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# 15-826: Multimedia Databases and Data Mining

*SVD - part III (more case studies)*

C. Faloutsos

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## Outline

Goal: 'Find similar / interesting things'

- Intro to DB
- Indexing - similarity search
- Data Mining



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## Indexing - Detailed outline

- primary key indexing
- secondary key / multi-key indexing
- spatial access methods
- fractals
- text
- Singular Value Decomposition (SVD)
- multimedia
- ...

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## SVD - Detailed outline

- Motivation
- Definition - properties
- Interpretation
- Complexity
- Case studies
- SVD properties
- More case studies
- Conclusions



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## SVD - detailed outline

- ...
- Case studies
- SVD properties
- more case studies
  - google/Kleinberg algorithms
  - query feedbacks
- Conclusions



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## SVD - Other properties - summary

- can produce orthogonal basis (obvious) (who cares?)
- can solve over- and under-determined linear problems (see C(1) property)
- can compute 'fixed points' (= 'steady state prob. in Markov chains') (see C(4) property)

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## SVD -outline of properties

- (A): obvious
- (B): less obvious
- (C): least obvious (and most powerful!)

  

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## Properties - by defn.:

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \mathbf{\Lambda}_{[r \times r]} \mathbf{V}^{\mathbf{T}}_{[r \times m]}$

A(1): $\mathbf{U}^{T}_{[r \times n]} \mathbf{U}_{[n \times r]} = \mathbf{I}_{[r \times r]}$ (identity matrix)

A(2): $\mathbf{V}^{T}_{[r \times n]} \mathbf{V}_{[n \times r]} = \mathbf{I}_{[r \times r]}$

A(3): $\mathbf{\Lambda}^{k} = \mathrm{diag}(\lambda_1^{k}, \lambda_2^{k}, \dots \lambda_r^{k})$ (k: ANY real number)

A(4): $\mathbf{A}^{\mathbf{T}} = \mathbf{V}\, \mathbf{\Lambda}\, \mathbf{U}^{\mathbf{T}}$

  

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## Less obvious properties

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \mathbf{\Lambda}_{[r \times r]} \mathbf{V}^{\mathbf{T}}_{[r \times m]}$

B(1): $\mathbf{A}_{[n \times m]} (\mathbf{A}^{T})_{[m \times n]} = ??$

  

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## Less obvious properties

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \mathbf{\Lambda}_{[r \times r]} \mathbf{V}^{\mathbf{T}}_{[r \times m]}$

B(1): $\mathbf{A}_{[n \times m]} (\mathbf{A}^T)_{[m \times n]} = \mathbf{U} \mathbf{\Lambda}^2 \mathbf{U}^T$

symmetric; Intuition?

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## Less obvious properties

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \mathbf{\Lambda}_{[r \times r]} \mathbf{V}^{\mathbf{T}}_{[r \times m]}$

B(1): $\mathbf{A}_{[n \times m]} (\mathbf{A}^T)_{[m \times n]} = \mathbf{U} \mathbf{\Lambda}^2 \mathbf{U}^T$

symmetric; Intuition?

'document-to-document' similarity matrix

B(2): symmetrically, for '**V**'

$(\mathbf{A}^T)_{[m \times n]} \mathbf{A}_{[n \times m]} = \mathbf{V} \mathbf{\Lambda}^2 \mathbf{V}^T$

Intuition?

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## Less obvious properties

A: term-to-term similarity matrix

B(3): $( (\mathbf{A}^T)_{[m \times n]} \mathbf{A}_{[n \times m]} )^k = \mathbf{V} \mathbf{\Lambda}^{2k} \mathbf{V}^T$

and

B(4): $(\mathbf{A}^T \mathbf{A})^k \sim \mathbf{v}_1 \lambda_1^{2k} \mathbf{v}_1^T$ for k>>1

where

$\mathbf{v}_1$: [m x 1] first column (singular-vector) of **V**

$\lambda_1$: strongest singular value

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## Proof of (B4)?



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## Less obvious properties

B(4): $(\mathbf{A}^T \mathbf{A})^k \sim \mathbf{v}_1 \lambda_1^{2k} \mathbf{v}_1^T$ for k>>1

$\boxed{\text{B(5): } (\mathbf{A}^T \mathbf{A})^k \mathbf{v'} \sim \text{(constant) } \mathbf{v}_1}$

ie., for (almost) any **v'**, it converges to a vector parallel to $\mathbf{v}_1$

Thus, useful to compute first singular vector/value (as well as the next ones, too...)



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## Proof of (B5)?

## Less obvious properties - repeated:

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \mathbf{\Lambda}_{[r \times r]} \mathbf{V}^{\mathbf{T}}_{[r \times m]}$

B(1): $\mathbf{A}_{[n \times m]} (\mathbf{A}^T)_{[m \times n]} = \mathbf{U} \mathbf{\Lambda}^2 \mathbf{U}^T$
B(2): $(\mathbf{A}^T)_{[m \times n]} \mathbf{A}_{[n \times m]} = \mathbf{V} \mathbf{\Lambda}^2 \mathbf{V}^T$
B(3): $( (\mathbf{A}^T)_{[m \times n]} \mathbf{A}_{[n \times m]} )^k = \mathbf{V} \mathbf{\Lambda}^{2k} \mathbf{V}^T$
B(4): $(\mathbf{A}^T \mathbf{A})^k \sim v_1 \lambda_1^{2k} v_1^T$
B(5): $(\mathbf{A}^T \mathbf{A})^k \mathbf{v}' \sim$ (constant) $\mathbf{v}_1$





## Least obvious properties

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \mathbf{\Lambda}_{[r \times r]} \mathbf{V}^{\mathbf{T}}_{[r \times m]}$

C(1): $\mathbf{A}_{[n \times m]} \mathbf{x}_{[m \times 1]} = \mathbf{b}_{[n \times 1]}$
let $\mathbf{x}_0 = \mathbf{V} \mathbf{\Lambda}^{(-1)} \mathbf{U}^T \mathbf{b}$
if under-specified, $\mathbf{x}_0$ gives ‘shortest’ solution
if over-specified, it gives the ‘solution’ with the smallest least squares error

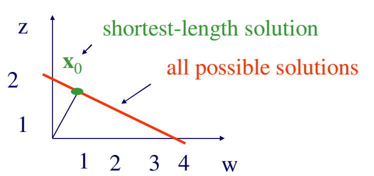
(see Num. Recipes, p. 62)





## Least obvious properties

Illustration: under-specified, eg
$[1\ 2]\ [w\ z]^T = 4$ (ie, $1\,w + 2\,z = 4$)



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## Verify formula:

$\mathbf{A}$ = [1 2]  $\mathbf{b}$ = [4]

$\mathbf{A} = \mathbf{U}\,\mathbf{\Lambda}\,\mathbf{V}^T$

$\mathbf{U}$ = ??

$\mathbf{\Lambda}$ = ??

$\mathbf{V}$= ??

$\mathbf{x}_0 = \mathbf{V}\,\mathbf{\Lambda}^{(-1)}\,\mathbf{U}^T\,\mathbf{b}$

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## Verify formula:

$\mathbf{A}$ = [1 2]  $\mathbf{b}$ = [4]

$\mathbf{A} = \mathbf{U}\,\mathbf{\Lambda}\,\mathbf{V}^T$

$\mathbf{U}$ = [1]

$\mathbf{\Lambda}$ = [ sqrt(5) ]

$\mathbf{V}$= [ 1/sqrt(5)  2/sqrt(5) ] $^T$

$\mathbf{x}_0 = \mathbf{V}\,\mathbf{\Lambda}^{(-1)}\,\mathbf{U}^T\,\mathbf{b}$

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## Verify formula:

$\mathbf{A}$ = [1 2]  $\mathbf{b}$ = [4]

$\mathbf{A} = \mathbf{U}\,\mathbf{\Lambda}\,\mathbf{V}^T$

$\mathbf{U}$ = [1]

$\mathbf{\Lambda}$ = [ sqrt(5) ]

$\mathbf{V}$= [ 1/sqrt(5)  2/sqrt(5) ] $^T$

$\mathbf{x}_0 = \mathbf{V}\,\mathbf{\Lambda}^{(-1)}\,\mathbf{U}^T\,\mathbf{b}$ = [ 1/5  2/5] $^T$ [4]

= [4/5  8/5] $^T$: w= 4/5, z = 8/5

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## Verify formula:

Show that w= 4/5, z = 8/5 is

(a) A solution to 1*w + 2*z = 4 and

(b) Minimal (wrt Euclidean norm)

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## Verify formula:

Show that w= 4/5, z = 8/5 is

(a) A solution to 1*w + 2*z = 4 and

A: easy

(b) Minimal (wrt Euclidean norm)

A: [4/5 8/5] is perpenticular to [2 -1]

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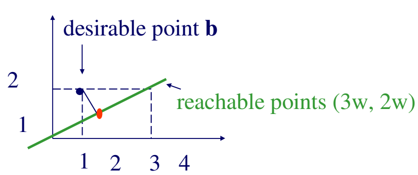
## Least obvious properties – cont'd

Illustration: over-specified, eg

$[3\ 2]^T\ [w] = [1\ 2]^T$ (ie, 3 w = 1; 2 w = 2 )

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## Verify formula:

$\mathbf{A} = [3\ 2]^T \quad \mathbf{b} = [\ 1\ \ 2]^T$

$\mathbf{A} = \mathbf{U}\ \mathbf{\Lambda}\ \mathbf{V}^T$

$\mathbf{U}$ = ??

$\mathbf{\Lambda}$ = ??

$\mathbf{V}$ = ??

$\mathbf{x_0} = \mathbf{V}\ \mathbf{\Lambda}^{(-1)}\ \mathbf{U}^T\ \mathbf{b}$



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## Verify formula:

$\mathbf{A} = [3\ 2]^T \quad \mathbf{b} = [\ 1\ \ 2]^T$

$\mathbf{A} = \mathbf{U}\ \mathbf{\Lambda}\ \mathbf{V}^T$

$\mathbf{U}$ = [ 3/sqrt(13)    2/sqrt(13) ] $^T$

$\mathbf{\Lambda}$ = [ sqrt(13) ]

$\mathbf{V}$ = [ 1 ]

$\mathbf{x_0} = \mathbf{V}\ \mathbf{\Lambda}^{(-1)}\ \mathbf{U}^T\ \mathbf{b}$ = [ 7/13 ]



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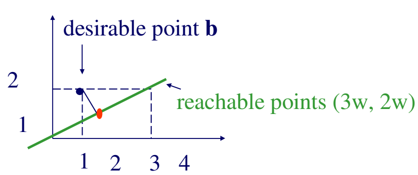
## Verify formula:

$[3\ 2]^T\ \ [7/13] = [1\ 2]^T$

$[21/13\ \ 14/13\ ]^T$ -> 'red point'



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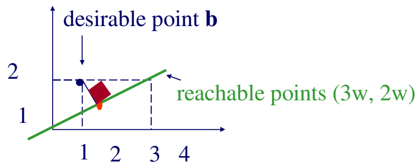
## Verify formula:

$[3\ 2]^T\ [7/13] = [1\ 2]^T$

$[21/13\ \ 14/13]^T$ -> 'red point' - perpenticular?



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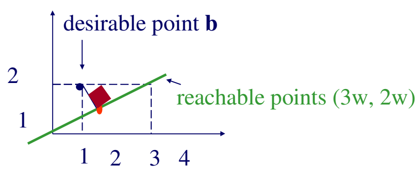
## Verify formula:

A: [3 2] . ( [1 2] – [21/13 14/13]) =

[3 2] . [ -8/13 12/13] = [3 2] . [ -2 3] = 0



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## Least obvious properties - cont'd

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]}\, \mathbf{\Lambda}_{[r \times r]}\, \mathbf{V}^{\mathbf{T}}_{[r \times m]}$

C(2): $\mathbf{A}_{[n \times m]}\, \mathbf{v_1}_{[m \times 1]} = \lambda_1\, \mathbf{u_1}_{[n \times 1]}$

where $\mathbf{v_1}$, $\mathbf{u_1}$ the first (column) vectors of **V**, **U**. ($\mathbf{v_1}$ == right-singular-vector)

C(3): symmetrically: $\mathbf{u_1}^T \mathbf{A} = \lambda_1 \mathbf{v_1}^T$

$\mathbf{u_1}$ == left-singular-vector

Therefore:

## Least obvious properties - cont'd

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \, \mathbf{\Lambda}_{[r \times r]} \, \mathbf{V}^{\mathbf{T}}_{[r \times m]}$

$\boxed{\text{C(4): } \mathbf{A}^{\mathbf{T}} \mathbf{A} \, \mathbf{v}_1 = \lambda_1^2 \, \mathbf{v}_1}$

(**fixed point** - the dfn of eigenvector for a symmetric matrix)





## Least obvious properties - altogether

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \, \mathbf{\Lambda}_{[r \times r]} \, \mathbf{V}^{\mathbf{T}}_{[r \times m]}$

C(1): $\mathbf{A}_{[n \times m]} \, \mathbf{x}_{[m \times 1]} = \mathbf{b}_{[n \times 1]}$

then, $\mathbf{x}_0 = \mathbf{V} \, \mathbf{\Lambda}^{(-1)} \, \mathbf{U}^{\mathrm{T}} \, \mathbf{b}$**:** shortest, actual or least-squares solution

C(2): $\mathbf{A}_{[n \times m]} \, \mathbf{v}_{1\,[m \times 1]} = \lambda_1 \, \mathbf{u}_{1\,[n \times 1]}$

C(3): $\mathbf{u}_1^{\mathbf{T}} \mathbf{A} = \lambda_1 \mathbf{v}_1^{\mathbf{T}}$

C(4): $\mathbf{A}^{\mathbf{T}} \mathbf{A} \, \mathbf{v}_1 = \lambda_1^2 \, \mathbf{v}_1$





## Properties - conclusions

A(0): $\mathbf{A}_{[n \times m]} = \mathbf{U}_{[n \times r]} \, \mathbf{\Lambda}_{[r \times r]} \, \mathbf{V}^{\mathbf{T}}_{[r \times m]}$

B(5): $(\mathbf{A}^{\mathrm{T}} \mathbf{A})^{k} \, \mathbf{v}' \sim$ (constant) $\mathbf{v}_1$

C(1): $\mathbf{A}_{[n \times m]} \, \mathbf{x}_{[m \times 1]} = \mathbf{b}_{[n \times 1]}$

then, $\mathbf{x}_0 = \mathbf{V} \, \mathbf{\Lambda}^{(-1)} \, \mathbf{U}^{\mathrm{T}} \, \mathbf{b}$**:** shortest, actual or least-squares solution

C(4): $\mathbf{A}^{\mathbf{T}} \mathbf{A} \, \mathbf{v}_1 = \lambda_1^2 \, \mathbf{v}_1$

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## SVD - detailed outline

- ...
- Case studies
- SVD properties
- more case studies
  - Kleinberg/google algorithms
  - query feedbacks
- Conclusions



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## Kleinberg's algorithm

- Problem dfn: given the web and a query
- find the most 'authoritative' web pages for this query

Step 0: find all pages containing the query terms

Step 1: expand by one move forward and backward



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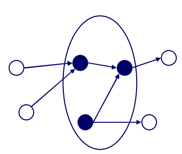
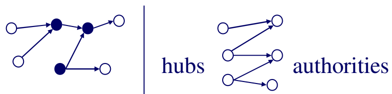
## Kleinberg's algorithm

- Step 1: expand by one move forward and backward



## Kleinberg's algorithm

- on the resulting graph, give high score (= 'authorities') to nodes that many important nodes point to
- give high importance score ('hubs') to nodes that point to good 'authorities')








## Kleinberg's algorithm

observations

- recursive definition!
- each node (say, '*i*'-th node) has both an authoritativeness score $a_i$ and a hubness score $h_i$





## Kleinberg's algorithm

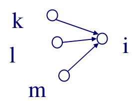
Let $E$ be the set of edges and **A** be the adjacency matrix:

the $(i,j)$ is 1 if the edge from $i$ to $j$ exists

Let $h$ and $a$ be [n x 1] vectors with the 'hubness' and 'authoritativiness' scores.

Then:

## Kleinberg's algorithm




Then:

$$a_i = h_k + h_l + h_m$$

that is

$a_i$ = Sum ($h_j$)   over all $j$ that ($j$,$i$) edge exists

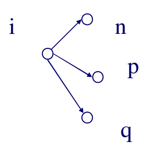
or

$\mathbf{a} = \mathbf{A}^T \mathbf{h}$





## Kleinberg's algorithm




symmetrically, for the 'hubness':

$$h_i = a_n + a_p + a_q$$

that is

$h_i$ = Sum ($q_j$)   over all $j$ that ($i$,$j$) edge exists

or

$\mathbf{h} = \mathbf{A}\ \mathbf{a}$



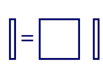


## Kleinberg's algorithm

In conclusion, we want vectors **h** and **a** such that:

$$\mathbf{h} = \mathbf{A}\ \mathbf{a}$$

$$\mathbf{a} = \mathbf{A}^T \mathbf{h}$$



Recall properties:

C(2): $\mathbf{A}_{[n \times m]}\ \mathbf{v_1}_{[m \times 1]} = \lambda_1\ \mathbf{u_1}_{[n \times 1]}$

C(3): $\mathbf{u_1}^T \mathbf{A} = \lambda_1 \mathbf{v_1}^T$

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## Kleinberg's algorithm

In short, the solutions to

$$\mathbf{h} = \mathbf{A}\ \mathbf{a}$$

$$\mathbf{a} = \mathbf{A}^T\ \mathbf{h}$$

are the left- and right- singular-vectors of the adjacency matrix **A.**

Starting from random **a'** and iterating, we'll eventually converge

(Q: to which of all the singular-vectors? why?)



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## Kleinberg's algorithm

(Q: to which of all the singular-vectors? why?)

A: to the ones of the strongest singular-value, because of property B(5):

B(5): $(\mathbf{A}^T\mathbf{A})^k\ \mathbf{v'} \sim (\text{constant})\ \mathbf{v}_1$



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## Kleinberg's algorithm - results

Eg., for the query 'java':

0.328 www.gamelan.com

0.251 java.sun.com

0.190 www.digitalfocus.com ("the java developer")

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## Kleinberg's algorithm - discussion

- 'authority' score can be used to find 'similar pages' (how?)
- closely related to 'citation analysis', social networs / 'small world' phenomena



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## google/page-rank algorithm

- closely related: imagine a particle randomly moving along the edges (*)
- compute its steady-state probabilities

(*) with occasional random jumps



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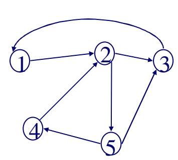
## google/page-rank algorithm

- ~identical problem: given a Markov Chain, compute the steady state probabilities p1 ... p5



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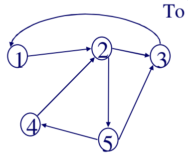
## (Simplified) PageRank algorithm

- Let **A** be the transition matrix (= adjacency matrix); let $\mathbf{A}^T$ become column-normalized - then




From / To

$$\mathbf{A}^T: \quad \begin{bmatrix} & & 1 & & \\ 1 & & & 1 & \\ & 1/2 & & & 1/2 \\ & & & & 1/2 \\ & 1/2 & & & \end{bmatrix} \begin{bmatrix} p1 \\ p2 \\ p3 \\ p4 \\ p5 \end{bmatrix} = \begin{bmatrix} p1 \\ p2 \\ p3 \\ p4 \\ p5 \end{bmatrix}$$

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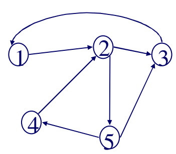
## (Simplified) PageRank algorithm

- $\mathbf{A}^T \mathbf{p} = \mathbf{p}$




$$\underset{\mathbf{A}^T}{\begin{bmatrix} & & 1 & & \\ 1 & & & 1 & \\ & 1/2 & & & 1/2 \\ & & & & 1/2 \\ & 1/2 & & & \end{bmatrix}} \underset{\mathbf{p}}{\begin{bmatrix} p1 \\ p2 \\ p3 \\ p4 \\ p5 \end{bmatrix}} = \underset{\mathbf{p}}{\begin{bmatrix} p1 \\ p2 \\ p3 \\ p4 \\ p5 \end{bmatrix}}$$

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## (Simplified) PageRank algorithm

- $\mathbf{A}^T \mathbf{p} = 1 * \mathbf{p}$
- thus, **p** is the **eigenvector** that corresponds to the highest **eigenvalue** (=1, since the matrix is column-normalized)
- formal definition of eigenvector/value: soon

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## (Simplified) PageRank algorithm

- In short: imagine a particle randomly moving along the edges
- compute its steady-state probabilities (ssp)

Full version of algo: with occasional random jumps



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## Formal definition

If **A** is a (n x n) square matrix
($\lambda$ , **x**) is an **eigenvalue/eigenvector** pair of **A** if

$$\boxed{\mathbf{A}\,\mathbf{x} = \lambda\,\mathbf{x}}$$

CLOSELY related to singular values:



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## Eigen- vs singular-values

if

$$\mathbf{B}_{[n \times m]} = \mathbf{U}_{[n \times r]}\, \mathbf{\Lambda}_{[r \times r]}\, (\mathbf{V}_{[m \times r]})^T$$

then $\mathbf{A} = (\mathbf{B}^T\mathbf{B})$ is symmetric and

$$C(4)\text{: } \mathbf{B}^T\,\mathbf{B}\,\mathbf{v}_i = \lambda_i^2\,\mathbf{v}_i$$

ie, $\mathbf{v}_1$ , $\mathbf{v}_2$ , ...: eigenvectors of $\mathbf{A} = (\mathbf{B}^T\mathbf{B})$

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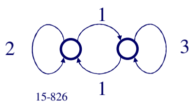
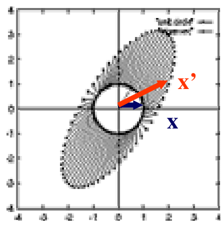
## Intuition

- **A** as vector transformation

$$\begin{array}{cccc} \mathbf{x'} & & \mathbf{A} & \mathbf{x} \\ \begin{bmatrix} 2 \\ 1 \end{bmatrix} & = & \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{array}$$

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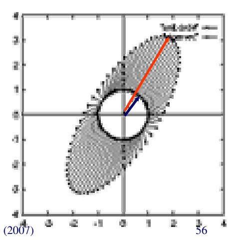
## Intuition

- By defn., eigenvectors remain parallel to themselves (‘**fixed points’**)

$$\begin{array}{ccccc} \lambda_1 & \mathbf{v_1} & & \mathbf{A} & \mathbf{v_1} \\ 3.62 \; * & \begin{bmatrix} 0.52 \\ 0.85 \end{bmatrix} & = & \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} & \begin{bmatrix} 0.52 \\ 0.85 \end{bmatrix} \end{array}$$



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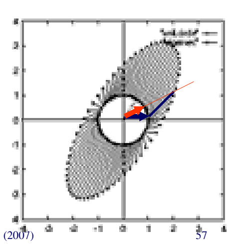
## Convergence

- Usually, fast:



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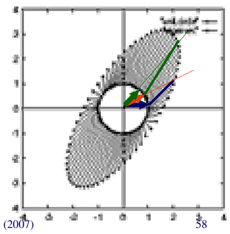
## Convergence

- Usually, fast:



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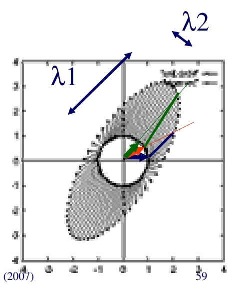
## Convergence

- Usually, fast:
- depends on ratio
  $\lambda 1 : \lambda 2$



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## Kleinberg/google - conclusions

**SVD** helps in graph analysis:

hub/authority scores: strongest left- and right-singular-vectors of the adjacency matrix

random walk on a graph: steady state probabilities are given by the strongest eigenvector of the transition matrix

  

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## SVD - detailed outline

- ...
- Case studies
- SVD properties
- more case studies
  - google/Kleinberg algorithms
  - query feedbacks
- Conclusions

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## Query feedbacks

[Chen & Roussopoulos, sigmod 94]

sample problem:

estimate selectivities (e.g., '*how many movies were made between 1940 and 1945?*'

for query optimization,

LEARNING from the query results so far!!

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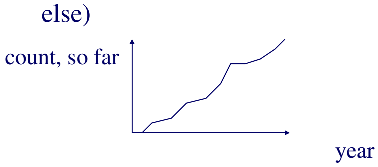
## Query feedbacks

Idea #1: consider a function for the CDF (cummulative distr. function), eg., 6-th degree polynomial (or splines, or anything else)

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## Query feedbacks

For example

F(x) = # movies made until year 'x'

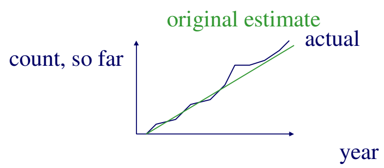
$= a_1 + a_2 * x + a_3 * x^2 + \ldots a_7 * x^6$



## Query feedbacks

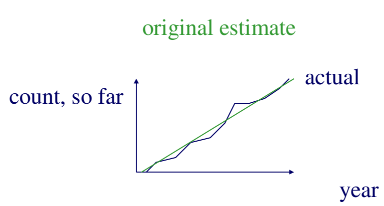
GREAT idea #2: adapt your model, as you see the actual counts of the actual queries





## Query feedbacks



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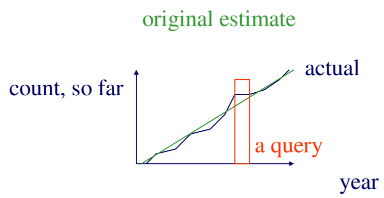
## Query feedbacks



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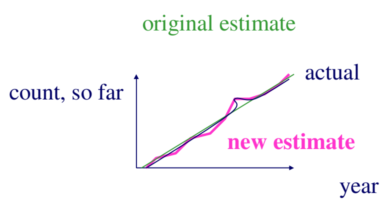
## Query feedbacks



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## Query feedbacks

Eventually, the problem becomes:

- estimate the parameters $a_1$, ... $a_7$ of the model
- to minimize the least squares errors from the real answers so far.

Formally:

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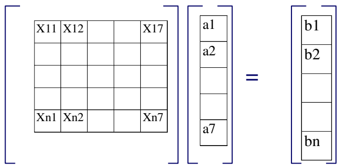
## Query feedbacks

Formally, with n queries and 6-th degree polynomials:



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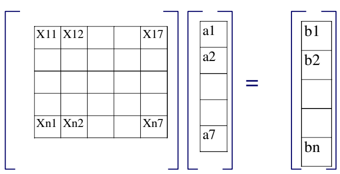
## Query feedbacks

where $x_{i,j}$ such that Sum ($x_{i,j}$ * $a_i$) = our estimate for the # of movies and $b_j$: the actual



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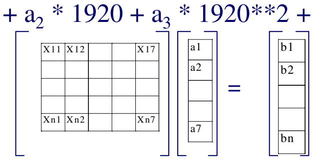
## Query feedbacks

For example, for query '*find the count of movies during (1920-1932)*':

$a_1 + a_2 * 1932 + a_3 * 1932**2 + \ldots$

-

$(a_1 + a_2 * 1920 + a_3 * 1920**2 + \ldots)$



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## Query feedbacks

And thus X11 = 0; X12 = 1932-1920, etc

$a_1 + a_2 * 1932 + a_3 * 1932{**}2 + \ldots$
-
$(a_1 + a_2 * 1920 + a_3 * 1920{**}2 + \ldots)$

$$\begin{bmatrix} X11 & X12 & & & X17 \\ & & & & \\ & & & & \\ & & & & \\ Xn1 & Xn2 & & & Xn7 \end{bmatrix} \begin{bmatrix} a1 \\ a2 \\ \\ \\ a7 \end{bmatrix} = \begin{bmatrix} b1 \\ b2 \\ \\ \\ bn \end{bmatrix}$$



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## Query feedbacks

In matrix form:

$$\begin{array}{lcccccc} & \mathbf{X} & & \mathbf{a} & = & \mathbf{b} \\ \begin{array}{l} \text{1st query} \\ \\ \\ \text{n-th query} \end{array} & \begin{bmatrix} X11 & X12 & & & X17 \\ & & & & \\ & & & & \\ Xn1 & Xn2 & & & Xn7 \end{bmatrix} & & \begin{bmatrix} a1 \\ a2 \\ \\ a7 \end{bmatrix} & = & \begin{bmatrix} b1 \\ b2 \\ \\ bn \end{bmatrix} \end{array}$$



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## Query feedbacks

In matrix form:

$$\mathbf{X}\,\mathbf{a} = \mathbf{b}$$

and the least-squares estimate for **a** is

$$\mathbf{a} = \mathbf{V}\,\mathbf{\Lambda}^{(-1)}\,\mathbf{U}^{\mathbf{T}}\,\mathbf{b}$$

according to property C(1)

(let $\mathbf{X} = \mathbf{U}\,\mathbf{\Lambda}\,\mathbf{V}^{\mathbf{T}}$ )

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## Query feedbacks - enhancements

the solution

$$\mathbf{a} = \mathbf{V}\, \mathbf{\Lambda}^{(-1)}\, \mathbf{U}^{\mathbf{T}}\, \mathbf{b}$$

works, but needs expensive SVD each time a new query arrives

GREAT Idea #3: Use ‘Recursive Least Squares’, to adapt **a** incrementally.

Details: in paper - intuition:



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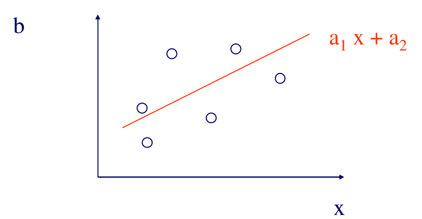
## Query feedbacks - enhancements

Intuition: least squares fit



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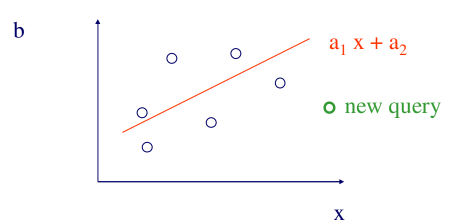
## Query feedbacks - enhancements

Intuition: least squares fit



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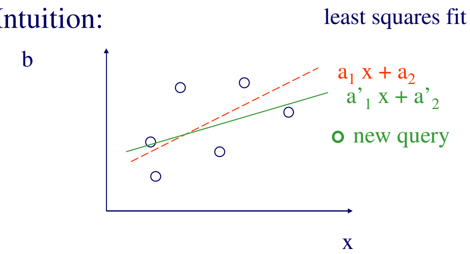
## Query feedbacks - enhancements

Intuition:

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## Query feedbacks - enhancements

the new coefficients can be quickly computed from the old ones, plus statistics in a (7x7) matrix

(no need to know the details, although the RLS is a brilliant method)

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## Query feedbacks - enhancements

GREAT idea #4: ‘forgetting’ factor - we can even down-play the weight of older queries, since the data distribution might have changed.

(comes for ‘free’ with RLS...)

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## Query feedbacks - conclusions

SVD helps find the Least Squares solution, to adapt to query feedbacks

(RLS = Recursive Least Squares is a great method to incrementally update least-squares fits)





## SVD - detailed outline

- ...
- Case studies
- SVD properties
- more case studies
  - google/Kleinberg algorithms
  - query feedbacks
- Conclusions





## Conclusions

- SVD: a **valuable** tool
- given a document-term matrix, it finds 'concepts' (LSI)
- ... and can reduce dimensionality (KL)
- ... and can find rules (PCA; RatioRules)

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## Conclusions cont'd

- ... and can find fixed-points or steady-state probabilities (google/ Kleinberg/ Markov Chains)
- ... and can solve optimally over- and under-constraint linear systems (least squares / query feedbacks)

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