CMU SCS

15-826: Multimedia Databases and Data Mining

Lecture#2: Primary key indexing – B-trees

Christos Faloutsos - CMU
www.cs.cmu.edu/~christos

CMU SCS

Reading Material

[Ramakrishnan & Gehrke, 3rd ed, ch. 10]

CMU SCS

Problem

Given a large collection of (multimedia) records, find similar/interesting things, ie:

- Allow fast, approximate queries, and
- Find rules/patterns

CMU SCS

Outline

Goal: 'Find similar / interesting things'

- Intro to DB
- Indexing - similarity search
- Data Mining

CMU SCS

Indexing - Detailed outline

- primary key indexing
  - B-trees and variants
  - (static) hashing
  - extendible hashing
- secondary key indexing
- spatial access methods
- text
- ...

CMU SCS

In even more detail:

- B – trees
- B+ - trees, B*-trees
- hashing

CMU SCS

Primary key indexing

- find employee with ssn=123

CMU SCS

B-trees

- the **most successful** family of index schemes (B-trees, B^{+}-trees, B^{*}-trees)
- Can be used for primary/secondary, clustering/non-clustering index.
- balanced "n-way" search trees

CMU SCS

Citation

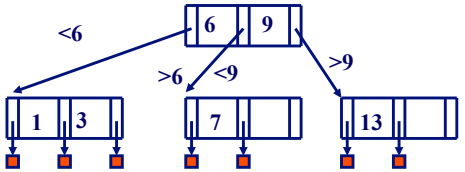
- Rudolf Bayer and Edward M. McCreight, *Organization and Maintenance of Large Ordered Indices*, Acta Informatica, 1:173-189, 1972.

- Received the *2001 SIGMOD innovations* award
- among the most cited db publications
  - www.informatik.uni-trier.de/~ley/db/about/top.html

CMU SCS

B-trees

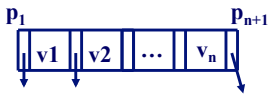
Eg., B-tree of order *d=1*:



CMU SCS

B - tree properties:

- each node, in a B-tree of order *d*:
  - Key order
  - at most *n=2d* keys
  - at least *d* keys (except root – it may have just 1 key)
  - all leaves at the same level
  - if number of pointers is *k*, then node has exactly *k-1* keys
  - (leaves are empty)



CMU SCS

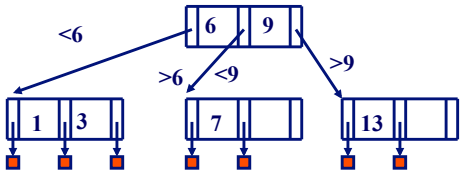
Properties

- "block aware" nodes: each node -> disk page
- O(log (N)) for everything! (ins/del/search)
- typically, if *n* = 50 - 100, then 2 - 3 levels
- utilization >= 50%, guaranteed; on average 69%

CMU SCS

Queries

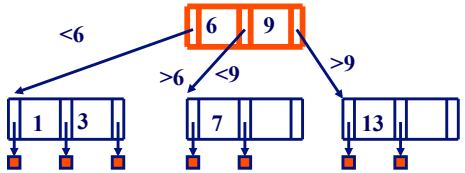
- Algo for exact match query? (eg., ssn=8?)



CMU SCS

Queries

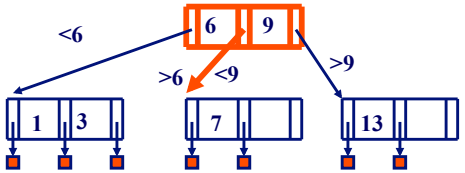
- Algo for exact match query? (eg., ssn=8?)



CMU SCS

Queries

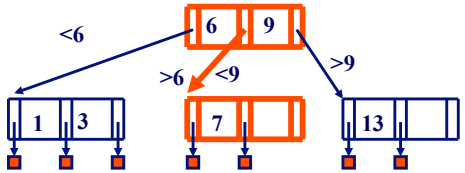
- Algo for exact match query? (eg., ssn=8?)



CMU SCS

Queries

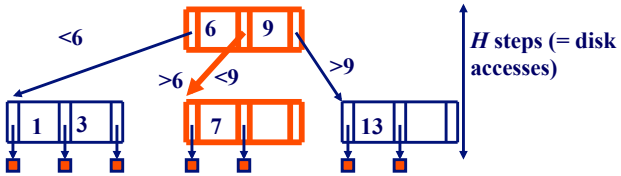
- Algo for exact match query? (eg., ssn=8?)



CMU SCS

Queries

- Algo for exact match query? (eg., ssn=8?)



CMU SCS

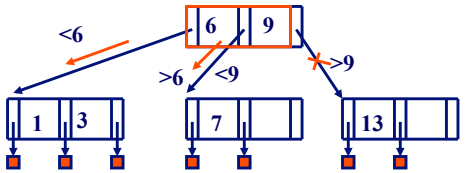
Queries

- what about range queries? (eg., *5<salary<8*)
- Proximity/ nearest neighbor searches? (eg., *salary ~ 8*)

CMU SCS

Queries

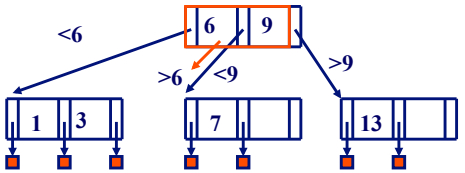
- what about range queries? (eg., ***5<salary<8***)
- Proximity/ nearest neighbor searches? (eg., *salary ~ 8*)



CMU SCS

Queries

- what about range queries? (eg., *5<salary<8*)
- Proximity/ nearest neighbor searches? (eg., ***salary ~ 8***)



CMU SCS

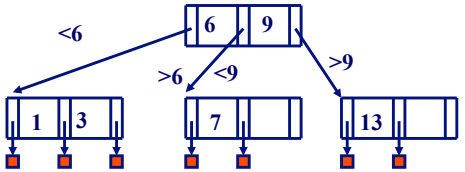
B-trees: Insertion

- Insert in leaf; on overflow, push middle up (recursively)
- split: preserves B - tree properties

CMU SCS

B-trees

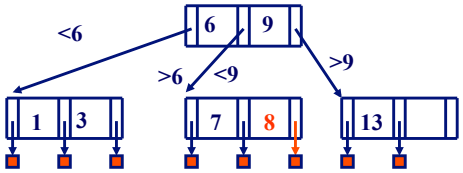
Easy case: Tree T0; insert '8'



CMU SCS

B-trees

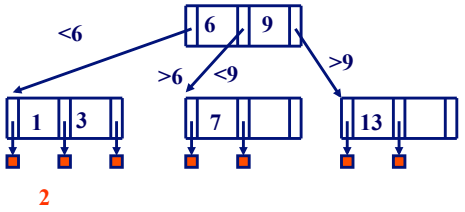
Tree T0; insert '8'



CMU SCS

B-trees

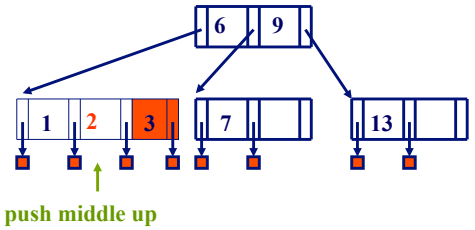
Hardest case: Tree T0; insert '2'



CMU SCS

B-trees

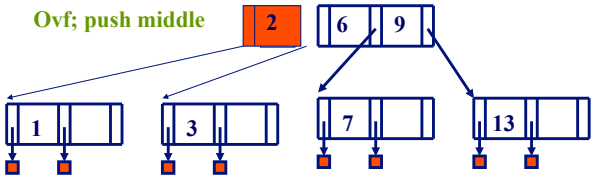
Hardest case: Tree T0; insert '2'



CMU SCS

B-trees

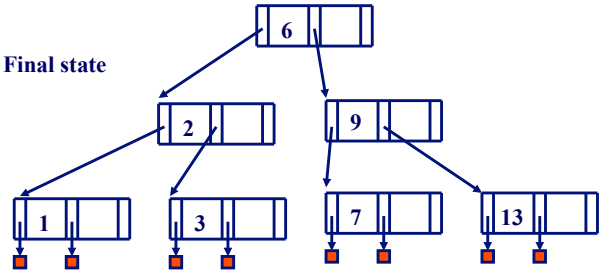
Hardest case: Tree T0; insert '2'



CMU SCS

B-trees

Hardest case: Tree T0; insert '2'



CMU SCS

B-trees: Insertion

- Insert in leaf; on overflow, push middle up (recursively – 'propagate split')
- split: preserves all B - tree properties (!!)
- notice how it grows: height increases when root overflows & splits
- Automatic, incremental re-organization

CMU SCS

Overview

- B – trees
  - Dfn, Search, insertion, **deletion**
- B+ - trees
- hashing

CMU SCS

Deletion

Rough outline of algo:

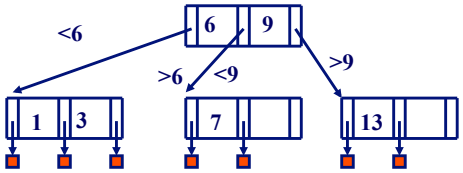
- Delete key;
- on underflow, may need to merge

In practice, some implementors just allow underflows to happen…

CMU SCS

B-trees – Deletion

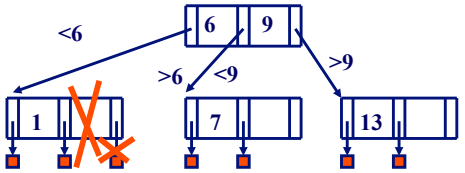
Easiest case: Tree T0; delete '3'



CMU SCS

B-trees – Deletion

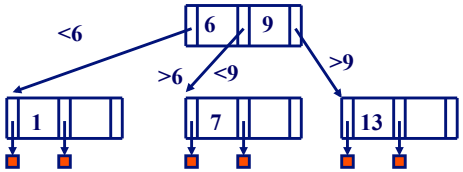
Easiest case: Tree T0; delete '3'



CMU SCS

B-trees – Deletion

Easiest case: Tree T0; delete '3'



CMU SCS

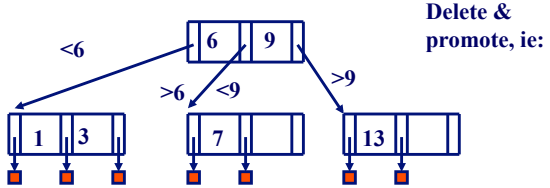
B-trees – Deletion

- Case1: delete a key at a leaf – no underflow
- Case2: delete non-leaf key – no underflow
- Case3: delete leaf-key; underflow, and 'rich sibling'
- Case4: delete leaf-key; underflow, and 'poor sibling'

CMU SCS

B-trees – Deletion

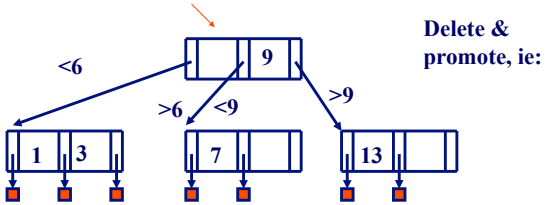
- Case2: delete a key at a non-leaf – no underflow (eg., delete 6 from T0)



CMU SCS

B-trees – Deletion

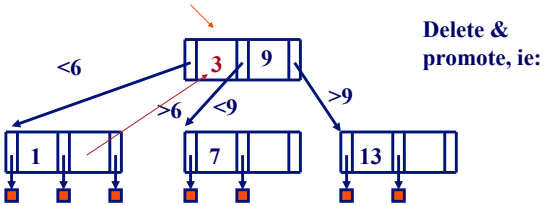
- Case2: delete a key at a non-leaf – no underflow (eg., delete 6 from T0)



CMU SCS

B-trees – Deletion

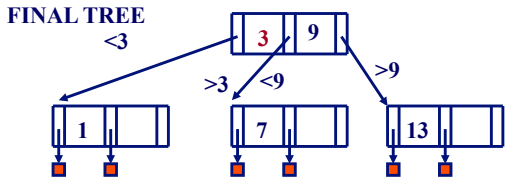
- Case2: delete a key at a non-leaf – no underflow (eg., delete 6 from T0)



CMU SCS

B-trees – Deletion

- Case2: delete a key at a non-leaf – no underflow (eg., delete 6 from T0)



CMU SCS

B-trees – Deletion

- Case2: delete a key at a non-leaf – no underflow (eg., delete 6 from T0)
- Q: How to promote?
- A: pick the largest key from the left sub-tree (or the smallest from the right sub-tree)
- Observation: every deletion eventually becomes a deletion of a leaf key

CMU SCS

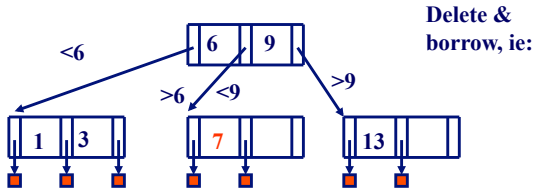
B-trees – Deletion

- Case1: delete a key at a leaf – no underflow
- Case2: delete non-leaf key – no underflow
- Case3: delete leaf-key; underflow, and 'rich sibling'
- Case4: delete leaf-key; underflow, and 'poor sibling'

CMU SCS

B-trees – Deletion

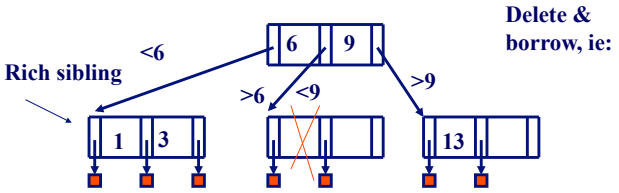
- Case3: underflow & 'rich sibling' (eg., delete 7 from T0)



CMU SCS

B-trees – Deletion

- Case3: underflow & 'rich sibling' (eg., delete 7 from T0)



CMU SCS

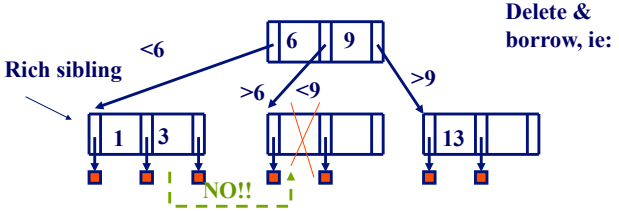
B-trees – Deletion

- Case3: underflow & 'rich sibling'
- 'rich' = can give a key, without underflowing
- 'borrowing' a key: THROUGH the PARENT!

CMU SCS

B-trees – Deletion

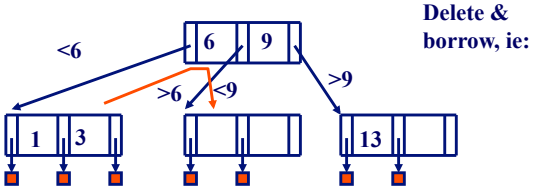
- Case3: underflow & 'rich sibling' (eg., delete 7 from T0)



CMU SCS

B-trees – Deletion

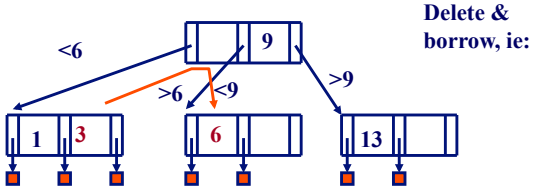
- Case3: underflow & 'rich sibling' (eg., delete 7 from T0)



CMU SCS

B-trees – Deletion

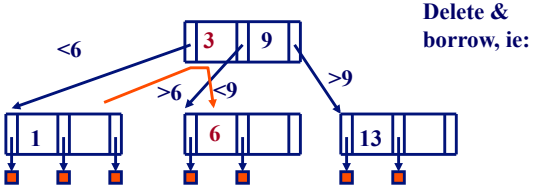
- Case3: underflow & 'rich sibling' (eg., delete 7 from T0)



CMU SCS

B-trees – Deletion

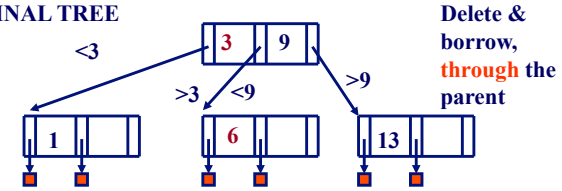
- Case3: underflow & 'rich sibling' (eg., delete 7 from T0)



CMU SCS

B-trees – Deletion

- Case3: underflow & 'rich sibling' (eg., delete 7 from T0)



CMU SCS

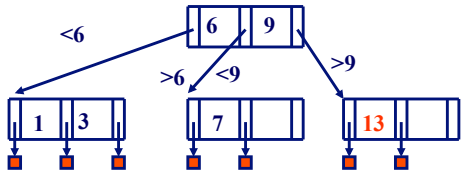
B-trees – Deletion

- Case1: delete a key at a leaf – no underflow
- Case2: delete non-leaf key – no underflow
- Case3: delete leaf-key; underflow, and 'rich sibling'
- Case4: delete leaf-key; underflow, and 'poor sibling'

CMU SCS

B-trees – Deletion

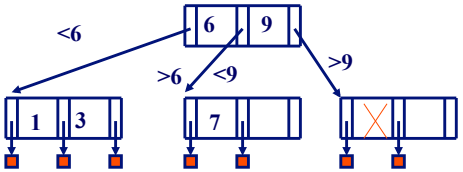
- Case4: underflow & 'poor sibling' (eg., delete 13 from T0)



CMU SCS

B-trees – Deletion

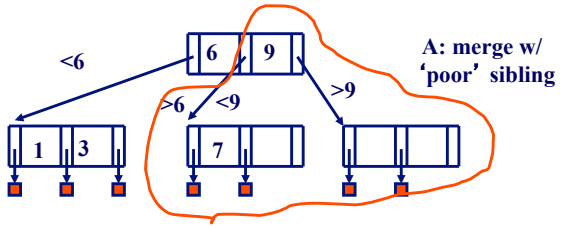
- Case4: underflow & 'poor sibling' (eg., delete 13 from T0)



CMU SCS

B-trees – Deletion

- Case4: underflow & 'poor sibling' (eg., delete 13 from T0)



CMU SCS

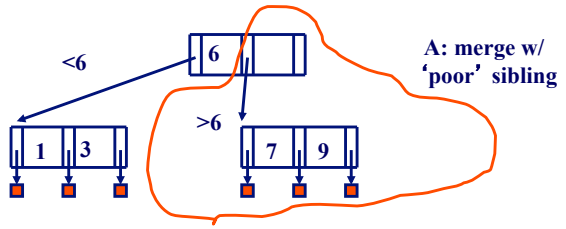
B-trees – Deletion

- Case4: underflow & 'poor sibling' (eg., delete 13 from T0)
- Merge, by pulling a key from the **parent**
- exact reversal from insertion: 'split and push up', vs. 'merge and pull down'
- Ie.:

CMU SCS

B-trees – Deletion

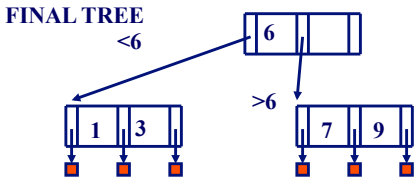
- Case4: underflow & 'poor sibling' (eg., delete 13 from T0)



CMU SCS

B-trees – Deletion

- Case4: underflow & 'poor sibling' (eg., delete 13 from T0)



CMU SCS

B-trees – Deletion

- Case4: underflow & 'poor sibling'
- -> 'pull key from parent, and merge'
- Q: What if the parent underflows?

CMU SCS

B-trees – Deletion

- Case4: underflow & 'poor sibling'
- -> 'pull key from parent, and merge'
- Q: What if the parent underflows?
- A: repeat recursively

CMU SCS

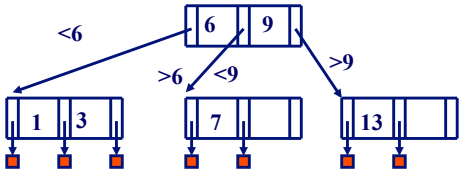
Overview

- B – trees
- **B+ - trees, B*-trees**
- hashing

CMU SCS

B+ trees - Motivation

if we want to store the whole record with the key –> problems (what?)



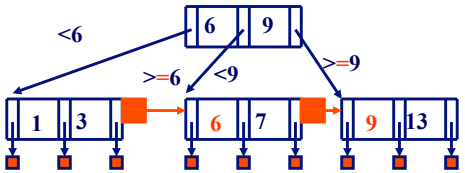
CMU SCS

Solution: B^+ - trees

- They string all leaf nodes together
- AND
- replicate keys from non-leaf nodes, to make sure every key appears at the leaf level

CMU SCS

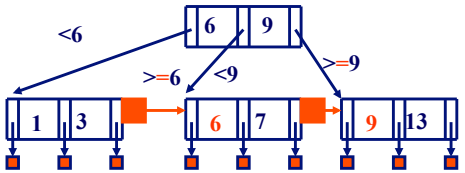
B+ trees



CMU SCS

B+ trees - insertion

Eg., insert '8'



CMU SCS

Overview

- B – trees
- B+ - trees, **B*-trees**
- hashing

CMU SCS

B*-trees

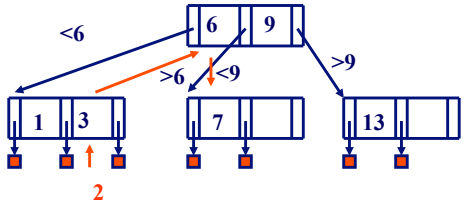
- splits drop util. to 50%, and maybe increase height
- How to avoid them?

CMU SCS

B*-trees: deferred split!

- Instead of splitting, LEND keys to sibling!

(through PARENT, of course!)



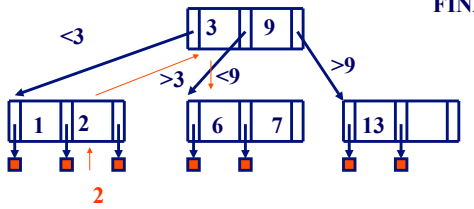
CMU SCS

B*-trees: deferred split!

- Instead of splitting, LEND keys to sibling!

(through PARENT, of course!)

FINAL TREE



CMU SCS

B*-trees: deferred split!

- Notice: shorter, more packed, faster tree
- It's a rare case, where space utilization and speed improve together
- BUT: What if the sibling has no room for our 'lending'?

CMU SCS

B*-trees: deferred split!

- BUT: What if the sibling has no room for our 'lending'?
- A: 2-to-3 split: get the keys from the sibling, pool them with ours (and a key from the parent), and split in 3.
- Details: too messy (and even worse for deletion)

CMU SCS

Conclusions

- Main ideas: recursive; block-aware; on overflow -> split; **defer** splits
- All B-tree variants have excellent, **O(logN) worst-case performance for ins/del/search**
- B+ tree is the prevailing indexing method
- More details: [Knuth vol 3.] or [Ramakrishnan & Gehrke, 3rd ed, ch. 10]

