white' nodes (ie., no volume')
 Assume that p_g is given - how many gray nodes at level i ?

A: 1 at level 0;

$4 * p_g$

$(4 * p_g) * (4 * p_g)$

...

$(4 * p_g)^i$

...

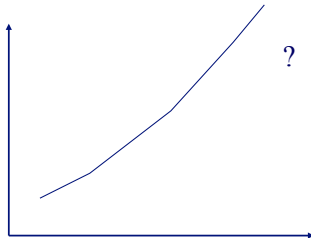
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Fractals and Quadrees

- I.e.:

of quadtree 'blocks'



? $(4 * p_g)^i$

level of quadtree ('i')

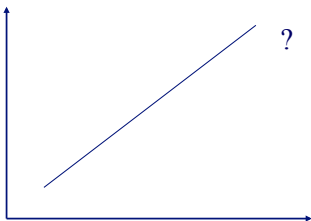
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Fractals and Quadrees

- I.e.:

$\log(\# \text{ of quadtree 'blocks'})$



? $\log[(4 * p_g)^i]$

level of quadtree

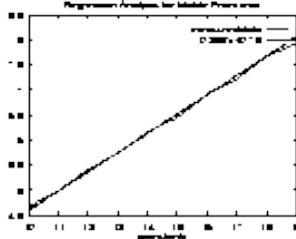
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Fractals and Quadrees

- Conclusion: Self-similarity leads to easy and accurate estimation

$\log_2(\# \text{ blocks})$





level

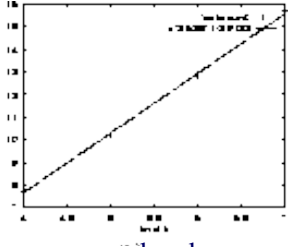
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Fractals and Quadrees

- Conclusion: Self-similarity leads to easy and accurate estimation

$\log_2(\# \text{ blocks})$





level

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Optional

Fractals and Quadrees

(a) 'I.I.E.' (b) 'MGcounty'

(c) 'LEconomy'

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Optional

Fractals and Quadrees

(a) 'I.I.E.' (b) 'MGcounty'

(c) 'LEconomy' (d) 'Pancreas'

(e) 'Sierpinski'

$\log(\#blocks)$ level

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Optional

Fractals and Quadrees

- Final observation: relationship between p_g and fractal dimension?

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Optional

Fractals and Quadrees

- Final observation: relationship between p_g and fractal dimension?
- A: very close:
 - $(4 * p_g)^i = \#$ of gray nodes at level $i =$
 - $\#$ of Hausdorff grid-cells of side $(1/2)^i = r$
 - Eventually: $D_H = 2 + \log_2(p_g)$
 - and, for E-d spaces: $D_H = E + \log_2(p_g)$

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Fractals and Quadrees

for E-d spaces: $D_H = E + \log_2(p_g)$

Sanity check:

- point in 2-d: $D_H = 0$ $p_g = ??$
- line in 2-d: $D_H = 1$ $p_g = ??$
- plane in 2-d: $D_H = 2$ $p_g = ??$
- point in 3-d: $D_H = 0$ $p_g = ??$

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Fractals and Quadrees

for E-d spaces: $D_H = E + \log_2(p_g)$

Sanity check:

- point in 2-d: $D_H = 0$ $p_g = 1/4$
- line in 2-d: $D_H = 1$ $p_g = 1/2$
- plane in 2-d: $D_H = 2$ $p_g = 1$
- point in 3-d: $D_H = 0$ $p_g = 1/8$

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Fractals and Quadrees

Final conclusions:

- self-similarity leads to estimates for # of z-values = # of quadtree/oct-tree blocks
- close dependence on the Hausdorff fractal dimension of the boundary

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- ➔ • nn queries [Belussi+]

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NN queries

- Q: in NN queries, what is the effect of the shape of the query region? [Belussi+95]

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NN queries

- Q: in NN queries, what is the effect of the shape of the query region?
- that is, for L2, and self-similar data:

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NN queries

- Q: What about L_1 , L_{inf} ?

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NN queries

- Q: What about L_1 , L_{inf} ?
- A: **Same slope**, different intercept

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NN queries

- Q: What about L_1 , L_{inf} ?
- A: **Same slope**, different intercept

SLOPE(square) = 1.598 SLOPE(circle) = 1.595 SLOPE(diamond) = 1.594

log(#neighbors) vs log(d)

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NN queries

- Q: what about the intercept? Ie., what can we say about N_2 and N_{inf}

N_2 neighbors

volume: V_2

N_{inf} neighbors

volume: V_{inf}

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NN queries

- Consider sphere with volume V_{inf} and r' radius

N_2 neighbors

volume: V_2

N_{inf} neighbors

volume: V_{inf}

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NN queries

- Consider sphere with volume V_{inf} and r' radius
- $(r/r')^E = V_2 / V_{inf}$
- $(r/r')^{D_2} = N_2 / N_2'$
- $N_2' = N_{inf}$ (since shape does not matter)
- and finally:

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NN queries

$$(N_2 / N_{\text{inf}})^{1/D_2} = (V_2 / V_{\text{inf}})^{1/E}$$

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NN queries

Conclusions: for self-similar datasets

- Avg # neighbors: grows like $(\text{distance})^{D_2}$, regardless of query shape (circle, diamond, square, e.t.c.)

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 - Conclusions


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Fractals - overall conclusions

- self-similar datasets: appear often
- powerful tools: correlation integral, NCDF, rank-frequency plot
- intrinsic/fractal dimension helps in
 - estimations (selectivities, quadtrees, etc)
 - dim. reduction / dim. curse
- (later: can help in image compression...)

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References

- ➔ 1. Belussi, A. and C. Faloutsos (Sept. 1995). Estimating the Selectivity of Spatial Queries Using the 'Correlation' Fractal Dimension. Proc. of VLDB, Zurich, Switzerland.
- ➔ 2. Faloutsos, C. and V. Gaede (Sept. 1996). Analysis of the z-ordering Method Using the Hausdorff Fractal Dimension. VLDB, Bombay, India.
3. Proietti, G. and C. Faloutsos (March 23-26, 1999). I/O complexity for range queries on region data stored using an R-tree. International Conference on Data Engineering (ICDE), Sydney, Australia.

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