

**15-826: Multimedia Databases
and Data Mining**

DSP tools: Fourier and Wavelets
C. Faloutsos




Outline

Goal: 'Find similar / interesting things'

- Intro to DB
- ➔ • Indexing - similarity search
- Data Mining

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Indexing - Detailed outline

- primary key indexing
- ..
- multimedia
- ➔ • Digital Signal Processing (DSP) tools
 - Discrete Fourier Transform (DFT)
 - Discrete Wavelet Transform (DWT)

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DSP - Detailed outline

- DFT
 - ➔ - what
 - why
 - how
 - Arithmetic examples
 - properties / observations
 - DCT
 - 2-d DFT
 - Fast Fourier Transform (FFT)

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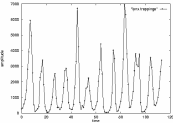
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Introduction

Goal: given a signal (eg., sales over time and/or space)

Find: patterns and/or compress

count



lynx caught per year

year

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What does DFT do?

A: highlights the periodicities

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Why should we care?

A: several real sequences are periodic
 Q: Such as?

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Why should we care?

A: several real sequences are periodic
 Q: Such as?
 A:

- sales patterns follow seasons;
- economy follows 50-year cycle
- temperature follows daily and yearly cycles

Many real signals follow (multiple) cycles


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Why should we care?

For example: human voice!

- Frequency analyzer
<http://www.relisoft.com/freeware/freq.html>
- speaker identification
- impulses/noise -> flat spectrum
- high pitch -> high frequency

 Freq.exe

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Sound Frequency Analyzer

This program requires a soundcard and a mike

Stop Bits per sample: 16

Speed (FFT's per sec): 10 x y z

Sampling Frequency: 11025

Points per Transform: 1024

Hz

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DFT and stocks

- Dow Jones Industrial index, 6/18/2001-12/21/2001

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DFT and stocks

- Dow Jones Industrial index, 6/18/2001-12/21/2001
- just 3 DFT coefficients give very good approximation

Log(ampl)

freq

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DFT: definition

- Discrete Fourier Transform (n-point):

$$X_f = 1/\sqrt{n} \sum_{t=0}^{n-1} x_t * \exp(-j 2\pi f t / n)$$

($j = \sqrt{-1}$) inverse DFT

$$x_t = 1/\sqrt{n} \sum_{f=0}^{n-1} X_f * \exp(+j 2\pi f t / n)$$

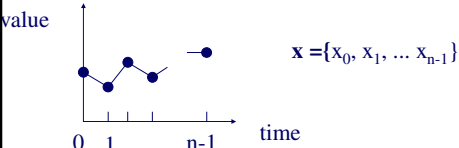
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How does it work? Skip

Decomposes signal to a sum of sine (and cosine) waves.

Q: How to assess 'similarity' of x with a wave?

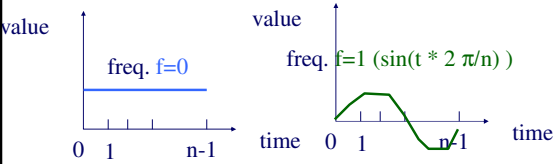


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How does it work? Skip

A: consider the waves with frequency 0, 1, ...;
use the inner-product (~cosine similarity)



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How does it work? Skip

A: consider the waves with frequency 0, 1, ...;
use the inner-product (~cosine similarity)

value

freq. $f=2$

time

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How does it work? Skip

'basis' functions

0 1 n-1

sine, freq = 1

0 1 n-1

0 1 n-1 cosine, f=1

sine, freq = 2

0 1 n-1

0 1 n-1 cosine, f=2

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How does it work? Skip

- Basis functions are actually n-dim vectors, **orthogonal** to each other
- 'similarity' of \mathbf{x} with each of them: inner product
- DFT: ~ all the similarities of \mathbf{x} with the basis functions

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Skip

How does it work?

Since $e^{jf} = \cos(f) + j \sin(f)$
 ($j = \text{sqrt}(-1)$),
 we finally have:

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DFT: definition

- Discrete Fourier Transform (n-point):

$$X_f = 1/\sqrt{n} \sum_{t=0}^{n-1} x_t * \exp(-j 2\pi f t / n)$$

$(j = \sqrt{-1})$

$$x_t = 1/\sqrt{n} \sum_{f=0}^{n-1} X_f * \exp(+j 2\pi f t / n)$$

inverse DFT

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DFT: definition

- Good news:** Available in **all** symbolic math packages, eg., in 'mathematica'

```
x = [1,2,1,2];
X = Fourier[x];
Plot[ Abs[X] ];
```

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DFT: definition

(variations:

- $1/n$ instead of $1/\sqrt{n}$
- $\exp(-\dots)$ instead of $\exp(+\dots)$

)

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DFT: definition

Observations:

- X_f : are complex numbers except X_0 , who is real
- $\text{Im}(X_f)$: ~ amplitude of sine wave of frequency f
- $\text{Re}(X_f)$: ~ amplitude of cosine wave of frequency f
- \mathbf{x} : is the sum of the above sine/cosine waves

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DFT: definition

Observation - SYMMETRY property:

$$X_f = (X_{n-f})^*$$

(“*” : complex conjugate: $(a + bj)^* = a - bj$)

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DFT: definition

Definitions

- $A_f = |X_f|$: amplitude of frequency f
- $|X_f|^2 = \text{Re}(X_f)^2 + \text{Im}(X_f)^2 = \text{energy of frequency } f$
- phase ϕ_f at frequency f

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DFT: definition

Amplitude spectrum: $|X_f|$ vs $f (f=0, 1, \dots, n-1)$

SYMMETRIC (Thus, we plot the first half only)

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DFT: definition

Phase spectrum $|\phi_f|$ vs $f (f=0, 1, \dots, n-1)$:

Anti-symmetric

(Rarely used)

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DFT: Amplitude spectrum

Amplitude: $A_f^2 = \text{Re}^2(X_f) + \text{Im}^2(X_f)$

count

year

Ampl.

Freq.

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DFT: examples

flat

time

Amplitude

freq

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DFT: examples

Low frequency sinusoid

time

freq

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DFT: examples

- Sinusoid - symmetry property: $X_f = X_{n-f}^*$

time freq

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DFT: examples

- Higher freq. sinusoid

time freq

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DFT: examples

examples

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DFT: examples

examples

Ampl.

Freq.

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DFT: Amplitude spectrum

Amplitude: $A_f^2 = \text{Re}^2(X_f) + \text{Im}^2(X_f)$

count

Ampl.

Freq.

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DFT: Amplitude spectrum

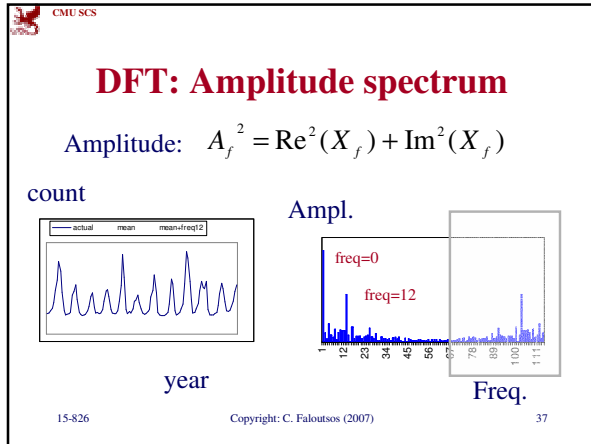
- Amplitude $A_f^2 = \text{Re}^2(X_f) + \text{Im}^2(X_f)$
- Intuition: strength of frequency ' f '

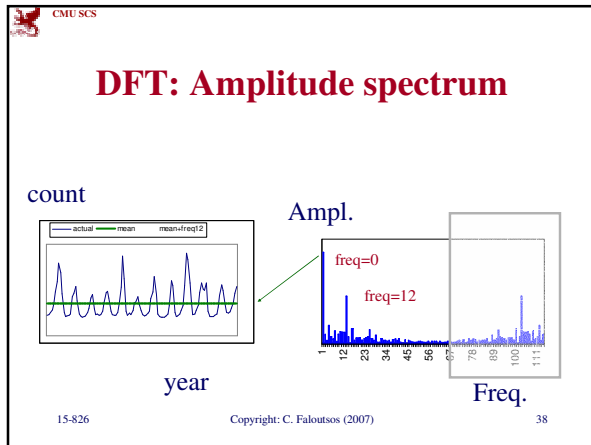
count

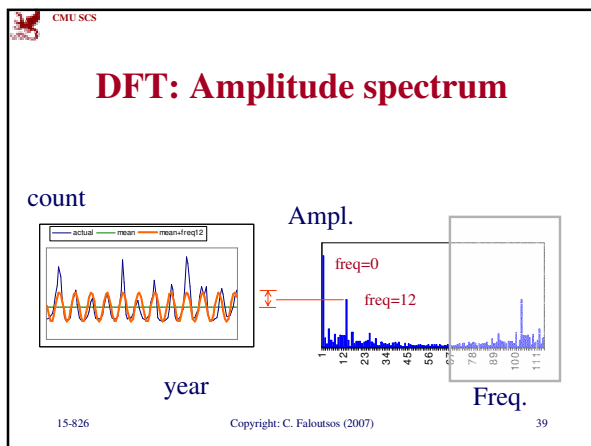
Ampl.

Freq.

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DFT: Amplitude spectrum

- excellent approximation, with only 2 frequencies!
- so what?

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DFT: Amplitude spectrum

- excellent approximation, with only 2 frequencies!
- so what?
- A1: compression
- A2: pattern discovery
- A3: forecasting

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DFT: Amplitude spectrum

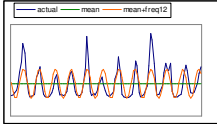
- excellent approximation, with only 2 frequencies!
- so what?
- A1: **(lossy) compression**
- A2: pattern discovery

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DFT: Amplitude spectrum

- excellent approximation, with only 2 frequencies!
- so what?
- A1: (lossy) compression
- A2: **pattern discovery**



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DFT: Amplitude spectrum

- Let's see it in action!
- <http://www.dsptutor.freeuk.com/jsanalyser/FFTSpectrumAnalyser.html>
- plain sine
- phase shift
- two sine waves
- the 'chirp' function

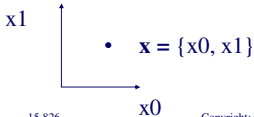
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DFT: Parseval's theorem

$$\sum(x_t^2) = \sum(|X_f|^2)$$

Ie., DFT preserves the 'energy'
or, alternatively: it does an axis rotation:



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DSP - Detailed outline

- DFT
 - what
 - why
 - how
 - ➔ - Arithmetic examples
 - properties / observations
 - DCT
 - 2-d DFT
 - Fast Fourier Transform (FFT)

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Arithmetic examples

- Impulse function: $\mathbf{x} = \{ 0, 1, 0, 0 \}$ ($n = 4$)
- $X_0 = ?$

value

1

0 1 time

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Arithmetic examples

- Impulse function: $\mathbf{x} = \{ 0, 1, 0, 0 \}$ ($n = 4$)
- $X_0 = ?$
- A: $X_0 = 1/\text{sqrt}(4) * 1 * \exp(-j 2 \pi 0 / n) = 1/2$
- $X_1 = ?$
- $X_2 = ?$
- $X_3 = ?$

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Arithmetic examples

- Impulse function: $\mathbf{x} = \{ 0, 1, 0, 0 \}$ ($n = 4$)
- $X_0 = ?$
- A: $X_0 = 1/\text{sqrt}(4) * 1 * \exp(-j 2 \pi 0 / n) = 1/2$
- $X_1 = -1/2 j$
- $X_2 = - 1/2$
- $X_3 = + 1/2 j$
- Q: does the 'symmetry' property hold?

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Arithmetic examples

- Impulse function: $\mathbf{x} = \{ 0, 1, 0, 0 \}$ ($n = 4$)
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- $X_1 = -1/2 j$
- $X_2 = - 1/2$
- $X_3 = + 1/2 j$
- Q: does the 'symmetry' property hold?
- A: Yes (of course)

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Arithmetic examples

- Impulse function: $\mathbf{x} = \{ 0, 1, 0, 0 \}$ ($n = 4$)
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- $X_3 = + 1/2 j$
- Q: check Parseval's theorem

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Arithmetic examples

- Impulse function: $\mathbf{x} = \{ 0, 1, 0, 0 \}$ ($n = 4$)
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- Q: (Amplitude) spectrum?

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Arithmetic examples

- Impulse function: $\mathbf{x} = \{ 0, 1, 0, 0 \}$ ($n = 4$)
- $X_0 = ?$
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- $X_1 = -1/2 j$
- $X_2 = - 1/2$
- $X_3 = + 1/2 j$
- Q: (Amplitude) spectrum?
- A: FLAT!

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Arithmetic examples

- Q: What does this mean?

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Arithmetic examples

- Q: What does this mean?
- A: All frequencies are equally important ->
 - we need n numbers in the frequency domain to represent just one non-zero number in the time domain!
 - “frequency leak”

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Observations

- DFT of ‘step’ function:
 $x = \{ 0, 0, \dots, 0, 1, 1, \dots, 1 \}$

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Observations

- DFT of 'step' function:
 $x = \{ 0, 0, \dots, 0, 1, 1, \dots, 1 \}$

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Observations

- DFT of 'step' function:
 $x = \{ 0, 0, \dots, 0, 1, 1, \dots, 1 \}$

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Observations

- DFT of 'step' function:
 $x = \{ 0, 0, \dots, 0, 1, 1, \dots, 1 \}$

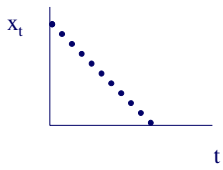
- the more frequencies,
the better the approx.
- 'ringing' becomes worse
- reason: discontinuities; trends

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Observations

- Ringing for trends: because DFT ‘sub-consciously’ replicates the signal

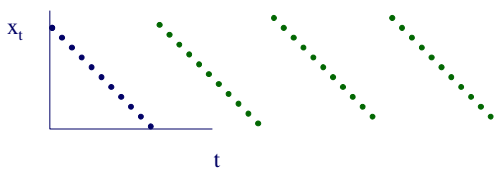


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Observations

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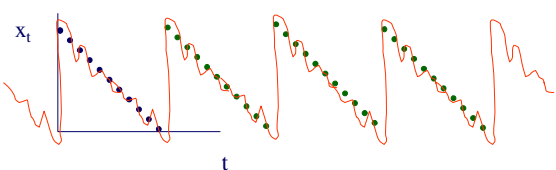


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Observations

- Ringing for trends: because DFT ‘sub-consciously’ replicates the signal



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Observations

- Q: DFT of a sinusoid, eg.

$$x_t = 3 \sin(2 \pi / 4 t)$$
 $(t = 0, \dots, 3)$
 - Q: $X_0 = ?$
 - Q: $X_1 = ?$
 - Q: $X_2 = ?$
 - Q: $X_3 = ?$

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Observations

- Q: DFT of a sinusoid, eg.

$$x_t = 3 \sin(2 \pi / 4 t)$$
 $(t = 0, \dots, 3)$
 - Q: $X_0 = 0$
 - Q: $X_1 = -3j$ •check 'symmetry'
 - Q: $X_2 = 0$ •check Parseval
 - Q: $X_3 = 3j$

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Observations

- Q: DFT of a sinusoid, eg.

$$x_t = 3 \sin(2 \pi / 4 t)$$
 $(t = 0, \dots, 3)$
 - Q: $X_0 = 0$
 - Q: $X_1 = -3j$
 - Q: $X_2 = 0$
 - Q: $X_3 = 3j$

•Does this make sense?

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Property

- Shifting x in time does NOT change the amplitude spectrum
- eg., $x = \{ 0 0 0 1 \}$ and $x' = \{ 0 1 0 0 \}$: same (flat) amplitude spectrum
- (only the phase spectrum changes)
- Useful property when we search for patterns that may 'slide'

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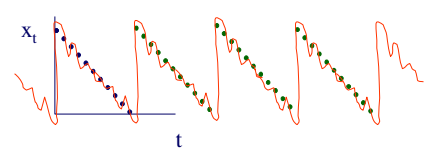
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DCT

Discrete Cosine Transform

- motivation#1: DFT gives complex numbers
- motivation#2: how to avoid the 'frequency leak' of DFT on trends?



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DCT

- brilliant solution to both problems: mirror the sequence, do DFT, and drop the redundant entries!

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DCT

- (see Numerical Recipes for exact formulas)

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DCT - properties

- it gives real numbers as the result
- it has no problems with trends
- it is very good when x_t and $x_{(t+1)}$ are correlated

(thus, is used in JPEG, for image compression)

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2-d DFT

- Definition:

$$X_{f_1, f_2} = \frac{1}{\sqrt{n_1}} \frac{1}{\sqrt{n_2}} \sum_{i_1=0}^{n_1-1} \sum_{i_2=0}^{n_2-1} x_{i_1, i_2} \exp(-2\pi j i_1 f_1 / n_1) \exp(-2\pi j i_2 f_2 / n_2)$$

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2-d DFT

- Intuition:

do 1-d DFT on each row

and then 1-d DFT on each column

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2-d DFT

- Quiz: how do the basis functions look like?
- for $f_1=f_2=0$
- for $f_1=1, f_2=0$
- for $f_1=1, f_2=1$

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2-d DFT

- Quiz: how do the basis functions look like?
- for $f_1=f_2=0$ flat
- for $f_1=1, f_2=0$ wave on x; flat on y
- for $f_1=1, f_2=1$ ~ egg-carton

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DSP - Detailed outline

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FFT

- What is the complexity of DFT?

$$X_f = 1/\sqrt{n} \sum_{t=0}^{n-1} x_t * \exp(-j2\pi tf/n)$$

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FFT

- What is the complexity of DFT?

$$X_f = 1/\sqrt{n} \sum_{t=0}^{n-1} x_t * \exp(-j2\pi tf/n)$$

- A: Naively, $O(n^2)$

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FFT

- However, if n is a power of 2 (or a number with many divisors), we can make it $O(n \log n)$

Main idea: if we know the DFT of the odd time-ticks, and of the even time-ticks, we can quickly compute the whole DFT

Details: in Num. Recipes

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DFT - Conclusions

- It spots periodicities (with the ‘**amplitude spectrum**’)
- can be quickly computed ($O(n \log n)$), thanks to the FFT algorithm.
- **standard** tool in signal processing (speech, image etc signals)

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Detailed outline

- primary key indexing
- ..
- multimedia
- Digital Signal Processing (DSP) tools
 - Discrete Fourier Transform (DFT)
 - – Discrete Wavelet Transform (DWT)

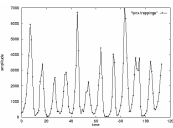
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Reminder: Problem:

Goal: given a signal (eg., #packets over time)
 Find: patterns, periodicities, and/or **compress**

count



year

lynx caught per year
 (packets per day;
 virus infections per month)

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Wavelets - DWT

- DFT is great - but, how about compressing a spike?

value

time

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Wavelets - DWT

- DFT is great - but, how about compressing a spike?
- A: Terrible - all DFT coefficients needed!

value

time

Ampl

Freq.

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Wavelets - DWT

- DFT is great - but, how about compressing a spike?
- A: Terrible - all DFT coefficients needed!

value

time

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Wavelets - DWT

- Similarly, DFT suffers on short-duration waves (eg., baritone, silence, soprano)

value

time

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Wavelets - DWT

- Solution#1: Short window Fourier transform (SWFT)
- But: how short should be the window?

freq

value

time

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Wavelets - DWT

- Answer: **multiple** window sizes! -> DWT

Time domain

freq

time

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Haar Wavelets

- subtract sum of left half from right half
- repeat recursively for quarters, eight-ths, ...

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Wavelets - construction

$x_0 \ x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7$

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Wavelets - construction

level 1 $d_{1,0}$ $s_{1,0}$ $d_{1,1}$ $s_{1,1}$

$x_0 \ x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7$

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Wavelets - construction

level 2

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etc ...

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Wavelets - construction

Q: map each coefficient on the time-freq. plane

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Wavelets - construction

Q: map each coefficient
on the time-freq. plane

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Haar wavelets - code

```

#!/usr/bin/perl5
# expects a file with numbers
# and prints the dwt transform
# The number of time-ticks should be a power of 2
# USAGE
# haar.pl <fname>

my @vals=();
my @smooth; # the smooth component of the signal
my @diff; # the high-freq. component

# collect the values into the array @val
while(<>){
    @vals = ( @vals , split );
}

my $len = scalar(@vals);
my $half = int($len/2);
while($half >= 1){
    for(my $i=0; $i<$half; $i++){
        $diff[$i] = ($vals[2*$i] - $vals[2*$i+1]) / sqrt(2);
        print "d", $diff[$i];
        $smooth[$i] = ($vals[2*$i] + $vals[2*$i+1]) / sqrt(2);
    }
    print "\n";
    @vals = @smooth;
    $half = int($half/2);
}
print "u", $vals[0], "\n"; # the final, smooth component
  
```

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Wavelets - construction

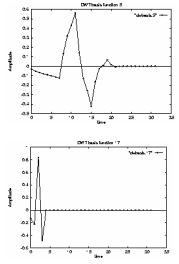
Observation1:
 '+' can be some weighted addition
 '-' is the corresponding weighted difference
 ('Quadrature mirror filters')

Observation2: unlike DFT/DCT,
 there are *many* wavelet bases: Haar, Daubechies-4, Daubechies-6, Coifman, Morlet, Gabor, ...

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Wavelets - how do they look like?

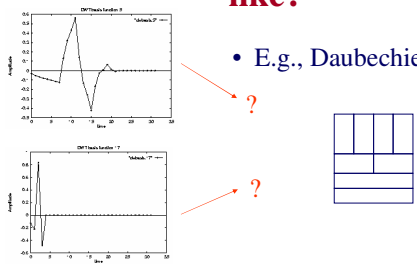


- E.g., Daubechies-4

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Wavelets - how do they look like?

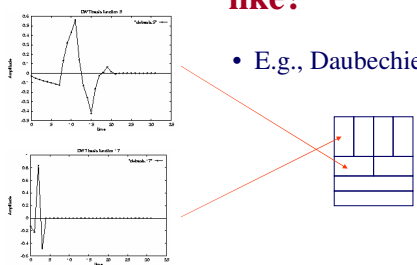


- E.g., Daubechies-4

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Wavelets - how do they look like?



- E.g., Daubechies-4

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Wavelets - Drill#1:

- Q: baritone/silence/soprano - DWT?

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Wavelets - Drill#1:

- Q: baritone/soprano - DWT?

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Wavelets - Drill#2:

- Q: spike - DWT?

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Wavelets - Drill#2:

- Q: spike - DWT?

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Wavelets - Drill#3:

- Q: weekly + daily periodicity, + spike - DWT?

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Wavelets - Drill#3:

- Q: weekly + daily periodicity, + spike - DWT?

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Wavelets - Drill#3:

- Q: weekly + **daily** periodicity, + spike - DWT?

f t

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Wavelets - Drill#3:

- Q: weekly + daily periodicity, + **spike** - DWT?

f t

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Wavelets - Drill#3:

- Q: weekly + daily periodicity, + spike - DWT?

f t

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Wavelets - Drill#3:

- Q: DFT?

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Wavelets - Drill:

Let's see it live:

<http://www-dsp.rice.edu/software/EDU/mra.shtml>

- delta; cosine; cosine2; chirp
- Haar vs Daubechies-4, -6, etc

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Wavelets - k-dimensions?

- easily defined for any dimensionality (like DFT, DCT)

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Advantages of Wavelets

- Better compression (better RMSE with same number of coefficients)
- closely related to the processing of the mammalian eye and ear
- Good for progressive transmission
- handle spikes well
- usually, fast to compute ($O(n)$!)

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Overall Conclusions

- DFT, DCT spot periodicities
- DWT : multi-resolution - matches processing of mammalian ear/eye better
- All three: powerful tools for compression, pattern detection in real signals
- All three: included in math packages (matlab, ...)

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Resources

- Numerical Recipes in C: great description, intuition and code for all three tools
- *xwpl*: open source wavelet package from Yale, with excellent GUI.


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Resources (cont'd)

- <http://www.dsptutor.freeuk.com/jsanalyser/FFTSpectrumAnalyser.html> : Nice java applets
- <http://www.relisoft.com/freeware/freq.html> : voice frequency analyzer (needs microphone)

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Resources (cont'd)

- www-dsp.rice.edu/software/EDU/mra.shtml (wavelets and other demos)

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