

15-826: Multimedia Databases and Data Mining

DSP tools: Fourier and Wavelets

C. Faloutsos

Outline

Goal: ‘Find similar / interesting things’

- Intro to DB
- • Indexing - similarity search
- Data Mining

15-826

Copyright: C. Faloutsos (2006)

2

Indexing - Detailed outline

- primary key indexing
- ..
- multimedia
- • Digital Signal Processing (DSP) tools
 - Discrete Fourier Transform (DFT)
 - Discrete Wavelet Transform (DWT)

15-826

Copyright: C. Faloutsos (2006)

3

DSP - Detailed outline

- DFT
 - – what
 - why
 - how
 - Arithmetic examples
 - properties / observations
 - DCT
 - 2-d DFT
 - Fast Fourier Transform (FFT)

15-826

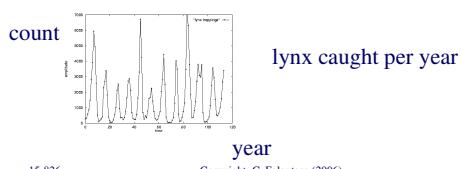
Copyright: C. Faloutsos (2006)

4

Introduction

Goal: given a signal (eg., sales over time and/or space)

Find: patterns and/or compress



15-826

Copyright: C. Faloutsos (2006)

5

What does DFT do?

A: highlights the periodicities

15-826

Copyright: C. Faloutsos (2006)

6

CMU SCS

Why should we care?

A: several real sequences are periodic
 Q: Such as?

15-826 Copyright: C. Faloutsos (2006) 7

CMU SCS

Why should we care?

A: several real sequences are periodic
 Q: Such as?
 A:

- sales patterns follow seasons;
- economy follows 50-year cycle
- temperature follows daily and yearly cycles

 Many real signals follow (multiple) cycles

15-826 Copyright: C. Faloutsos (2006) 8

CMU SCS

Why should we care?

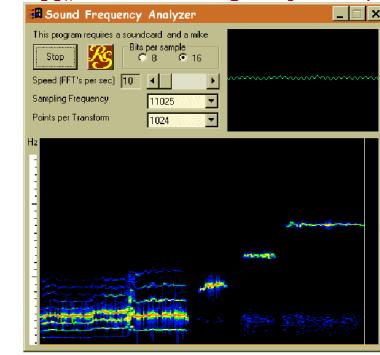
For example: human voice!

- Frequency analyzer
<http://www.relisoft.com/freeware/freq.html>
- speaker identification
- impulses/noise -> flat spectrum
- high pitch -> high frequency

Freq.exe

15-826 Copyright: C. Faloutsos (2006) 9

CMU SCS



Sound Frequency Analyzer
 This program requires a soundcard and a mike.
 Bits per sample: 8 16
 Speed (IFFT's per sec): 10
 Sampling Frequency: 11025
 Points per Transform: 1024

Hz

15-826 Copyright: C. Faloutsos (2006) 10

CMU SCS

DFT and stocks

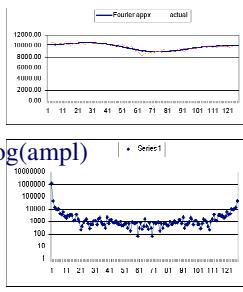


- Dow Jones Industrial index, 6/18/2001-12/21/2001

15-826 Copyright: C. Faloutsos (2006) 11

CMU SCS

DFT and stocks



- Dow Jones Industrial index, 6/18/2001-12/21/2001
- just 3 DFT coefficients give very good approximation

freq

15-826 Copyright: C. Faloutsos (2006) 12



DFT: definition

- Discrete Fourier Transform (n-point):

$$X_f = 1/\sqrt{n} \sum_{t=0}^{n-1} x_t * \exp(-j2\pi tf/n)$$

$(j = \sqrt{-1})$

$$x_t = 1/\sqrt{n} \sum_{f=0}^{n-1} X_f * \exp(+j2\pi tf/n)$$

inverse DFT

15-826

Copyright: C. Faloutsos (2006)

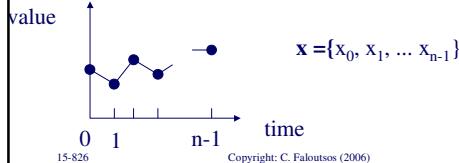
13

Skip

How does it work?

Decomposes signal to a sum of sine (and cosine) waves.

Q: How to assess ‘similarity’ of \mathbf{x} with a wave?

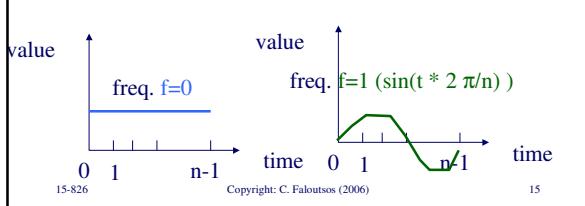


14


Skip

How does it work?

- A: consider the waves with frequency 0, 1, ...; use the inner-product (~cosine similarity)

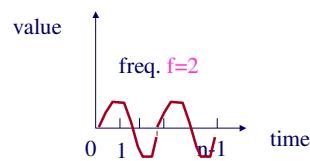


15


Skip

How does it work?

- A: consider the waves with frequency 0, 1, ...; use the inner-product (~cosine similarity)

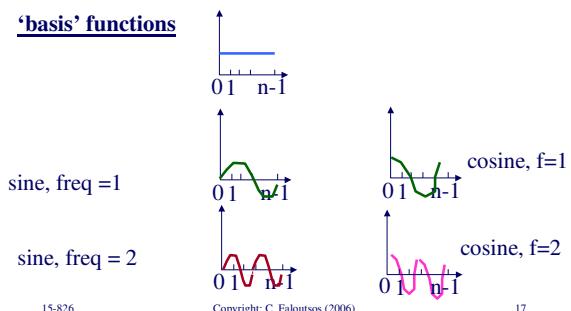


16


Skip

How does it work?

basis’ functions



17


Skip

How does it work?

- Basis functions are actually n-dim vectors, **orthogonal** to each other
- ‘similarity’ of \mathbf{x} with each of them: inner product
- DFT: ~ all the similarities of \mathbf{x} with the basis functions

15-826

Copyright: C. Faloutsos (2006)

18

CMU SCS

How does it work?

Since $e^{j\theta} = \cos(\theta) + j \sin(\theta)$
 $(j=\sqrt{-1})$,
we finally have:

15-826 Copyright: C. Faloutsos (2006) 19

CMU SCS

DFT: definition

- Discrete Fourier Transform (n-point):

$$X_f = 1/\sqrt{n} \sum_{t=0}^{n-1} x_t * \exp(-j2\pi tf/n)$$

$$(j=\sqrt{-1}) \quad \text{inverse DFT}$$

$$x_t = 1/\sqrt{n} \sum_{f=0}^{n-1} X_f * \exp(+j2\pi tf/n)$$

15-826 Copyright: C. Faloutsos (2006) 20

CMU SCS

DFT: definition

- **Good news:** Available in **all** symbolic math packages, eg., in ‘mathematica’
 $x = [1,2,1,2];$
 $X = \text{Fourier}[x];$
 $\text{Plot}[\text{Abs}[X]];$

15-826 Copyright: C. Faloutsos (2006) 21

CMU SCS

DFT: definition

(variations:

- $1/n$ instead of $1/\sqrt{n}$
- $\exp(-...)$ instead of $\exp(+...)$
-)

15-826 Copyright: C. Faloutsos (2006) 22

CMU SCS

DFT: definition

Observations:

- X_f : are complex numbers except $-X_0$, who is real
- $\text{Im}(X_f)$: ~ amplitude of sine wave of frequency f
- $\text{Re}(X_f)$: ~ amplitude of cosine wave of frequency f
- \mathbf{x} : is the sum of the above sine/cosine waves

15-826 Copyright: C. Faloutsos (2006) 23

CMU SCS

DFT: definition

Observation - SYMMETRY property:

$$X_f = (X_{n-f})^*$$

(“*”: complex conjugate: $(a + b j)^* = a - b j$)

15-826 Copyright: C. Faloutsos (2006) 24

DFT: definition

Definitions

- $A_f = |X_f|$: amplitude of frequency f
- $|X_f|^2 = \text{Re}(X_f)^2 + \text{Im}(X_f)^2$ = energy of frequency f
- phase ϕ_f at frequency f

15-826 Copyright: C. Faloutsos (2006) Re 25

DFT: definition

Amplitude spectrum: $|X_f|$ vs f ($f=0, 1, \dots, n-1$)

SYMMETRIC (Thus, we plot the first half only)

15-826 Copyright: C. Faloutsos (2006) 26

DFT: definition

Phase spectrum $|\phi_f|$ vs f ($f=0, 1, \dots, n-1$):
Anti-symmetric

(Rarely used)

15-826 Copyright: C. Faloutsos (2006) 27

DFT: Amplitude spectrum

Amplitude: $A_f^2 = \text{Re}^2(X_f) + \text{Im}^2(X_f)$

count

15-826 Copyright: C. Faloutsos (2006) 28

DFT: examples

flat

Amplitude

time freq

15-826 Copyright: C. Faloutsos (2006) 29

DFT: examples

Low frequency sinusoid

time freq

15-826 Copyright: C. Faloutsos (2006) 30

DFT: examples

- Sinusoid - symmetry property: $X_f = X^*_{n-f}$

15-826 Copyright: C. Faloutsos (2006) 31

DFT: examples

- Higher freq. sinusoid

15-826 Copyright: C. Faloutsos (2006) 32

DFT: examples

examples

15-826 Copyright: C. Faloutsos (2006) 33

DFT: examples

examples

Ampl.

Freq.

15-826 Copyright: C. Faloutsos (2006) 34

DFT: Amplitude spectrum

Amplitude: $A_f^2 = \text{Re}^2(X_f) + \text{Im}^2(X_f)$

count

year

Ampl.

Freq.

15-826 Copyright: C. Faloutsos (2006) 35

DFT: Amplitude spectrum

• Amplitude $A_f^2 = \text{Re}^2(X_f) + \text{Im}^2(X_f)$

• Intuition: strength of frequency ' f '

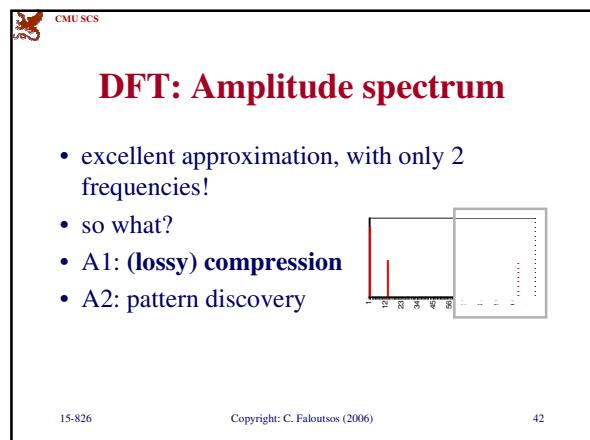
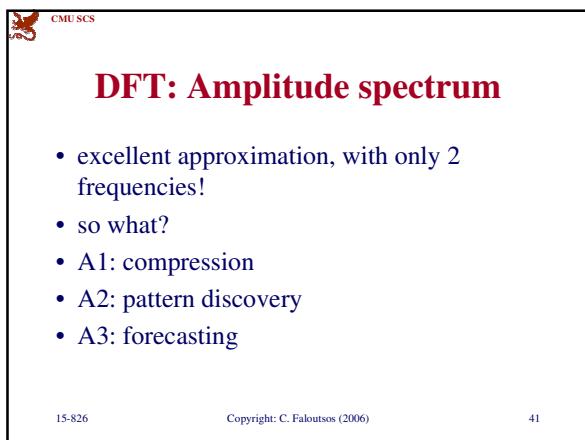
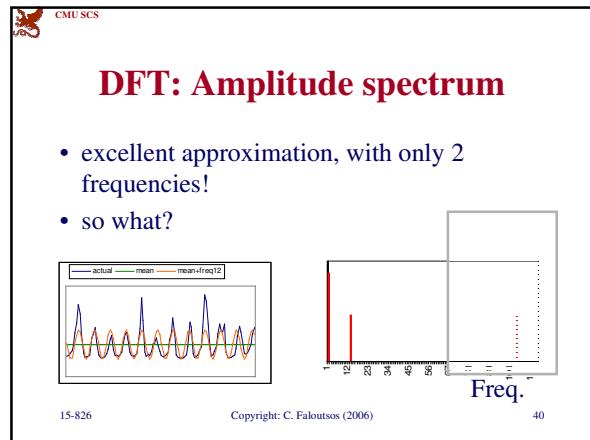
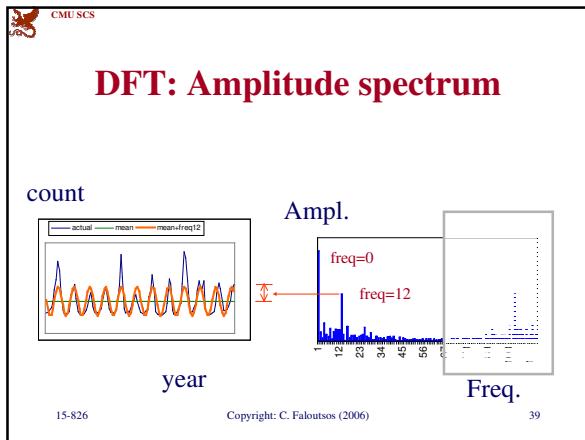
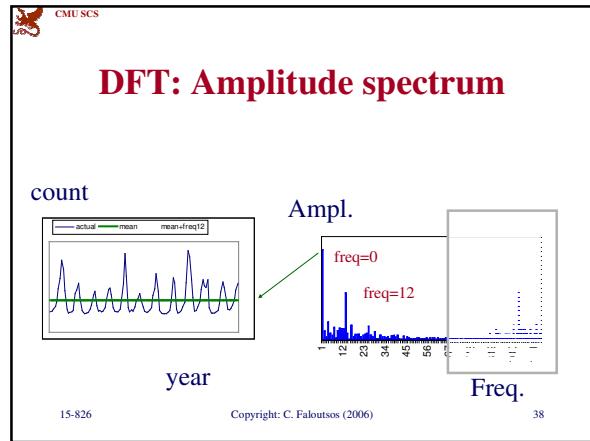
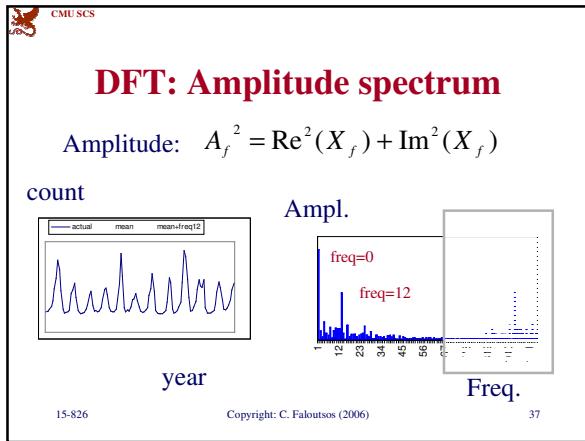
count

A_f

time

freq. f

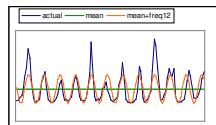
15-826 Copyright: C. Faloutsos (2006) 36





DFT: Amplitude spectrum

- excellent approximation, with only 2 frequencies!
- so what?
- A1: (lossy) compression
- A2: **pattern discovery**



15-826

Copyright: C. Faloutsos (2006)

43



DFT: Amplitude spectrum

- Let's see it in action!
- <http://www.dsptutor.freeuk.com/jsanalyser/FFTSpectrumAnalyser.html>
- plain sine
- phase shift
- two sine waves
- the 'chirp' function

15-826

Copyright: C. Faloutsos (2006)

44

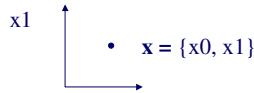


DFT: Parseval's theorem

$$\sum(x_t^2) = \sum(|X_f|^2)$$

Ie., DFT preserves the 'energy'

or, alternatively: it does an axis rotation:



15-826

Copyright: C. Faloutsos (2006)

45



DSP - Detailed outline

- DFT
 - what
 - why
 - how
- Arithmetic examples
 - properties / observations
 - DCT
 - 2-d DFT
 - Fast Fourier Transform (FFT)

15-826

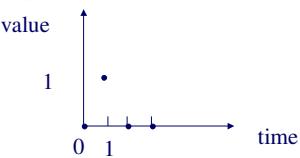
Copyright: C. Faloutsos (2006)

46



Arithmetic examples

- Impulse function: $\mathbf{x} = \{0, 1, 0, 0\}$ ($n = 4$)
- $X_0 = ?$



15-826

Copyright: C. Faloutsos (2006)

47



Arithmetic examples

- Impulse function: $\mathbf{x} = \{0, 1, 0, 0\}$ ($n = 4$)
- $X_0 = ?$
- A: $X_0 = 1/\sqrt{4} * 1 * \exp(-j 2 \pi 0/n) = 1/2$
- $X_1 = ?$
- $X_2 = ?$
- $X_3 = ?$

15-826

Copyright: C. Faloutsos (2006)

48



Arithmetic examples

- Impulse function: $\mathbf{x} = \{ 0, 1, 0, 0 \}$ ($n = 4$)
- $X_0=?$
- A: $X_0 = 1/\sqrt{4} * 1 * \exp(-j 2 \pi 0 / n) = 1/2$
- $X_1 = -1/2 j$
- $X_2 = -1/2$
- $X_3 = +1/2 j$
- Q: does the ‘symmetry’ property hold?

15-826

Copyright: C. Faloutsos (2006)

49



Arithmetic examples

- Impulse function: $\mathbf{x} = \{ 0, 1, 0, 0 \}$ ($n = 4$)
- $X_0=?$
- A: $X_0 = 1/\sqrt{4} * 1 * \exp(-j 2 \pi 0 / n) = 1/2$
- $X_1 = -1/2 j$
- $X_2 = -1/2$
- $X_3 = +1/2 j$
- Q: does the ‘symmetry’ property hold?
- A: Yes (of course)

15-826

Copyright: C. Faloutsos (2006)

50



Arithmetic examples

- Impulse function: $\mathbf{x} = \{ 0, 1, 0, 0 \}$ ($n = 4$)
- $X_0=?$
- A: $X_0 = 1/\sqrt{4} * 1 * \exp(-j 2 \pi 0 / n) = 1/2$
- $X_1 = -1/2 j$
- $X_2 = -1/2$
- $X_3 = +1/2 j$
- Q: check Parseval’s theorem

15-826

Copyright: C. Faloutsos (2006)

51



Arithmetic examples

- Impulse function: $\mathbf{x} = \{ 0, 1, 0, 0 \}$ ($n = 4$)
- $X_0=?$
- A: $X_0 = 1/\sqrt{4} * 1 * \exp(-j 2 \pi 0 / n) = 1/2$
- $X_1 = -1/2 j$
- $X_2 = -1/2$
- $X_3 = +1/2 j$
- Q: (Amplitude) spectrum?

15-826

Copyright: C. Faloutsos (2006)

52



Arithmetic examples

- Impulse function: $\mathbf{x} = \{ 0, 1, 0, 0 \}$ ($n = 4$)
- $X_0=?$
- A: $X_0 = 1/\sqrt{4} * 1 * \exp(-j 2 \pi 0 / n) = 1/2$
- $X_1 = -1/2 j$
- $X_2 = -1/2$
- $X_3 = +1/2 j$
- Q: (Amplitude) spectrum?
- A: FLAT!

15-826

Copyright: C. Faloutsos (2006)

53



Arithmetic examples

- Q: What does this mean?

15-826

Copyright: C. Faloutsos (2006)

54



Arithmetic examples

- Q: What does this mean?
- A: All frequencies are equally important ->
 - we need n numbers in the frequency domain to represent just one non-zero number in the time domain!
 - “frequency leak”

15-826

Copyright: C. Faloutsos (2006)

55



DSP - Detailed outline

- DFT
 - what
 - why
 - how
 - Arithmetic examples
 - properties / observations
 - DCT
 - 2-d DFT
 - Fast Fourier Transform (FFT)

15-826

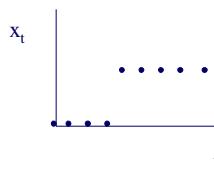
Copyright: C. Faloutsos (2006)

56



Observations

- DFT of ‘step’ function:
 $x = \{ 0, 0, \dots, 0, 1, 1, \dots 1 \}$



15-826

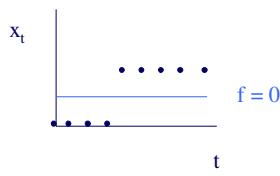
Copyright: C. Faloutsos (2006)

57



Observations

- DFT of ‘step’ function:
 $x = \{ 0, 0, \dots, 0, 1, 1, \dots 1 \}$



15-826

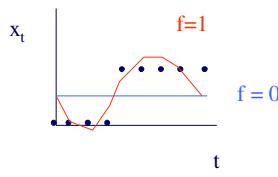
Copyright: C. Faloutsos (2006)

58



Observations

- DFT of ‘step’ function:
 $x = \{ 0, 0, \dots, 0, 1, 1, \dots 1 \}$



15-826

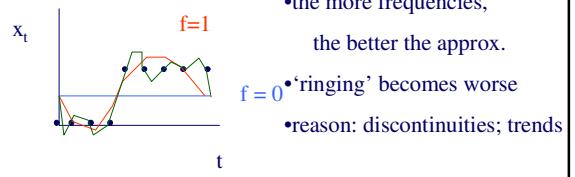
Copyright: C. Faloutsos (2006)

59



Observations

- DFT of ‘step’ function:
 $x = \{ 0, 0, \dots, 0, 1, 1, \dots 1 \}$



15-826

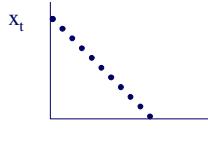
Copyright: C. Faloutsos (2006)

60



Observations

- Ringing for trends: because DFT ‘sub-consciously’ replicates the signal



15-826

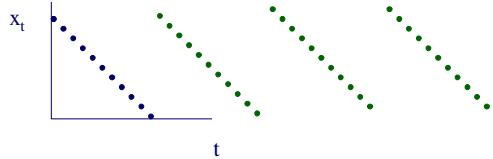
Copyright: C. Faloutsos (2006)

61



Observations

- Ringing for trends: because DFT ‘sub-consciously’ replicates the signal



15-826

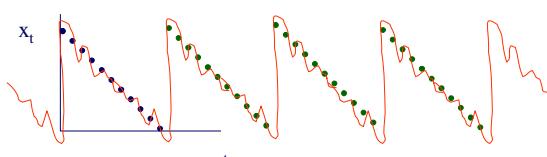
Copyright: C. Faloutsos (2006)

62



Observations

- Ringing for trends: because DFT ‘sub-consciously’ replicates the signal



15-826

Copyright: C. Faloutsos (2006)

63



Observations

- Q: DFT of a sinusoid, eg.
 $x_t = 3 \sin(2\pi/4t)$
 $(t = 0, \dots, 3)$
- Q: $X_0 = ?$
- Q: $X_1 = ?$
- Q: $X_2 = ?$
- Q: $X_3 = ?$

15-826

Copyright: C. Faloutsos (2006)

64



Observations

- Q: DFT of a sinusoid, eg.
 $x_t = 3 \sin(2\pi/4t)$
 $(t = 0, \dots, 3)$
- Q: $X_0 = 0$
- Q: $X_1 = -3j$
- Q: $X_2 = 0$
- Q: $X_3 = 3j$
- check ‘symmetry’
- check Parseval

15-826

Copyright: C. Faloutsos (2006)

65



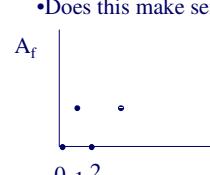
Observations

- Q: DFT of a sinusoid, eg.
 $x_t = 3 \sin(2\pi/4t)$
 $(t = 0, \dots, 3)$
- Q: $X_0 = 0$
- Q: $X_1 = -3j$
- Q: $X_2 = 0$
- Q: $X_3 = 3j$
- Does this make sense?

15-826

Copyright: C. Faloutsos (2006)

66





Property

- Shifting x in time does NOT change the amplitude spectrum
- eg., $x = \{ 0 \ 0 \ 0 \ 1 \}$ and $x' = \{ 0 \ 1 \ 0 \ 0 \}$: same (flat) amplitude spectrum
- (only the phase spectrum changes)
- Useful property when we search for patterns that may ‘slide’

15-826

Copyright: C. Faloutsos (2006)

67



DSP - Detailed outline

- DFT
 - what
 - why
 - how
 - Arithmetic examples
 - properties / observations
 - DCT
 - 2-d DFT
 - Fast Fourier Transform (FFT)

15-826

Copyright: C. Faloutsos (2006)

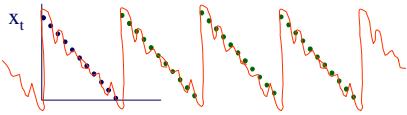
68



DCT

Discrete Cosine Transform

- motivation#1: DFT gives complex numbers
- motivation#2: how to avoid the ‘frequency leak’ of DFT on trends?



15-826

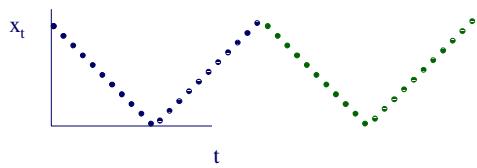
Copyright: C. Faloutsos (2006)

69



DCT

- brilliant solution to both problems: mirror the sequence, do DFT, and drop the redundant entries!



15-826

Copyright: C. Faloutsos (2006)

70



DCT

- (see Numerical Recipes for exact formulas)

15-826

Copyright: C. Faloutsos (2006)

71



DCT - properties

- it gives real numbers as the result
- it has no problems with trends
- it is very good when x_t and $x_{(t+1)}$ are correlated

(thus, is used in JPEG, for image compression)

15-826

Copyright: C. Faloutsos (2006)

72



DSP - Detailed outline

- DFT
 - what
 - why
 - how
 - Arithmetic examples
 - properties / observations
 - DCT
 - 2-d DFT
 - Fast Fourier Transform (FFT)

15-826

Copyright: C. Faloutsos (2006)

73



2-d DFT

- Definition:

$$X_{f_1, f_2} = \frac{1}{\sqrt{n_1}} \frac{1}{\sqrt{n_2}} \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} a_{i_1, i_2} \exp(-2\pi j i_1 f_1 / n_1) \exp(-2\pi j i_2 f_2 / n_2)$$

15-826

Copyright: C. Faloutsos (2006)

74



2-d DFT

- Intuition:

do 1-d DFT on each row

and then
1-d DFT
on each
column

15-826

Copyright: C. Faloutsos (2006)

75



2-d DFT

- Quiz: how do the basis functions look like?
 - for $f_1=f_2=0$
 - for $f_1=1, f_2=0$
 - for $f_1=1, f_2=1$

15-826

Copyright: C. Faloutsos (2006)

76



2-d DFT

- Quiz: how do the basis functions look like?
- for $f_1=f_2=0$ flat
- for $f_1=1, f_2=0$ wave on x; flat on y
- for $f_1=1, f_2=1$ ~ egg-carton

15-826

Copyright: C. Faloutsos (2006)

77



DSP - Detailed outline

- DFT
 - what
 - why
 - how
 - Arithmetic examples
 - properties / observations
 - DCT
 - 2-d DFT
 - Fast Fourier Transform (FFT)

15-826

Copyright: C. Faloutsos (2006)

78



FFT

- What is the complexity of DFT?

$$X_f = 1/\sqrt{n} \sum_{t=0}^{n-1} x_t * \exp(-j2\pi tf/n)$$

15-826

Copyright: C. Faloutsos (2006)

79



FFT

- What is the complexity of DFT?

$$X_f = 1/\sqrt{n} \sum_{t=0}^{n-1} x_t * \exp(-j2\pi tf/n)$$

- A: Naively, $O(n^2)$

15-826

Copyright: C. Faloutsos (2006)

80



FFT

- However, if n is a power of 2 (or a number with many divisors), we can make it $O(n \log n)$

Main idea: if we know the DFT of the odd time-ticks, and of the even time-ticks, we can quickly compute the whole DFT

Details: in Num. Recipes

15-826

Copyright: C. Faloutsos (2006)

81



DFT - Conclusions

- It spots periodicities (with the ‘amplitude spectrum’)
- can be quickly computed ($O(n \log n)$), thanks to the FFT algorithm.
- standard tool in signal processing (speech, image etc signals)

15-826

Copyright: C. Faloutsos (2006)

82



Detailed outline

- primary key indexing
- ..
- multimedia
- Digital Signal Processing (DSP) tools
 - Discrete Fourier Transform (DFT)
 - Discrete Wavelet Transform (DWT)



15-826

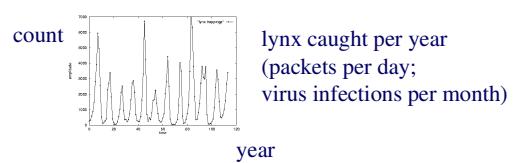
Copyright: C. Faloutsos (2006)

83



Reminder: Problem:

Goal: given a signal (eg., #packets over time)
 Find: patterns, periodicities, and/or compress



15-826

Copyright: C. Faloutsos (2006)

84

Wavelets - DWT

- DFT is great - but, how about compressing a spike?

value

time

15-826 Copyright: C. Faloutsos (2006) 85

Wavelets - DWT

- DFT is great - but, how about compressing a spike?
- A: Terrible - all DFT coefficients needed!

value

time

15-826 Copyright: C. Faloutsos (2006)

Ampl.

Freq.

86

Wavelets - DWT

- DFT is great - but, how about compressing a spike?
- A: Terrible - all DFT coefficients needed!

value

time

15-826 Copyright: C. Faloutsos (2006) 87

Wavelets - DWT

- Similarly, DFT suffers on short-duration waves (eg., baritone, silence, soprano)

value

time

15-826 Copyright: C. Faloutsos (2006) 88

Wavelets - DWT

- Solution#1: Short window Fourier transform (SWFT)
- But: how short should be the window?

freq

value

time

15-826 Copyright: C. Faloutsos (2006) 89

Wavelets - DWT

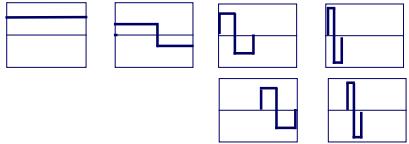
- Answer: **multiple** window sizes! -> DWT

Time domain	DFT	SWFT	DWT
freq			time

15-826 Copyright: C. Faloutsos (2006) 90

Haar Wavelets

- subtract sum of left half from right half
- repeat recursively for quarters, eighth-ths, ...



15-826

Copyright: C. Faloutsos (2006)

91

Wavelets - construction

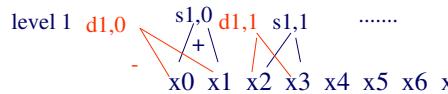
 $x_0 \ x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7$

15-826

Copyright: C. Faloutsos (2006)

92

Wavelets - construction

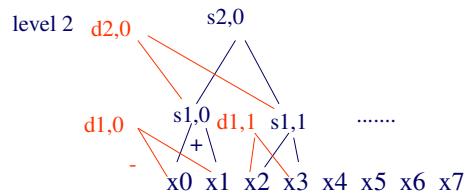


15-826

Copyright: C. Faloutsos (2006)

93

Wavelets - construction

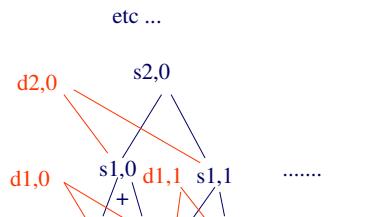


15-826

Copyright: C. Faloutsos (2006)

94

Wavelets - construction



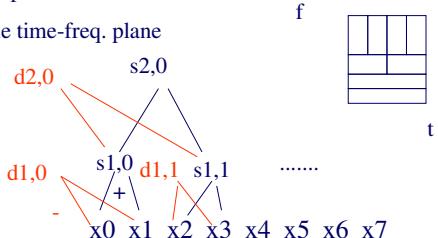
15-826

Copyright: C. Faloutsos (2006)

95

Wavelets - construction

Q: map each coefficient
on the time-freq. plane



15-826

Copyright: C. Faloutsos (2006)

96

Wavelets - construction

Q: map each coefficient on the time-freq. plane

15-826 Copyright: C. Faloutsos (2006) 97

Haar wavelets - code

```
#!/usr/bin/perl
# expects a file with numbers
# and prints the dwt transform
# The number of time-ticks should be a power of 2
# USAGE
#   haarr.pl <filename>
my $len = scalar(@vals);
my $shift = int($len/2);
while($shift >= 1) {
    for my $i(0..$shift-1) {
        $diff[$i] = ($vals[2*$i]-$vals[2*$i+1])/sqrt(2);
        print "u", $diff[$i];
        $smooth[$i] = ($vals[2*$i]+$vals[2*$i+1])/sqrt(2);
    }
    print "\n";
    @vals = (@vals, split);
    $shift = int($shift/2);
}
print "u", $vals[0], "\n"; # the final, smooth component
```

15-826 Copyright: C. Faloutsos (2006) 98

Wavelets - construction

Observation1:

- ‘+’ can be some weighted addition
- ‘-’ is the corresponding weighted difference
(‘Quadrature mirror filters’)

Observation2: unlike DFT/DCT,
there are *many* wavelet bases: Haar, Daubechies-4, Daubechies-6, Coifman, Morlet, Gabor, ...

15-826 Copyright: C. Faloutsos (2006) 99

Wavelets - how do they look like?

- E.g., Daubechies-4

15-826 Copyright: C. Faloutsos (2006) 100

Wavelets - how do they look like?

- E.g., Daubechies-4

15-826 Copyright: C. Faloutsos (2006) 101

Wavelets - how do they look like?

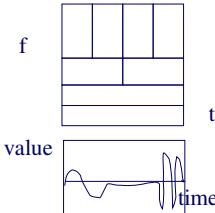
- E.g., Daubechies-4

15-826 Copyright: C. Faloutsos (2006) 102



Wavelets - Drill#1:

- Q: baritone/silence/soprano - DWT?



15-826

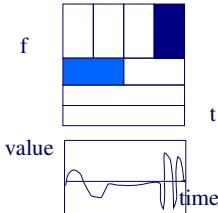
Copyright: C. Faloutsos (2006)

103



Wavelets - Drill#1:

- Q: baritone/soprano - DWT?



15-826

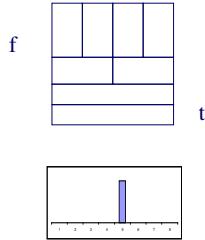
Copyright: C. Faloutsos (2006)

104



Wavelets - Drill#2:

- Q: spike - DWT?



15-826

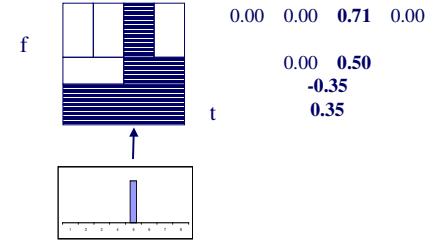
Copyright: C. Faloutsos (2006)

105



Wavelets - Drill#2:

- Q: spike - DWT?



15-826

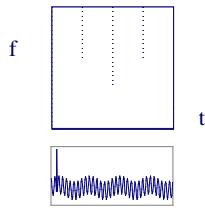
Copyright: C. Faloutsos (2006)

106



Wavelets - Drill#3:

- Q: weekly + daily periodicity, + spike - DWT?



15-826

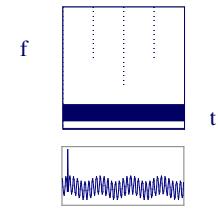
Copyright: C. Faloutsos (2006)

107



Wavelets - Drill#3:

- Q: weekly + daily periodicity, + spike - DWT?



15-826

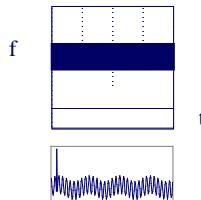
Copyright: C. Faloutsos (2006)

108



Wavelets - Drill#3:

- Q: weekly + **daily** periodicity, + spike - DWT?



15-826

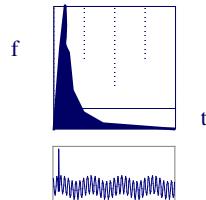
Copyright: C. Faloutsos (2006)

109



Wavelets - Drill#3:

- Q: weekly + daily periodicity, + **spike** - DWT?



15-826

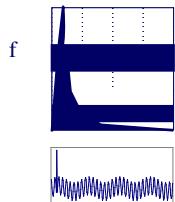
Copyright: C. Faloutsos (2006)

110



Wavelets - Drill#3:

- Q: weekly + daily periodicity, + spike - DWT?



15-826

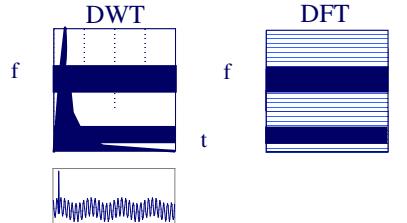
Copyright: C. Faloutsos (2006)

111



Wavelets - Drill#3:

- Q: DFT?



15-826

Copyright: C. Faloutsos (2006)

112



Wavelets - Drill:

Let's see it live:

<http://monet.me.ic.ac.uk/people/gavin/java/waveletDemos.html>

- delta; cosine; cosine2; chirp
- Haar vs Daubechies-4, -6, etc
- OR
- <http://www-dsp.rice.edu/~harry/class/wavelet/wavejava.html>

15-826

Copyright: C. Faloutsos (2006)

113



Wavelets - k-dimensions?

- easily defined for any dimensionality (like DFT, DCT)

15-826

Copyright: C. Faloutsos (2006)

114



Advantages of Wavelets

- Better compression (better RMSE with same number of coefficients)
- closely related to the processing of the mammalian eye and ear
- Good for progressive transmission
- handle spikes well
- usually, fast to compute ($O(n)!$)

15-826

Copyright: C. Faloutsos (2006)

115



Overall Conclusions

- DFT, DCT spot periodicities
- DWT : multi-resolution - matches processing of mammalian ear/eye better
- All three: powerful tools for compression, pattern detection in real signals
- All three: included in math packages (matlab, ...)

15-826

Copyright: C. Faloutsos (2006)

116



Resources

- Numerical Recipes in C: great description, intuition and code for all three tools
- *xwpl*: open source wavelet package from Yale, with excellent GUI.

15-826

Copyright: C. Faloutsos (2006)

117



Resources (cont'd)

- <http://www.dsptutor.freeuk.com/janalyser/FFTSpectrumAnalyser.html> : Nice java applets
- <http://www.relisoft.com/freeware/freq.html> : voice frequency analyzer (needs microphone)

15-826

Copyright: C. Faloutsos (2006)

118



Resources (cont'd)

- <http://monet.me.ic.ac.uk/people/gavin/java/waveletDemos.html> : wavelets and scalograms

15-826

Copyright: C. Faloutsos (2006)

119