

SAMPLE FINAL EXAMINATION

- **3 hours** duration
- **No aids allowed** - only 2 pages of your notes, and calculator.
- You should have **4 non-empty pages, including this cover**
- Graded out of **100** points (*actually, this sample has less than 100 points*)
- Numbers in square brackets indicate points.

LAST NAME	
First Name	
andrew login	

## Contents

Q1	DFT [10 pts]	2
Q2	DWT [10 pts]	2
Q3	IFS [15 pts]	2
Q4	Graphs - Kronecker [10 pts]	3
Q5	Text [20 pts]	3
Q6	Other questions [35 pts]	4

## Q1 DFT [10 pts]

Recall that the  $n$ -point DFT of a sequence  $\vec{x} = [x_i]$  ( $i = 1, \dots, n$ ) is defined as the sequence  $[X_f]$ , with

$$X_f = 1/\sqrt{n} \sum_{i=0}^{n-1} x_i \exp(-j2\pi fi/n) \quad f = 0, 1, \dots, n-1 \quad (1)$$

where  $j = \sqrt{-1}$  is the imaginary unit.

Consider a real-valued time sequence of  $n=32$  time ticks ( $t = 0, 1, \dots, 31$ ). It has only one non-zero value in its amplitude spectrum (or two, if we count its mirror). Specifically, we have  $|X_3|=2$ .

1. ([1 pts] ) How many sequences  $x_t$  ( $t = 0, \dots, 31$ ) can lead to such a spectrum?
2. ([3 pts] ) Could the sequence  $x_t = 2 \cos(3\pi t/32)$  give that spectrum? Justify briefly.
3. ([6 pts] ) Give the formula  $x_t$  that generates a sequence with the above spectrum

## Q2 DWT [10 pts]

Consider time sequences like  $\vec{x} = [x_i]$  ( $i = 1, \dots, n$ ) with  $n=32$  time-ticks. Let  $n_{nz}$  be the number of non-zero values it has.

1. ([4 pts] ) Give such a sequence, such that
  - it has as many zeros as possible (i.e., minimum  $n_{nz}$ )
  - it has only 2 non-zero Haar wavelet coefficients
2. ([6 pts] ) Is it possible to find a time sequence with only 1 non-zero value in the time domain, and 2 non-zero values in the (Haar) DWT domain? Explain briefly.

## Q3 IFS [15 pts]

Consider Barnsley's method of Iterated Function Systems (IFS). The transformations of Figure 1 lead to the Cantor dust on the diagonal of the unit square, where all its points are disconnected.

1. ([4 pts] ) Is it possible to have a fractal set, with fractal dimension  $D > 1$ , and with all its points disconnected?
2. ([5 pts] ) If yes, describe or draw the corresponding transformations, and give the resulting fractal dimension, up to 2 decimal points. If not, explain briefly.

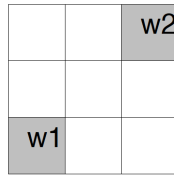


Figure 1: Barnsley’s method: IFS transformations for the Cantor dust.

## Q4 Graphs - Kronecker [10 pts]

Consider the two matrices  $\mathbf{A}$  and  $\mathbf{B}$ :

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 10 & 20 & 30 \end{bmatrix}$$

- [3 pts] What are the dimensions of the Kronecker product matrix  $\mathbf{C} = \mathbf{A} \otimes \mathbf{B}$ ? A, possibly correct answer, could be: 3 rows, 4 columns.
- [3 pts] What is the value of the top left entry  $c_{1,1}$  of  $\mathbf{C}$ ?
- [4 pts] What is the value of the bottom right entry  $c_{\maxRow, \maxCol}$ ?

## Q5 Text [20 pts]

Consider the string editing distance and let the cost for insertion and deletion be  $C_i = C_d = 1$ ; let the cost for substitution of any pair of differing letters be  $C_s = 0.5$ . Consider the strings ‘‘TOM’’ and ‘‘STUMP’’.

	-	T	O	M
-	0	1	2	3
S	1			
T	2			
U	3			
M	4			
P	5			

Table 1: Cost matrix for string editing distance

- ([15 pts] ) Give the  $cost(i, j)$  matrix as in the course foils. Also see Table 1, which is partially filled for your convenience.
- ([3 pts] ) Show the lowest-cost path.
- ([2 pts] ) What is the string editing cost for the two input strings?

## **Q6      Other questions [35 pts]**

And 4-5 more questions, on SVD, pageRank, power laws, forecasting (and, even 1 or 2, on z-ordering, signature files, etc), adding up to 100 points.

The exam questions will be similar to the quiz questions on the web:

[www.cs.cmu.edu/~christos/courses/826-resources/REVIEW-QUESTIONS](http://www.cs.cmu.edu/~christos/courses/826-resources/REVIEW-QUESTIONS)

END OF EXAM QUESTIONS \_\_\_\_\_GOOD LUCK