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15-826: Multimedia Databases and Data Mining

Lecture #11: Fractals - case studies Part III
(regions, quadtrees, knn queries)
C. Faloutsos

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Must-read Material

- Alberto Belussi and Christos Faloutsos, Estimating the Selectivity of Spatial Queries Using the `Correlation' Fractal Dimension Proc. of VLDB, p. 299-310, 1995

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Optional Material

Optional, but very useful: Manfred Schroeder *Fractals, Chaos, Power Laws: Minutes from an Infinite Paradise* W.H. Freeman and Company, 1991

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Optional

Outline

Goal: 'Find similar / interesting things'

- Intro to DB
- Indexing - similarity search
- Data Mining

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Indexing - Detailed outline

- primary key indexing
- secondary key / multi-key indexing
- spatial access methods
  - z-ordering
  - R-trees
  - misc
- fractals
  - intro
  - applications
- text

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Indexing - Detailed outline

- fractals
  - intro
  - applications
    - ✔ disk accesses for R-trees (range queries)
    - ✔ dimensionality reduction
    - ✔ selectivity in M-trees
    - ✔ dim. curse revisited
    - "fat fractals"
    - quad-tree analysis [Gaede+]
    - nn queries [Belussi+]

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'Fat' fractals & R-tree performance on region data

- Problem [Proietti+,'99]
- Given
  - N (# of data regions)
- estimate how many of them will qualify for the average range query (q1 x q2 x ... qE)

Of course, we need more info

Q: what?

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R-tree performance on region data

A: the distributions of their sizes

Q: do we also need some info about the locations?

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R-tree performance on region data

A: the distributions of their sizes

Q: do we also need some info about the locations?

A: no (not for range queries)

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R-tree performance on region data

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A: the distributions of their sizes

Q: what exactly would we need?

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R-tree performance on region data

Optional

A: the distributions of their sizes

Q: what exactly would we need?

A: for self-similar regions (~ ‘fat’ fractals), we just need the slope of the Korcak law! (and the total area) [Proietti+]

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More power laws: areas – Korcak’s law

Optional

Scandinavian lakes

Any pattern?

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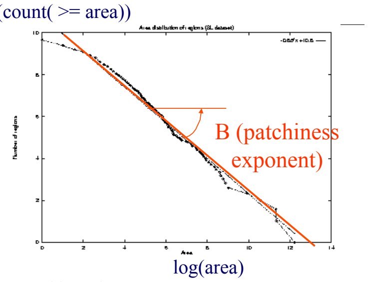
Optional

More power laws: areas – Korcak's law



Scandinavian lakes area vs complementary cumulative count (log-log axes)

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R-tree performance on regions

- Once we know 'B' (and the total area)
- we can second-guess the individual sizes
- and then apply the [Pagel+93] formula
- Bottom line:

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R-tree performance on regions

Dataset	N	A	B
LAKES	816	75,910	0.85
ISLANDS	470	136,893	0.60
REGIONS	757	190,526	0.70

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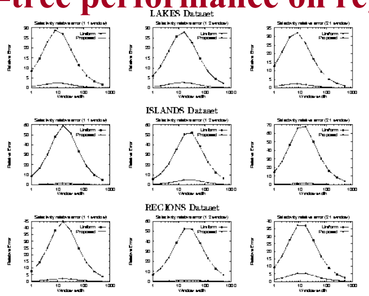
R-tree performance on regions

sel. error



query side

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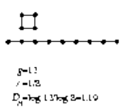
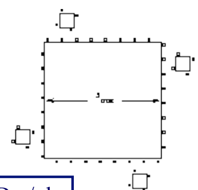
‘Fat’ fractals - observation





$\boxed{B = D_H / d}$

B: patchiness exp; d: embedding dim

D_H: Hausdorff of periphery

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‘Fat’ fractals - observation

Dataset	D_H	B	$D_H - 2B$
LAKES	1.78	0.85	0.08
ISLANDS	1.23	0.60	0.03
REGIONS	1.48	0.70	0.08
Aegean Island	1.08	0.52	0.04
Japan archipelago	1.19	0.59	0.01
Italy plains	1.32	0.63	0.06
Whole Earth	1.2	0.6	0
Cypress vegetation	0.62	1.23	0.01

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Optional

'Fat' fractals

- intuition behind $B = D_H / d$?

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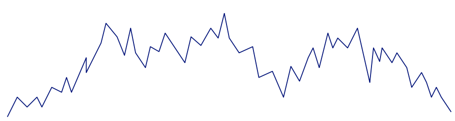
'Fat' fractals

- intuition behind $B = D_H / d$?
- A: consider 'flooding':



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'Fat' fractals

- intuition behind $B = D_H / d$?
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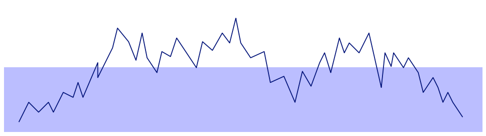

Optional

Conclusions

- 'Fat' fractals model regions well
- patchiness exp.: $B = D_H / d$
- can help us estimate selectivities

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Optional

Indexing - Detailed outline

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    - ✔dimensionality reduction
    - ✔selectivity in M-trees
    - ✔dim. curse revisited
    - ✔"fat fractals"
    - quad-tree analysis [Gaede+]
    - nn queries [Belussi+]

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Fractals and Quadtrees

- Problem: how many quadtree nodes will we need, to store a region in some level of approximation? [Gaede+96]

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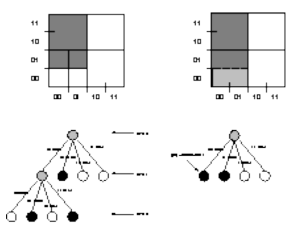
Fractals and Quadtrees

- I.e.:



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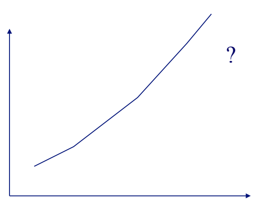
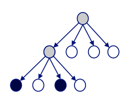
Fractals and Quadtrees

- I.e.:

# of quadtree
'blocks'
(= # gray nodes)





level of quadtree

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Optional

Fractals and Quadtrees

- Datasets:

Franconia

Brain Atlas

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Fractals and Quadtrees

- Hint:
  - assume that the boundary is self-similar, with a given fd
  - how will the quad-tree (oct-tree) look like?

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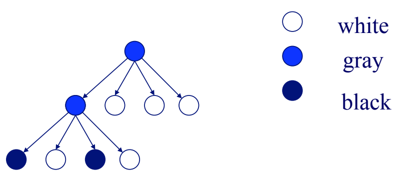
Fractals and Quadtrees



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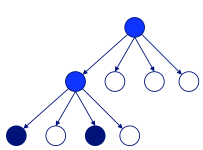
Fractals and Quadtrees

Let $p_g(i)$ the prob. to find a gray node at level i.

If self-similar, what can we say for $p_g(i)$?



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Fractals and Quadtrees

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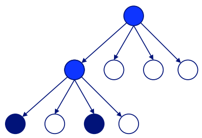

A: $p_g(i) = p_g$ = constant

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Fractals and Quadtrees

Assume only 'gray' and 'white' nodes (ie., no volume')

Assume that p_g is given - how many gray nodes at level i?

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Fractals and Quadtrees

Assume only 'gray' and 'white' nodes (ie., no volume')

Assume that p_g is given - how many gray nodes at level i?

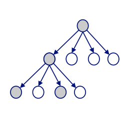
A: 1 at level 0;

$4*p_g$

$(4*p_g)*(4*p_g)$

...

$(4*p_g)^i$



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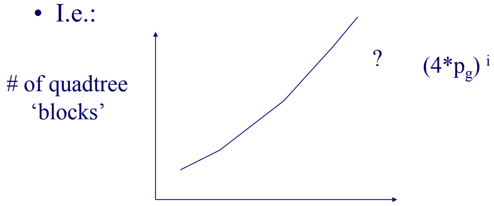

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Fractals and Quadtrees

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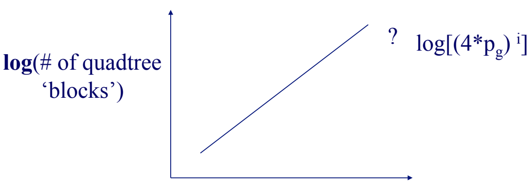

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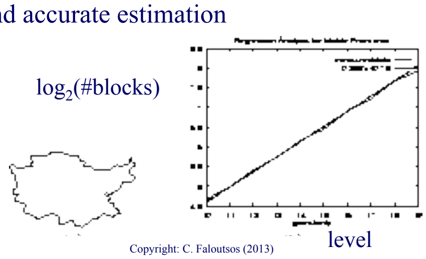
Fractals and Quadtrees

- Conclusion: Self-similarity leads to easy and accurate estimation



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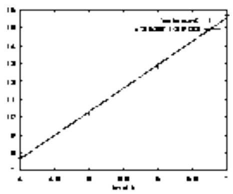
Fractals and Quadtrees

- Conclusion: Self-similarity leads to easy and accurate estimation

$\log_2$(#blocks)



level

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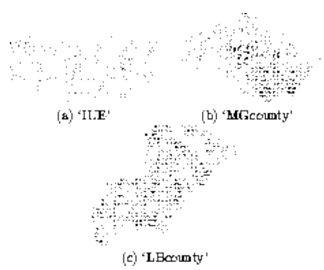
Fractals and Quadtrees



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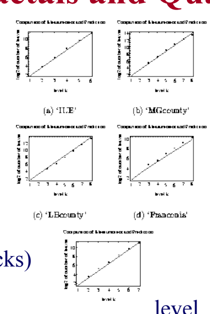
Fractals and Quadtrees



log(#blocks)

level

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Fractals and Quadtrees

- Final observation: relationship between p_g and fractal dimension?

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Fractals and Quadtrees

- Final observation: relationship between p_g and fractal dimension?
- A: very close:

  $(4*p_g)^i$ = # of gray nodes at level i =

  # of Hausdorff grid-cells of side $(1/2)^i = r$

  Eventually: $D_H = 2 + \log_2(p_g)$

  and, for E-d spaces: $\boxed{D_H = E + \log_2(p_g)}$

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Fractals and Quadtrees

for E-d spaces: $D_H = E + \log_2(p_g)$

Sanity check:

- point in 2-d: $D_H = 0$ p_g = ??
- line in 2-d: $D_H = 1$ p_g = ??
- plane in 2-d: D_H=2 p_g = ??
- point in 3-d: $D_H = 0$ p_g = ??

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Fractals and Quadtrees

for E-d spaces: $D_H = E + \log_2(p_g)$

Sanity check:

- point in 2-d: $D_H = 0$ $p_g = 1/4$
- line in 2-d: $D_H = 1$ $p_g = 1/2$
- plane in 2-d: $D_H=2$ $p_g= 1$
- point in 3-d: $D_H = 0$ $p_g = 1/8$

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Fractals and Quadtrees

Final conclusions:

- self-similarity leads to estimates for # of z-values = # of quadtree/oct-tree blocks
- close dependence on the Hausdorff fractal dimension of the boundary

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    - nn queries [Belussi+]

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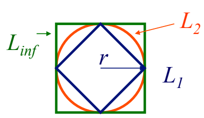
NN queries

- Q: in NN queries, what is the effect of the shape of the query region? [Belussi+95]



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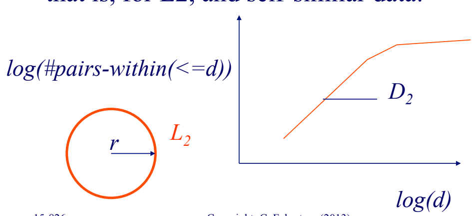
NN queries

- Q: in NN queries, what is the effect of the shape of the query region?
- that is, for L2, and self-similar data:



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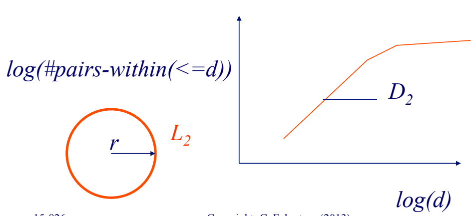
NN queries

- Q: What about L_1, L_{inf}?



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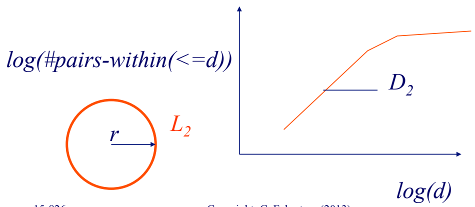
NN queries

- Q: What about L_1, L_{inf}?
- A: **Same slope**, different intercept



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NN queries

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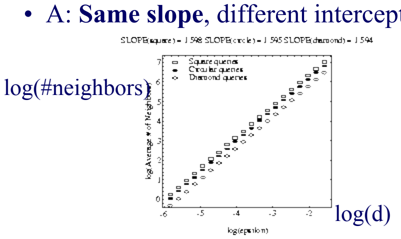

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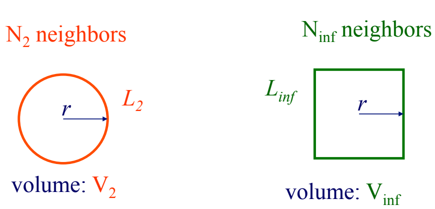
NN queries

- Q: what about the intercept? Ie., what can we say about N_2 and N_{inf}



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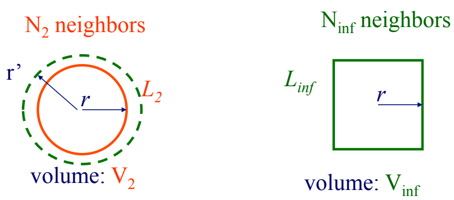
NN queries

- Consider sphere with volume V_{inf} and r' radius



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NN queries

- Consider sphere with volume V_{inf} and r' radius
- (r/r')^E $= V_2 / V_{inf}$
- (r/r')^$D_2 = N_2 / N_2'$
- $N_2' = N_{inf}$ (since shape does not matter)
- and finally:

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NN queries

(N_2 / N_{inf}) ^ $1/D_2 = (V_2 / V_{inf})$ ^ $1/E$

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NN queries

Conclusions: for self-similar datasets

- Avg # neighbors: grows like $\boxed{(distance)\text{^}D_2}$, regardless of query shape (circle, diamond, square, e.t.c.)

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    - nn queries [Belussi+]
  - Conclusions

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Fractals - overall conclusions

- self-similar datasets: appear often
- powerful tools: correlation integral, NCDF, rank-frequency plot
- intrinsic/fractal dimension helps in
  - estimations (selectivities, quadtrees, etc)
  - dim. reduction / dim. curse
- (later: can help in image compression...)

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References

1. Belussi, A. and C. Faloutsos (Sept. 1995). Estimating the Selectivity of Spatial Queries Using the `Correlation' Fractal Dimension. Proc. of VLDB, Zurich, Switzerland.
2. Faloutsos, C. and V. Gaede (Sept. 1996). Analysis of the z-ordering Method Using the Hausdorff Fractal Dimension. VLDB, Bombay, India.
3. Proietti, G. and C. Faloutsos (March 23-26, 1999). I/O complexity for range queries on region data stored using an R-tree. International Conference on Data Engineering (ICDE), Sydney, Australia.

