


**15-826: Multimedia Databases
and Data Mining**


Lecture #11: Fractals: M-trees and dim.
course (case studies – Part II)
C. Faloutsos



Reading Material

- Alberto Belussi and Christos Faloutsos, [Estimating the Selectivity of Spatial Queries Using the 'Correlation' Fractal Dimension](#) Proc. of VLDB, p. 299-310, 1995
- (optional, but very useful: Manfred Schroeder *Fractals, Chaos, Power Laws: Minutes from an Infinite Paradise* W.H. Freeman and Company, 1991)

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Outline

Goal: 'Find similar / interesting things'

- Intro to DB
- ➔ • Indexing - similarity search
- Data Mining

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Indexing - Detailed outline

- primary key indexing
- secondary key / multi-key indexing
- spatial access methods
 - z-ordering
 - R-trees
 - misc
 - ➔ fractals
 - intro
 - applications
- text

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Indexing - Detailed outline

- fractals
 - intro
 - applications
 - disk accesses for R-trees (range queries)
 - dimensionality reduction
 - ➔ • selectivity in M-trees
 - dim. curse revisited
 - "fat fractals"
 - quad-tree analysis [Gaede+]


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What else can they solve?

- ✓ separability [KDD'02]
 - forecasting [CIKM'02]
- ✓ dimensionality reduction [SBBD'00]
 - non-linear axis scaling [KDD'02]
- ✓ disk trace modeling [Wang+'02]
 - ➔ • selectivity of spatial/multimedia queries [PODS'94, VLDB'95, ICDE'00]
 - ...


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Metric trees - analysis

- Problem: How many disk accesses, for an M-tree?
- Given:
 - N (# of objects)
 - C (fanout of disk pages)
 - r (radius of range query - BIASED model)


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Metric trees - analysis

- Problem: How many disk accesses, for an M-tree?
- Given:
 - N (# of objects)
 - C (fanout of disk pages)
 - r (radius of range query - BIASED model)
- NOT ENOUGH - what else do we need?

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Metric trees - analysis

- A: something about the distribution



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Metric trees - analysis

- A: something about the distribution

[Ciaccia, Patella, Zezula, PODS98]: assumed that the distance distribution is the same, for every object:

Paolo Ciaccia Marco Patella

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Metric trees - analysis

- A: something about the distribution

[Ciaccia+, PODS98]: assumed that the distance distribution is the same, for every object:

$F1(d) = \text{Prob}(\text{an object is within } d \text{ from object \#1})$
 $= F2(d) = \dots = F(d)$

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Metric trees - analysis

- A: something about the distribution
- Given our 'fractal' tools, we could try them - which one?

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Metric trees - analysis

- A: something about the distribution
- Given our 'fractal' tools, we could try them - which one?
- A: Correlation integral [Traina+, ICDE2000]

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Metric trees - analysis

English dictionary Portuguese dictionary

$\log(\#\text{pairs})$ $\log(\#\text{pairs})$

Slope = 4.754 Slope = 6.68664

EnglishWords dataset Portuguese dataset

$\log(d)$ $\log(d)$

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Metric trees - analysis

Divina Comedia Eigenfaces

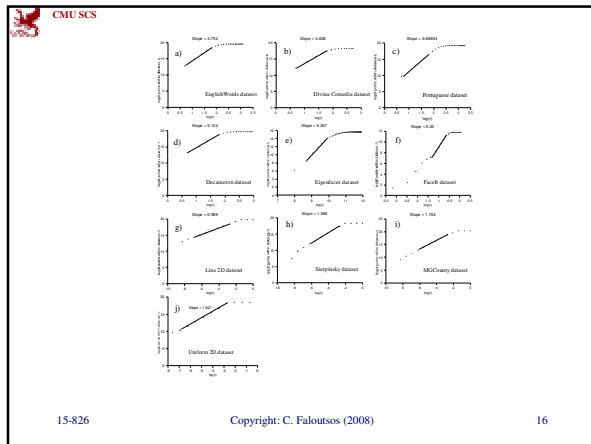
$\log(\#\text{pairs})$ $\log(\#\text{pairs})$

Slope = 4.828 Slope = 5.267

Divina Comedia dataset Eigenfaces dataset

$\log(d)$ $\log(d)$

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Metric trees - analysis

	Data Set	N (# Objects)	Dimension	Distance Function	Distance Exponent D
Real Metric datasets	English	25,143	NA	L_{edit}	4.753
	Divina Commedia	12,701	NA	L_{edit}	4.827
	Decamerone	18,719	NA	L_{edit}	5.124
	Portuguese	21,473	NA	L_{edit}	6.686
	Facell	1,056	NA	Not divulged	6.821
	MGCCounty	15,559	2	L_2	1.752
Real vector datasets	Eigenfaces	11,900	16	L_2	5.267
	Serpinsky	9,841	2	L_2	1.584
Synthetic datasets	2D Line	20,000	2	L_2	0.989
	Uniform 2D	10,000	2	L_2	1.947

Table 2 - Datasets used in the experiments.

Metric trees - analysis

- So, what is the # of disk accesses, for a node of radius r_d , on a query of radius r_q ?

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Metric trees - analysis

- So, what is the # of disk accesses, for a node of radius r_d , on a query of radius r_q ?
- A: $\sim (r_d+r_q)$

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Metric trees - analysis

- So, what is the # of disk accesses, for a node of radius r_d , on a query of radius r_q ?
- A: $\sim (r_d+r_q)^D$

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Accuracy of selectivity formulas

$\log(\#d.a.)$

Legend: $D=4$ Minimum $\Delta=10^3$ $D=16$ Max $\Delta=10^3$ $D=4$ Max $\Delta=10^3$ $D=16$ Min $\Delta=10^3$ $D=4$ Min $\Delta=10^3$ $D=16$ Max $\Delta=10^3$

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Fast estimation of D

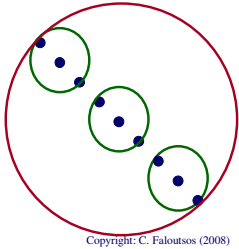
- Normally, D takes $O(N^2)$ time
- Anything faster? suppose we have already built an M-tree

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Fast estimation of D

- Hint:



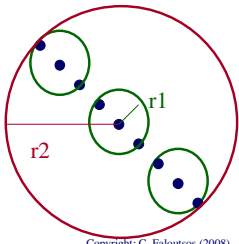
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Fast estimation of D

- Hint:

ratio of radii:
 $r1^D * C = r2^D$



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Dim. curse revisited

- (Q: how serious is the dim. curse, e.g.):
- Q: what is the search effort for k-nn?
 - given N points, in E dimensions, in an R-tree, with k-nn queries (‘biased’ model)

[Pagel, Korn + ICDE 2000]

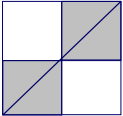


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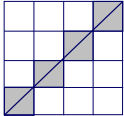
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Reminder: Hausdorff Dimension (D_0)

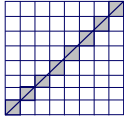
- r = side length (each dimension)
- $B(r)$ = # boxes containing points $\propto r^{D_0}$



$r = 1/2 \quad B = 2$
 $\log r = -1$
 $\log B = 1$



$r = 1/4 \quad B = 4$
 $\log r = -2$
 $\log B = 2$



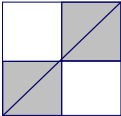
$r = 1/8 \quad B = 8$
 $\log r = -3$
 $\log B = 3$

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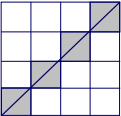
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Reminder: Correlation Dimension (D_2)

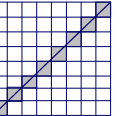
- $S(r) = \sum p_i^2$ (squared % pts in box) $\propto r^{-D_2}$
 $\propto \text{\#pairs(within } \leq r)$



$r = 1/2 \quad S = 1/2$
 $\log r = -1$
 $\log S = -1$



$r = 1/4 \quad S = 1/4$
 $\log r = -2$
 $\log S = -2$



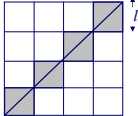
$r = 1/8 \quad S = 1/8$
 $\log r = -3$
 $\log S = -3$

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Observation #1

- How to determine avg MBR side l ?
 - $N = \text{\#pts}$, $C = \text{MBR capacity}$



Hausdorff dimension: $B(r) \propto r^{-D_0}$

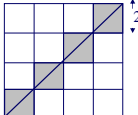
$B(l) = N/C = l^{-D_0} \Rightarrow l = (N/C)^{-1/D_0}$

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Observation #2

- k -NN query $\rightarrow \epsilon$ -range query
 - For k pts, what radius ϵ do we expect?



Correlation dimension: $S(r) \propto r^{-D_2}$

$S(\epsilon) = \frac{k}{N-1} = (2\epsilon)^{-D_2}$

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Observation #3

- Estimate avg # query-sensitive anchors:
 - How many **expected** q will touch **avg** page?
 - Page touch: q stabs ϵ -dilated MBR(p)

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Asymptotic Formula

- k -NN page accesses as $N \rightarrow \infty$
 - C = capacity
 - D = fractal dimension ($=D0 \sim D2$)

$$P_{all}^{L\infty}(k) \approx \sum_{j=0}^h \left\{ \frac{1}{C^{h-j}} + \left[1 + \left(\frac{k}{C^{h-j}} \right)^{1/D} \right]^D \right\}$$

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Asymptotic Formula

$$P_{all}^{L\infty}(k) \approx \sum_{j=0}^h \left\{ \frac{1}{C^{h-j}} + \left[1 + \left(\frac{k}{C^{h-j}} \right)^{1/D} \right]^D \right\}$$

- NO mention of the embedding dimensionality!!
- Still have dim. curse, but on f.d. D

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Synthetic Data

- plane
 - $D_0 = D_2 = 2$
 - embedded in E -space
 - $N = 100K$
- manifold
 - $E = 8$
 - $D_0 = D_2$ varies from 1-6
 - line, plane, etc. (in 8-d)

plane in 3-space ($E=3, D_0=D_2=2$)

line in 3-space ($E=3, D_0=D_2=1$)

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Accuracy of L_∞ Formula

MGcounty: Leaf Accesses

leaf accesses

nearest neighbors (k)

predicted observed

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Embedding Dimension

plane: Leaf Accesses

E	uni/ind	fractal	leaf
2	3.49	3.49	4.75
5	28.26	3.45	6.40
10	847.26	3.34	6.42
20	All	3.36	6.9
50	All	3.32	6.37
100	All	3.32	5.43

leaf accesses

embedding dimension (E)

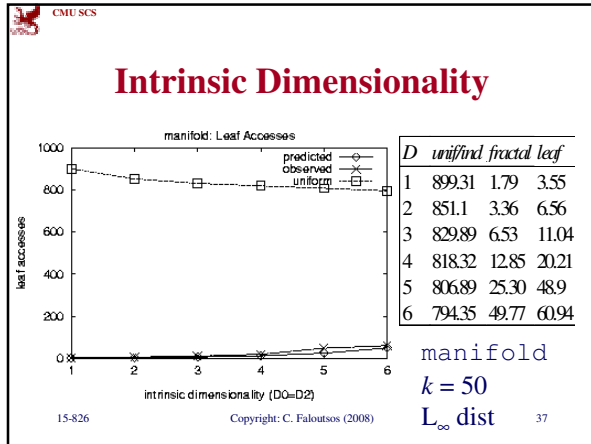
predicted observed uniform

plane
 $k = 50$
 L_∞ dist

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Non-Euclidean Data Set

E	unif/ind	fractal	leaf
2	3.49	2.53	4.72±1.81
10	847.26	2.53	6.42±2.11
20	all	2.53	7.76±4.12
50	all	2.53	6.15±2.82
100	all	2.53	5.64±2.32

sierpinski, $k = 50$, L_∞ dist

-
- Conclusions**
- Worst-case theory is **over-pessimistic**
 - High dimensional data can exhibit good performance if **correlated, non-uniform**
 - Many real data sets are **self-similar**
 - Determinant is **intrinsic** dimensionality
 - multiple fractal dimensions (D_0 and D_2)
 - indication of how far one can go

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References

- Ciaccia, P., M. Patella, et al. (1998). *A Cost Model for Similarity Queries in Metric Spaces*. PODS.
- Pagel, B.-U., F. Korn, et al. (2000). *Deflating the Dimensionality Curse Using Multiple Fractal Dimensions*. ICDE, San Diego, CA.
- Traina, C., A. J. M. Traina, et al. (2000). *Distance Exponent: A New Concept for Selectivity Estimation in Metric Trees*. ICDE, San Diego, CA.

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