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15-826: Multimedia Databases and Data Mining

Approximate Counting

C. Faloutsos

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Outline

Goal: 'Find similar / interesting things'

- Intro to DB
- Indexing - similarity search
- Data Mining

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Data Mining - Detailed outline

- Statistics
- AI - decision trees
- DB
  - data warehouses; data cubes; OLAP
  - classifiers
  - association rules
  - misc. topics:
    - ...
    - approximate counting

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Outline

- **Flajolet-Martin (and Cohen) – vocabulary size (Problem #1)**
- Application: Approximate Neighborhood function (ANF)
- other, powerful approximate counting tools (Problem #2, #3)

Problem #1

- Given a multiset (eg., words in a document)
- find the vocabulary size (#, after dup. elimination)

Thanks to

- Chris Palmer (Vivisimo)

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Problem #2

- Given a multiset
- compute approximate high-end histogram = hot-list query = (*k* most common words, and their counts)

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Problem #3

- Given two documents
- compute quickly their similarity (#common words/ #total-words) == Jaccard coefficient

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Problem #1

- Given a multiset (eg., words in a document)
- find the vocabulary size V (#, after dup. elimination)
- using space O(V), or O(log(V))

(Q1: Applications?)

(Q2: How would you solve it?)

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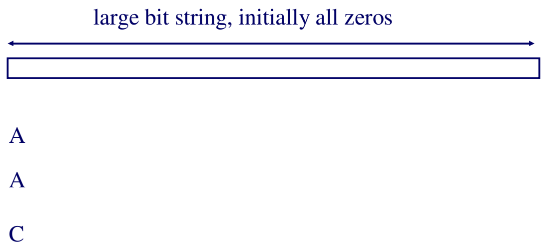
Basic idea (Cohen)



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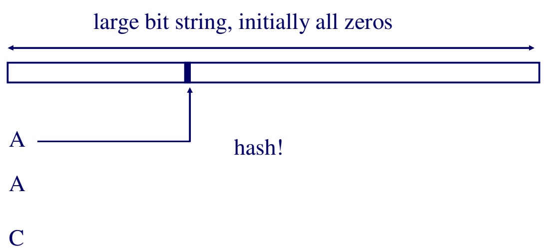

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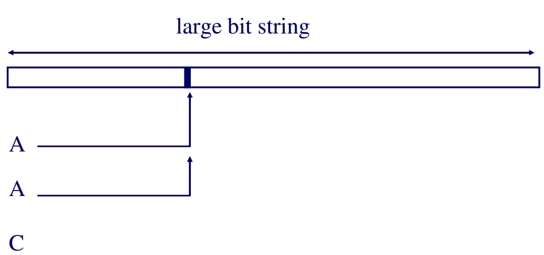

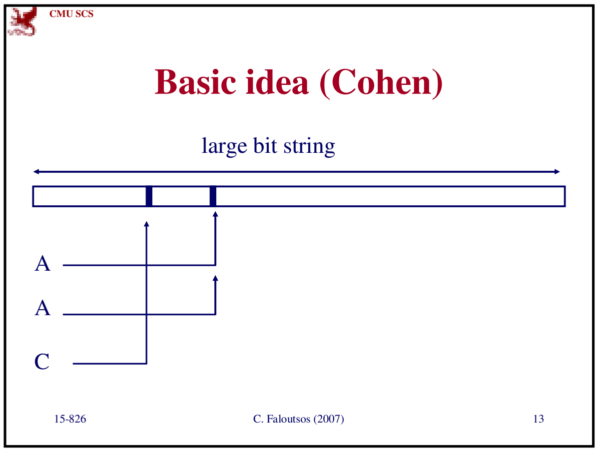
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Basic idea (Cohen)
large bit string
A
A
C
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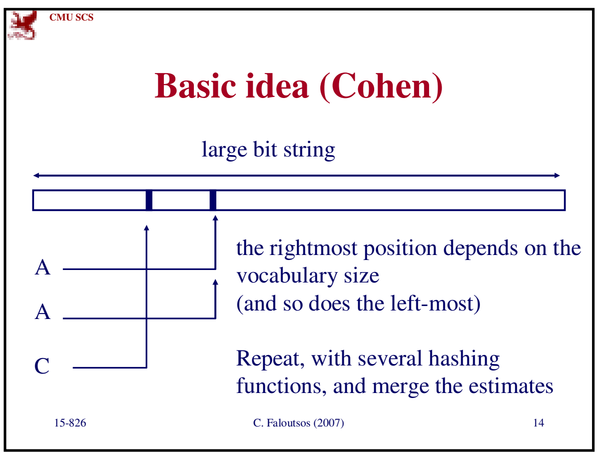
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Basic idea (Cohen)
large bit string
A
A
C
the rightmost position depends on the vocabulary size
(and so does the left-most)
Repeat, with several hashing functions, and merge the estimates
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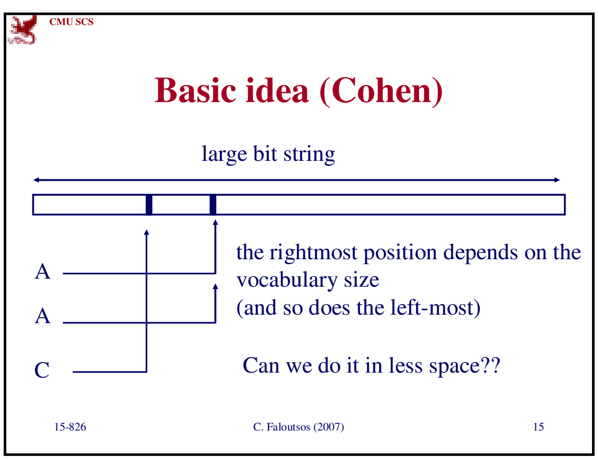
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Basic idea (Cohen)
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(and so does the left-most)
Can we do it in less space??
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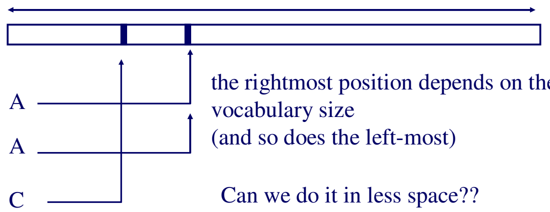
Basic idea (Cohen)

large bit string



the rightmost position depends on the vocabulary size
(and so does the left-most)

Can we do it in less space??
YES

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How?

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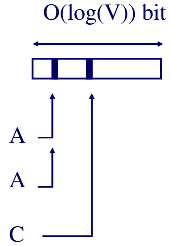
Basic idea (Flajolet-Martin)

O(log(V)) bit string (V: voc. size)



first bit: with prob. ½
second: with prob. ¼
...
i-th: with prob. ½**i

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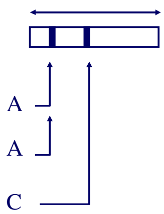
Basic idea (Flajolet-Martin)

O(log(V)) bit string (V: voc. size)



again, the rightmost bit 'reveals' the vocabulary size

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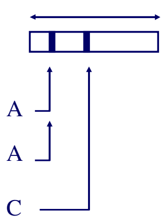
Basic idea (Flajolet-Martin)

O(log(V)) bit string (V: voc. size)



again, the rightmost bit 'reveals' the vocabulary size

Eg.: V=4, will probably set the 2nd bit, etc

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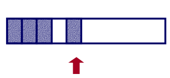
Flajolet-Martin

- Hash multiple values of *X* to same signature
  - Hash each *x* to a bit, using exponential distr.
  - ½ map to bit 0, ¼ map to bit 1, …
- Do several different mappings and average
  - Gives better accuracy
  - Estimate is: *2^b / .77351 / BIAS*
    - *b ~ rightmost '1', and actually:*



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Flajolet-Martin

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- Do several different mappings and average
  - Gives better accuracy
  - Estimate is: 2^b / .77351 / *BIAS*
    - *b : average least zero bit in the bitmask*
    - *bias : 1+.31/k for k different mappings*

- Flajolet & Martin prove this works

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FM Approx. Counting Alg.

Assume $X = \{ 0, 1, \ldots, V\text{-}1 \}$
FOR *i = 1* to *k* DO *bitmask[i] = 0000…00*
Create *k* random hash functions, $hash_i$
FOR each element *x* of *M* DO
  FOR *i = 1* to *k* DO
    $h = hash_i(x)$
    bitmask[i] = bitmask[i] LOR *h*
Estimate: b = average least zero bit in *bitmask[i]*
  $2^b/.77351/(1+.31/k)$

- How many bits? *log V* + small constant
- What hash functions?

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Random Hash Functions

- Can use linear hash functions. Pick random (a_i, b_i) and then the hash function is:
  - $lhash_i(x) = a_i * x + b_i$
- Gives uniform distribution over the bits
- To make this exponential, define
  - $hash_i(x)$ = least zero bit in $lhash_i(x)$
- Hash functions easy to create and fast to use

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Conclusions

- Want to measure # of distinct elements
- Approach #1: (Flajolet-Martin)
  - Map elements to random bits
  - Keep bitmask of bits
  - Estimate is $O(2^b)$ for least zero-bit *b*
- Approach #2: (Cohen)
  - Create random permutation of elements
  - Keep least element seen
  - Estimate is: $O(1/le)$ for least rank *le*

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Approximate counting

- Flajolet-Martin (and Cohen) – vocabulary size
- **Application: Approximate Neighborhood function (ANF)**
- other, powerful approximate counting tools

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Fast Approximation of the "neighborhood" Function for Massive Graphs

Christopher R. Palmer
Phillip B. Gibbons
Christos Faloutsos

KDD 2001

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Motivation

- What is the diameter of the Web?
- What is the effective diameter of the Web?
- Are the telephone caller-callee graphs for the U.S. similar to the ones in Europe?
- Is the citation graph for physics different from the one for computer science?
- Are users in India further away from the core of the Internet than those in the U.S.?

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Proposed Tool: neighborhood

Given graph $G=(V,E)$

$N(h)$ = # pairs within h hops or less

= **neighborhood function**

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Proposed Tool: neighborhood

Given graph $G=(V,E)$

$N(h)$ = # pairs within h hops or less

= **neighborhood function**

$N(u,h)$ = # neighbors of node u, within h hops or less

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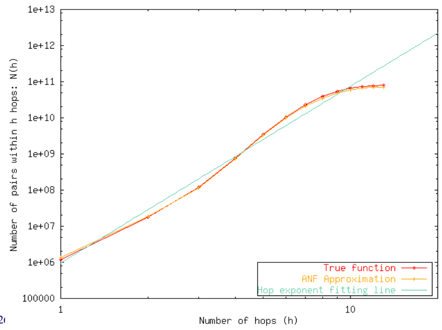
Example of neighborhood



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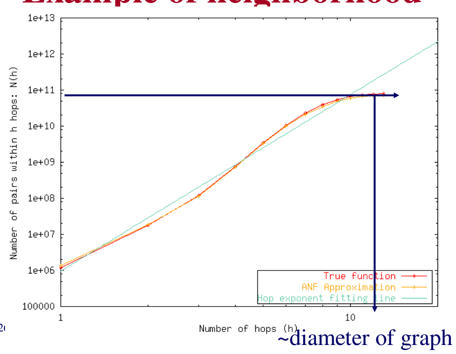
Example of neighborhood



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Requirements (for massive graphs)

- *Error guarantees*
- *Fast*: (and must scale linearly with graph)
- *Low storage requirements*: massive graphs!
- *Adapts to available memory*
- *Sequential scans of the edges*
- Also estimates *individual neighborhood functions* |S(u,h)|
  - These are actually quite useful for mining

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How would you compute it?

- Repeated matrix multiply
  - Too slow $O(n^{2.38})$ at the very least
  - Too much memory $O(n^2)$
- Breadth-first search

  FOR each node *u* DO

  bf-search to compute *S(u,h)* for each *h*
  - Best known exact solution!
  - We will use this as a reference
- Approximations? Only 1 that we know of which we will discuss when we evaluate it.

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Intuition

- Guess what we'll use?
  - Approximate Counting!
- Use very simple algorithm:

FOR each node *u* DO *S(u,0) = { (u,u) }* — initialize to self-only

FOR *h = 1* to *diameter* of *G* DO

FOR each node *u* DO *S(u,h) = S(u,h-1)* — can reach same things

FOR each edge *(u,v)* in *G* DO — and add one more step

S(u,h) = S(u,h) U { (u,v') : (v,v') ∈ S(v,h-1) }

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Intuition

(distinct) neighbors of u, within h hops

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- Too slow and requires too much memory
- Replace expensive set ops with bit ops

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ANF Algorithm #1

FOR each node, *u*, DO
  M(u,0) = concatenation of ***k*** bitmasks of length *log n* + ***r***
    each bitmask has 1 bit set (exp. distribution)
DONE

FOR *h = 1* to *diameter* of *G* DO
  FOR each node, u, DO *M(u,h) = M(u,h-1)*
  FOR each edge *(u,v)* in G DO
    M(u,h) = (*M(u,h)* OR *M(v,h-1)*)

  Estimate $N(h) = Sum(N(u,h)) = Sum\ 2^{b(u)} / .77351 / (1+.31/k)$
    where *b(u)* = average least zero bit in *M(u,it)*
DONE

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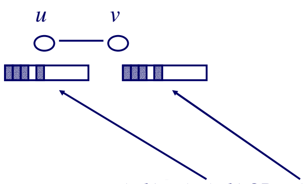
ANF Algorithm #1



whatever *u* can reach with h hops plus whatever *v* can reach with h-1 hops
Duplicates: automatically eliminated!

M(u,h) = (M(u,h) OR *M(v,h-1))*

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Properties

- **Has error guarantees**: (from F&M)
- **Is fast**: $O((n+m)d)$ for n nodes, m edges, diameter d (which is typically small)
- **Has low storage requirements**: $O(n)$
- **Easily parallelizable**: Partition nodes among processors, communicate after full iteration
- **Does sequential scans of edges**.
- **Estimates individual neighborhood functions**
- DOES NOT work with limited memory

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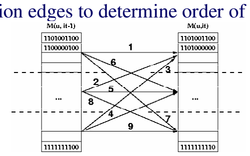
SKIP

Using limited memory

- Idea
  - edges determine access into 2 large tables
  - partition edges to determine order of accesses



- Use prefetching/async writing to hide I/O costs
- Details in the paper

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Experiments – What are the Qs?

- What scheme gives the best results?
  - Us? A Cohen based scheme? Sampling?
- How big a value of ***k*** do we need?
  - Will try 32, 64 and 128
- Are the results sensitive to ***r***?
- How fast is our approximation?
- How well does this performance scale?

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What is the data?

Name	#nodes	#edges	Max. degree	Avg. degree	Eff. Diam.	Orient.	Real?
cornell	844	1,647	131	1.95	8	Dir.	**Y**
cycle	1,000	1,000	2	2.00	450	Undir.	N
grid	10,000	19,800	4	3.96	89	Undir.	N
uniform	65,378	199,996	20	**6.12**	7	Undir.	N
cora	127,083	330,198	457	2.60	28	Dir.	**Y**
80-20	166,946	449,832	723	5.39	8	Undir.	N
router	284,805	430,342	**1,978**	3.15	10	Undir.	**Y**

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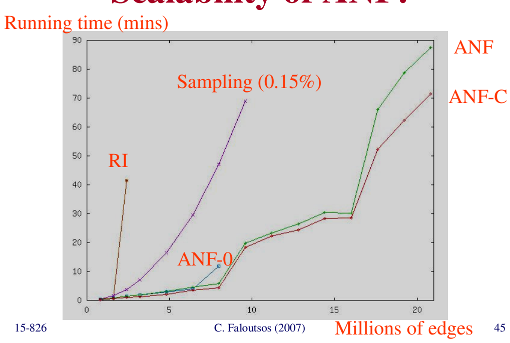
Scalability of ANF!



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We are much faster than BF

Data	BF (Exact)	ANF	Speedup
Uniform	92	0.5	184x
Cora	6	1.5	4x
80-20	680	1.5	**453x**
Router	1,200	2.75	**436x**

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Conclusions

- **Very accurate**
  - less than *10%* error for *k=64*
- **Orders of magnitude faster**
  - up to 450x (on our experiments)
- **Low storage requirements**
  - Only *O(n)* additional memory needed
- **Adapts to available memory**
  - see paper
- **May be parallelized**
  - very few synchronization points are needed
- **Employs sequential scans**
  - May run on graphs larger than memory
- **Estimates Individual neighborhood functions**

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Outline

- Flajolet-Martin (and Cohen) – vocabulary size
- Application: Approximate Neighborhood function (ANF)
  - **putting ANF to work**
- other, powerful approximate counting tools

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The Connectivity and Fault-Tolerance of the Internet Topology

Christopher R. Palmer
Georgos Siganos (UC Riverside)
Michalis Faloutsos (UC Riverside)
Phillip B. Gibbons (Bell-Labs)
Christos Faloutsos

NRDM 2001

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Understanding the Internet

- Large (285K nodes, 430K edges)
  – Hard to process using existing tools
- Yet, Internet very important in daily life
- We want to
  – Identify interesting nodes (routers)
  – **Want to understand network failures**
  – Identify errors / suspicious routers

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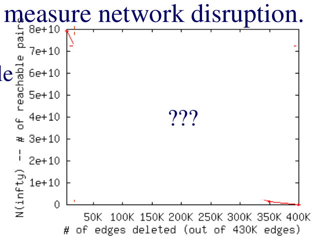
Link Failures

Experiment: Pick an edge at random, delete it and measure network disruption.

reachable pairs



edges deleted

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Link Failures

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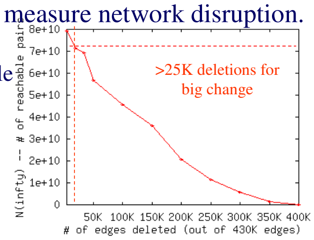
reachable pairs



Internet very resilient to link failures

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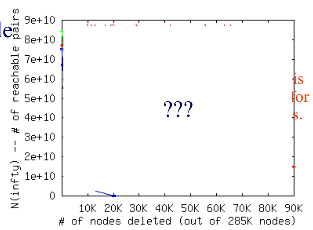
Effect of node deletions

- Random failures
- Targeted failures

reachable pairs



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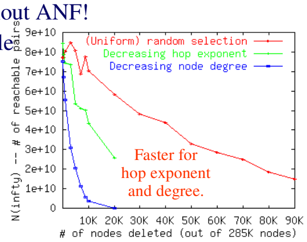
Effect of node deletions

- Robust to random failures, focussed failures are a problem
- ALL these runs would take **>100x** times longer without ANF!

reachable pairs



Disconnection is relatively slow for random failures.

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Conclusions

- Approximate counting (ANF / Martin-Flajolet) take minutes, instead of hours
- and discover internet facts quickly

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Outline

- Flajolet-Martin (and Cohen) – vocabulary size (Problem #1)
- Application: Approximate Neighborhood function (ANF)
- other, powerful approximate counting tools (**Problem #2**, #3)

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Problem #2

- Given a multiset
- compute approximate high-end histogram = hot-list query = (k most common words, and their counts)

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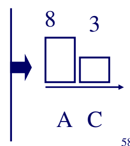
Hot-list queries

- Given a stream of product ids (with duplicates)
- Compute
  - the *k* most frequent products,
  - and their counts
- with a SINGLE PASS and O(*k*) memory

A A B A C A B C A A D E A C A

k=2



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Applications?

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Applications?

- Best selling products
- most common words
- most busy IP destinations/sources (DoS attacks)
- summarization / synopses of datasets
- high-end histograms for DBMS query optimization

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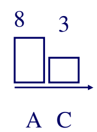
Hot-list queries

- Given a stream of product ids (with duplicates)
- Compute
  - the *k* most frequent products,
  - and their counts
- with a SINGLE PASS and O(*k*) memory

A A B A C A B C A A D E A C A

Exact: impossible
Thus: **approximate**

k=2



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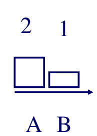
Hot-list queries - idea

- Keep the (approx.) *k* best so far, plus counts
- for a new item, if it is in the hot list
  - increment its count

A A B A C A B C A A D E A C A

k=2



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Hot-list queries - idea

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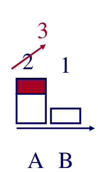

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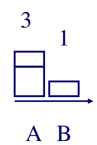
Hot-list queries - idea

- Keep the (approx.) *k* best so far, plus counts
- for a new item, if it is in the hot list
  - increment its count
  - else ??

A A B A C A B C A A D E A C A

k=2



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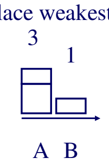
Hot-list queries - idea

- Keep the (approx.) *k* best so far, plus counts
- for a new item, if it is in the hot list
  - increment its count
  - else TOSS a coin, and possibly displace weakest

A A B A C A B C A A D E A C A

k=2



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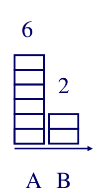
Hot-list queries - idea

- Biased coin - what are the Head/Tail prob.?

A A B A C A B C A A D E A C A

k=2



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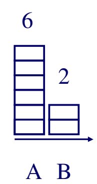
Hot-list queries - idea

- Biased coin - what are the Head/Tail prob.?
- A: depends on count(weakest)

A A B A C A B C A A D E A C A

k=2



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Hot-list queries - idea

- Biased coin - what are the Head/Tail prob.?
- A: depends on count(weakest)
- and the new item (‘D’), if it wins, it gets the count of the item it displaced.

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Hot-list queries - idea

- See [Gibbons+Matias 98] for proofs

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Outline

- Flajolet-Martin (and Cohen) – vocabulary size (Problem #1)
- Application: Approximate Neighborhood function (ANF)
- other, powerful approximate counting tools (**Problem** #2, **#3**)

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Problem #3

- Given two documents
- compute quickly their similarity (#common words/ #total-words) == Jaccard coefficient

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Problem #3'

- Given a query document *q*
- and many other documents
- compute quickly the *k* nearest neighbors of *q*, using the Jaccard coefficient

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Applications?

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Applications?

- Set comparisons eg.,
  - snail-mail address (set of trigrams)
- search engines - 'similar pages'
- social networks: people with many joint friends

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Problem #3'

- Given a query document *q*
- and many other documents
- compute quickly the *k* nearest neighbors of *q*, using the Jaccard coefficient
- Q: how to extract a fixed set of numerical features, to index on?

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Answer

- Approximation / hashing - Cohen:

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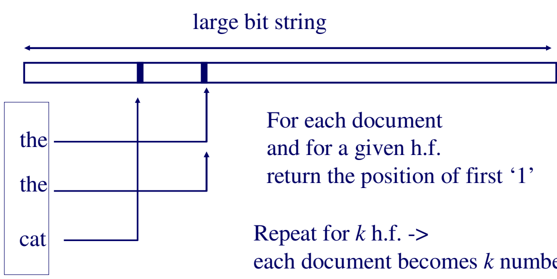
Basic idea (Cohen)



For each document
and for a given h.f.
return the position of first ‘1’

Repeat for *k* h.f. ->
each document becomes *k* numbers

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Idea

- Doc1: n1, n2, ... nk
- Doc2: n1’, n2’, ... nk’

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Idea

- Doc1: n1, n2, ... nk
- Doc2: n1', n2', ... nk'

1 *m*

- say they agree on *m* values

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Idea

- Doc1: n1, n2, ... nk
- Doc2: n1', n2', ... nk'
- say they agree on *m* values,
- then

  Jaccard(Doc1, Doc2) ~ *m/k*

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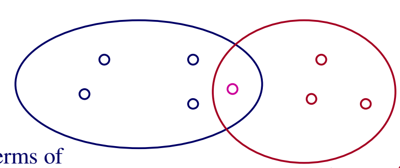
Intuition behind proof

- Venn diagram



voc. terms of Doc.#1

voc. terms of Doc.#2

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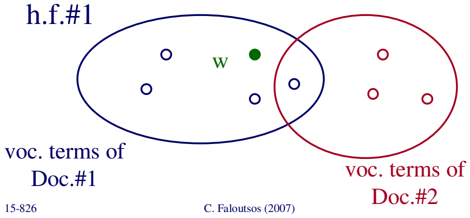
Intuition behind proof

- Venn diagram - let w be the voc. word with the overal smallest hash value, for h.f.#1



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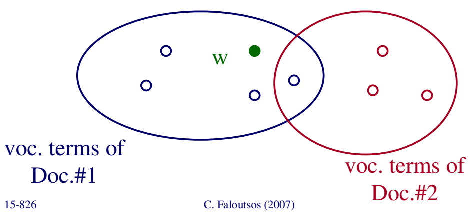
Intuition behind proof

- Prob. that w is smallest on both is exactly Jaccard: #common / #union



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Conclusions

- Approximations can achieve the impossible!
- MF and ANF for neighborhood function
- hot-lists
- Jaccard coeff. / ‘similar pages’

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References

E. Cohen. Size-estimation framework with applications to transitive closure and reachability. *Journal of Computer and System Sciences*, 55(3):441-453, December 1997. http://www.research.att.com/~edith/Papers/tcest.ps.Z

Phillip B. Gibbons, Yossi Matias, *New sampling-based summary statistics for improving approximate query answers*, ACM SIGMOD, 1998 Seattle, Washington, pp 331 - 342

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References (cont'd)

Aristides Gionis, Dimitrios Gunopulos, Nikos Koudas, *Efficient and Tunable Similar Set Retrieval,* ACM SIGMOD 2001, Santa Barbara, California

M. Faloutsos, P. Faloutsos, and C. Faloutsos. *On power-law relationships for the internet topology*. SIGCOMM, 1999.

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References (cont'd)

P. Flajolet and G. N. Martin. Probabilistic counting algorithms for data base applications. *Journal of Computer and System Sciences*, 31:182-209, 1985.

C. R. Palmer and C. Faloutsos. *Density biased sampling: an improved method for data mining and cluster*. In SIGMOD, 2000.

C. R. Palmer, P. B. Gibbons and C. Faloutsos. *Fast approximation of the "neighborhood" function for massive graphs*. KDD 2002

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References (cont'd)

C. R. Palmer, G. Siganos, M. Faloutsos, P. B. Gibbons and C. Faloutsos. *The connectivity and fault-tolerance of the internet topology*. NRDM 2001.