CMU SCS

15-826: Multimedia Databases and Data Mining

Lecture #8: Fractals - introduction

C. Faloutsos

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Must-read Material

- Christos Faloutsos and Ibrahim Kamel, *Beyond Uniformity and Independence: Analysis of R-trees Using the Concept of Fractal Dimension*, Proc. ACM SIGACT-SIGMOD-SIGART PODS, May 1994, pp. 4-13, Minneapolis, MN.

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Recommended Material

optional, but **very** useful:

- Manfred Schroeder *Fractals, Chaos, Power Laws: Minutes from an Infinite Paradise* W.H. Freeman and Company, 1991 (on reserve in the library)
  - Chapter 10: boxcounting method
  - Chapter 1: Sierpinski triangle

Outline

Goal: 'Find similar / interesting things'

- Intro to DB
- ➡ Indexing - similarity search
- Data Mining

Indexing - Detailed outline

- primary key indexing
- secondary key / multi-key indexing
- spatial access methods
  - z-ordering
  - R-trees
  - misc
- fractals
  - ➡ intro
  - applications
- text

Intro to fractals - outline

- ➡ Motivation – 3 problems / case studies
- Definition of fractals and power laws
- Solutions to posed problems
- More examples and tools
- Discussion - putting fractals to work!
- Conclusions – practitioner's guide
- Appendix: gory details - boxcounting plots

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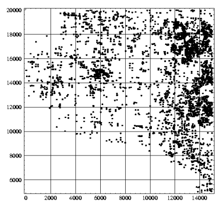
Problem #1: GIS - points



Road end-points of Montgomery county:

- •Q1: how many d.a. for an R-tree?
- •Q2 : distribution?
  - •not uniform
  - •not Gaussian
  - •no rules??

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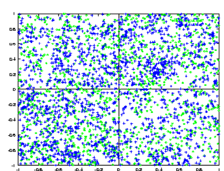
Problem #2 - spatial d.m.

Galaxies (Sloan Digital Sky Survey w/ B. Nichol)



- 'spiral' and 'elliptical' galaxies

(stores and households ...)

- patterns?

- attraction/repulsion?

- how many 'spi' within r from an 'ell'?

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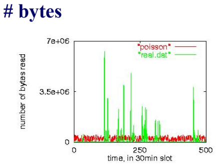
Problem #3: traffic

 # bytes



time

- disk trace (from HP - J. Wilkes); Web traffic - fit a model
- how many explosions to expect?
- queue length distr.?

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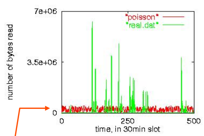
Problem #3: traffic

bytes



time

Poisson
indep.,
ident. distr

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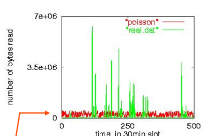
Problem #3: traffic

bytes



time

~~Poisson~~
indep.,
ident. distr

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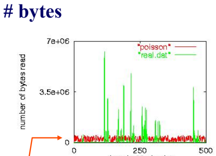
Problem #3: traffic

bytes



time

~~Poisson~~
~~indep.,~~
ident. distr

Q: Then, how to generate such bursty traffic?

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Common answer:

- Fractals / self-similarities / power laws
- Seminal works from Hilbert, Minkowski, Cantor, Mandelbrot, (Hausdorff, Lyapunov, Ken Wilson, …)

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Road map

- Motivation – 3 problems / case studies
- Definition of fractals and power laws
- Solutions to posed problems
- More examples and tools
- Discussion - putting fractals to work!
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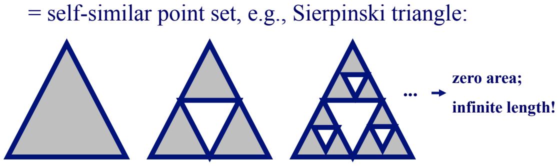
What is a fractal?

= self-similar point set, e.g., Sierpinski triangle:



Dimensionality??

Definitions (cont'd)

- Paradox: Infinite perimeter ; Zero area!
- 'dimensionality': between 1 and 2
- actually: Log(3)/Log(2) = 1.58...

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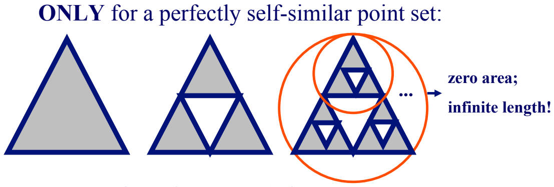
Dfn of fd:

ONLY for a perfectly self-similar point set:



=log(n)/log(f) = log(3)/log(2) = 1.58

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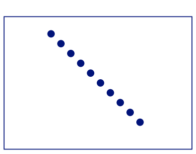
Intrinsic ('fractal') dimension

- Q: fractal dimension of a line?
- A: 1 (= log(2)/log(2)!)



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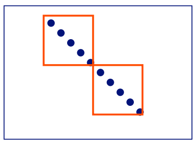

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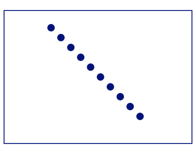
Intrinsic ('fractal') dimension

- Q: dfn for a given set of points?



x	y
5	1
4	2
3	3
2	4

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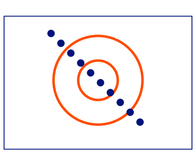
Intrinsic ('fractal') dimension

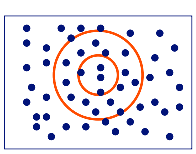
- Q: fractal dimension of a line?
- A: nn (<= r) ~ r^1

('power law': y=x^a)



- Q: fd of a plane?
- A: nn (<= r) ~ r^2

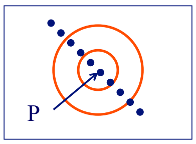
fd== slope of (log(nn) vs log (r))



EXPLANATIONS

Intrinsic ('fractal') dimension

- **Local** fractal dimension of point 'P'?
- A: nn_P (<= r) ~ r^1

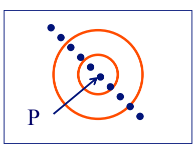


- If this equation holds for several values of r,
- Then, the **local fractal dimension** of point P:
- Local fd = exp = 1

EXPLANATIONS

Intrinsic ('fractal') dimension

- **Local** fractal dimension of point 'A'?
- A: nn_P (<= r) ~ r^1

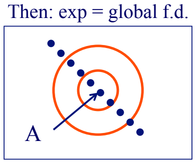


- If this is true for all points of the cloud
- Then the exponent is the **global** f.d.
- Or simply the f.d.

EXPLANATIONS

Intrinsic ('fractal') dimension

- **Global** fractal dimension?
- A: if
- sum_{all_P} [nn_P (<= r)] ~ r^1
- Then: exp = global f.d.



- If this is true for all points of the cloud
- Then the exponent is the **global** f.d.
- Or simply the f.d.

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EXPLANATIONS

Intrinsic ('fractal') dimension

- Algorithm, to estimate it?

Notice

- Sum_{all_P} *[* nn_P *(<=r)]* is exactly *tot#pairs(<=r)*

including 'mirror' pairs

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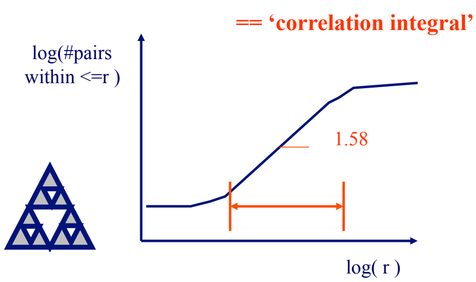
Sierpinsky triangle



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Observations:

- Euclidean objects have **integer** fractal dimensions
  - point: 0
  - lines and smooth curves: 1
  - smooth surfaces: 2
- fractal dimension -> roughness of the periphery

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Important properties

- fd = embedding dimension -> uniform pointset
- a point set may have several fd, depending on scale

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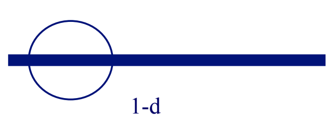

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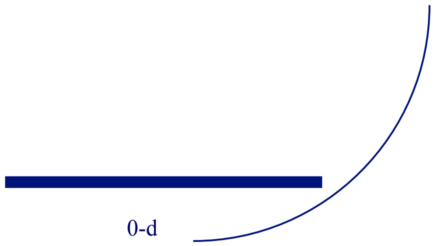
Important properties



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Road map

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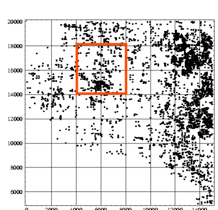
Problem #1: GIS points



Cross-roads of Montgomery county:

•any rules?

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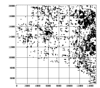
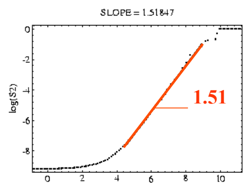
Solution #1



log(#pairs(within <= r))



log(r)

A: self-similarity ->

- <=> fractals
- <=> scale-free
- <=> power-laws (y=x^a, F=C*r^(-2))
- avg#neighbors(<= r) = r^D

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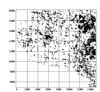
Solution #1



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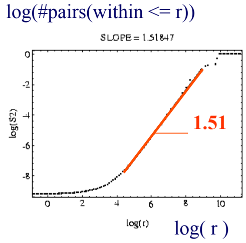

log(r)

A: self-similarity

- avg#neighbors(<= r) ~ r^(1.51)

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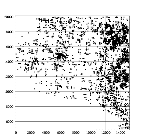
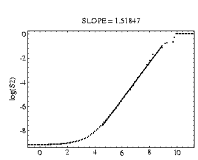
Examples:MG county

- Montgomery County of MD (road end-points)





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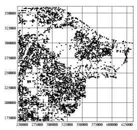
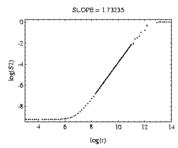
Examples:LB county

- Long Beach county of CA (road end-points)





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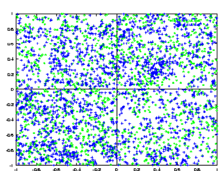
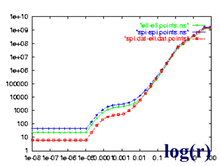
Solution#2: spatial d.m.

Galaxies (‘BOPS’ plot - [sigmod2000])





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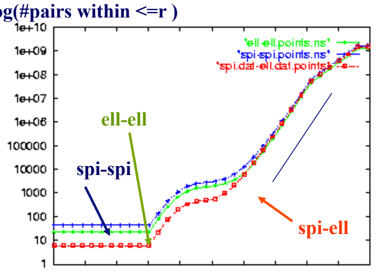
Solution#2: spatial d.m.



- 1.8 slope
- plateau!
- repulsion!

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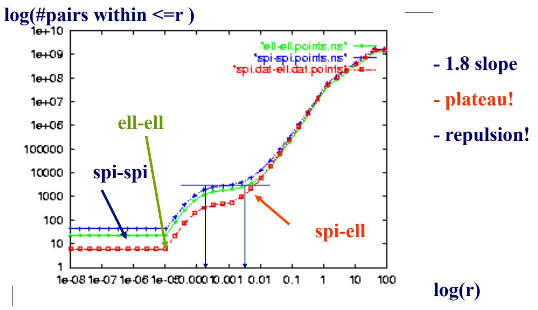
Spatial d.m.



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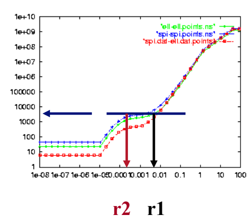
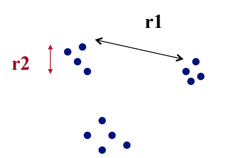
Spatial d.m.





Heuristic on choosing # of clusters

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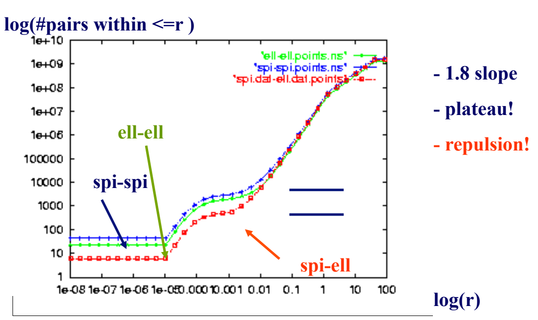
Spatial d.m.



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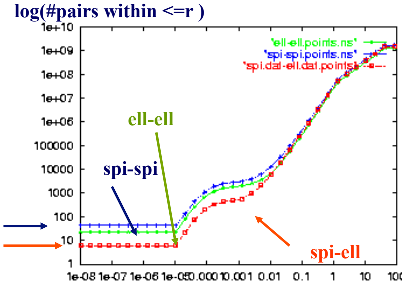
Spatial d.m.



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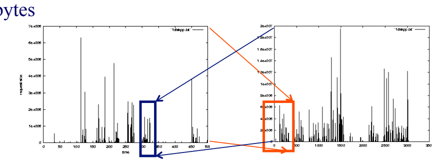
Solution #3: traffic

- disk traces: self-similar:

#bytes



time

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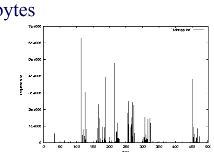
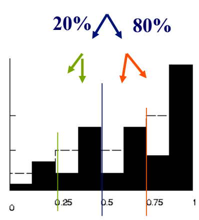
Solution #3: traffic

- disk traces (80-20 ‘law’ = ‘multifractal’)

#bytes



time



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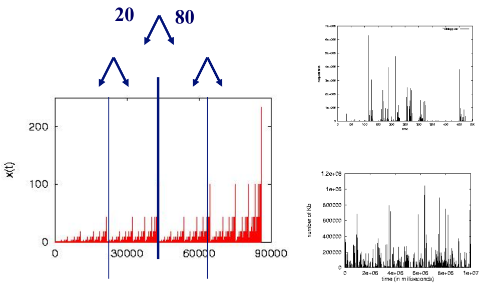
80-20 / multifractals



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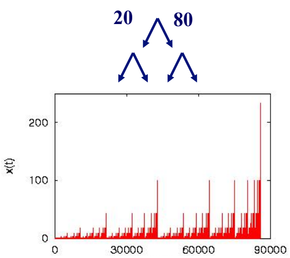
80-20 / multifractals



- p ; (1-p) in general
- **yes, there are dependencies**

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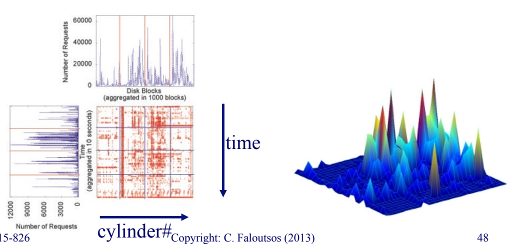
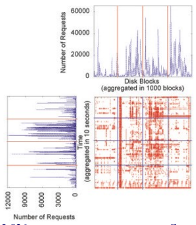
More on 80/20: PQRS

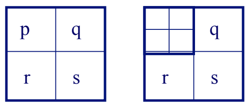
- Part of 'self-* storage' project [Wang+'02]



More on 80/20: PQRS

- Part of 'self-* storage' project [Wang+'02]





Solution#3: traffic

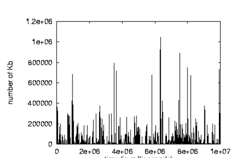
Clarification:

- fractal: a set of points that is self-similar
- multifractal: a probability density function that is self-similar

Many other time-sequences are bursty/ clustered: (such as?)

Example:

- network traffic



http://repository.cs.vt.edu/lbl-conn-7.tar.Z

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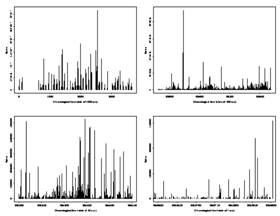
Web traffic

- [Crovella Bestavros, SIGMETRICS'96]

1000 sec; 100sec
10sec; 1sec



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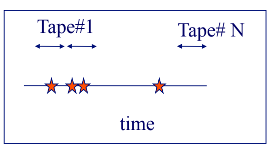
Tape accesses



tapes needed, to retrieve *n* records?

(# days down, due to failures / hurricanes / communication noise...)

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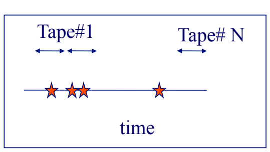
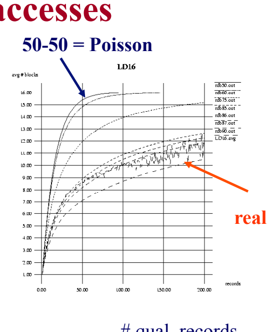
Tape accesses





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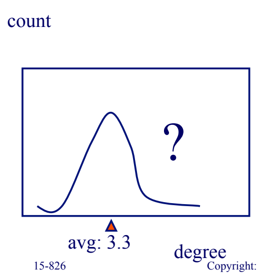
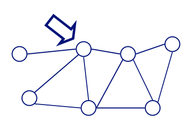
A counter-intuitive example



- avg degree is, say 3.3
- pick a node at random – guess its degree, exactly (-> "mode")



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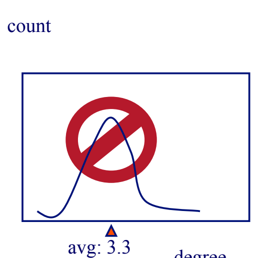
A counter-intuitive example



- avg degree is, say 3.3
- pick a node at random – guess its degree, exactly (-> "mode")
- A: 1!!

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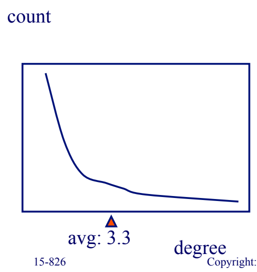
A counter-intuitive example



- avg degree is, say 3.3
- pick a node at random - what is the degree you expect it to have?
- A: 1!!
- A': very skewed distr.
- Corollary: **the mean is meaningless**!
- (and std -> infinity (!))

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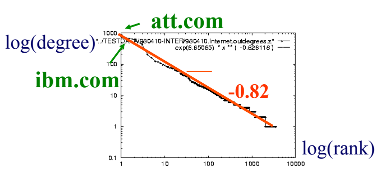
Rank exponent *R*

- Power law in the degree distribution [SIGCOMM99]

internet domains



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More tools

- Zipf's law
- Korcak's law / "fat fractals"

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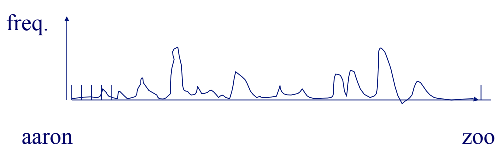
A famous power law: Zipf's law

- Q: vocabulary word frequency in a document - any pattern?



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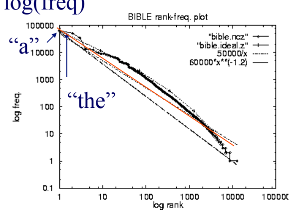
A famous power law: Zipf's law

log(freq)



log(rank)

- Bible - **rank** vs **frequency** (log-log)

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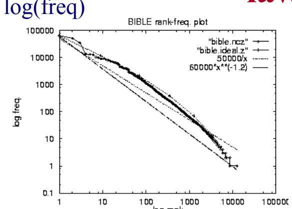
A famous power law: Zipf's law

log(freq)



log(rank)

- Bible - rank vs frequency (log-log)
- similarly, in **many other** languages; for customers and sales volume; city populations etc etc

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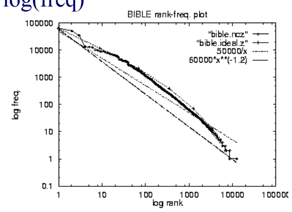
A famous power law: Zipf's law

log(freq)



•Zipf distr:

freq = 1/ rank

•generalized Zipf:

freq = 1 / (rank)^a

log(rank)

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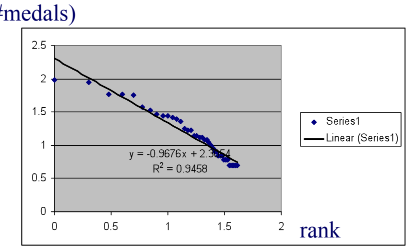
Olympic medals (Sydney):

log(#medals)



rank

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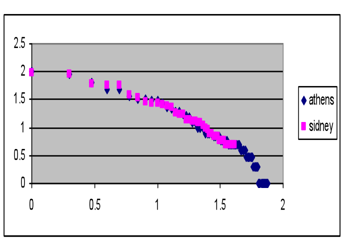
Olympic medals (Sydney'00, Athens'04):

log(#medals)



log(rank)

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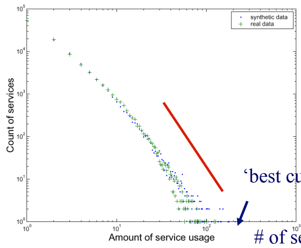
TELCO data

count of customers



'best customer'

# of service units

Count-frequency plot of real and synthetic data

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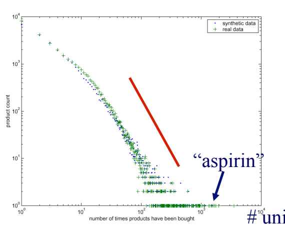
SALES data – store#96

count of products



"aspirin"

units sold

Count-frequency plot for store no. 96.

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More power laws: areas – Korcak's law

Scandinavian lakes

Any pattern?

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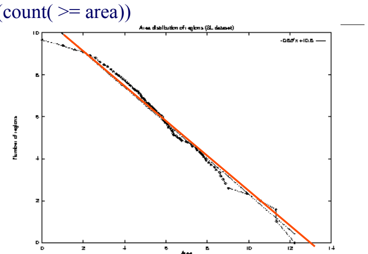
More power laws: areas – Korcak's law

log(count(>= area))



log(area)

Scandinavian lakes area vs complementary cumulative count (log-log axes)

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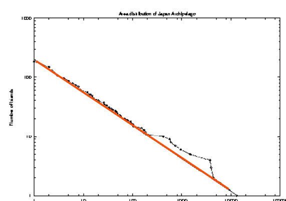
More power laws: Korcak

log(count(>= area))



log(area)

Japan islands;

area vs cumulative count (log-log axes)

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(Korcak's law: Aegean islands)

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Korcak's law & "fat fractals"

How to generate such regions?

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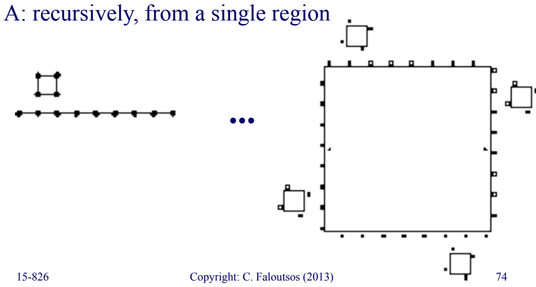
Korcak's law & "fat fractals"

Q: How to generate such regions?
A: recursively, from a single region



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so far we've seen:

- concepts:
  - fractals, multifractals and fat fractals
- tools:
  - correlation integral (= pair-count plot)
  - rank/frequency plot (Zipf's law)
  - CCDF (Korcak's law)

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so far we've seen:

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same info

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Road map

- Motivation – 3 problems / case studies
- Definition of fractals and power laws
- Solutions to posed problems
- More tools and **examples**
- Discussion - putting fractals to work!
- Conclusions – practitioner's guide
- Appendix: gory details - boxcounting plots

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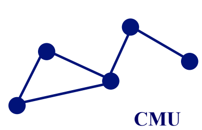
Other applications: Internet

- How does the internet look like?



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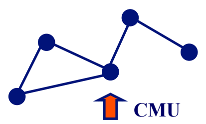
Other applications: Internet

- How does the internet look like?
- Internet routers: how many neighbors within *h* hops?



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(reminder: our tool-box:)

- concepts:
  - fractals, multifractals and fat fractals
- tools:
  - correlation integral (= pair-count plot)
  - rank/frequency plot (Zipf's law)
  - CCDF (Korcak's law)

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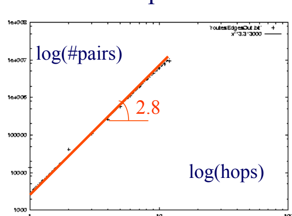
Internet topology

- Internet routers: how many neighbors within *h* hops?



Reachability function: number of neighbors within r hops, vs r (log-log).

Mbone routers, 1995

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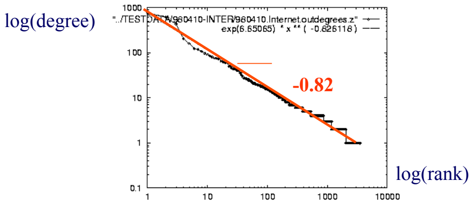
More power laws on the Internet



degree vs rank, for Internet domains (log-log) [sigcomm99]

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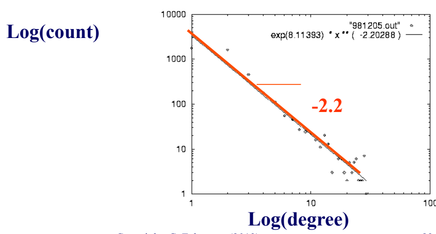
More power laws - internet

- pdf of degrees: (slope: 2.2)



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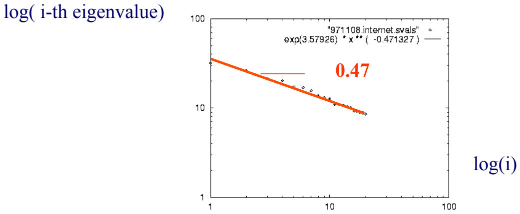
Even more power laws on the Internet



Scree plot for Internet domains (log-log) [sigcomm99]

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Fractals & power laws:

appear in numerous settings:

- **medical**
- geographical / geological
- social
- computer-system related

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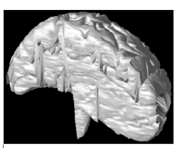
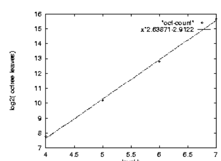
More apps: Brain scans

- Oct-trees; brain-scans

Log(#octants)





octree levels

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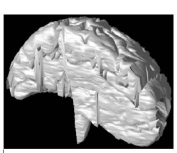
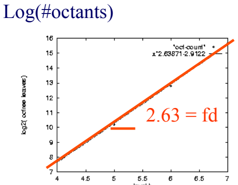
More apps: Brain scans

- Oct-trees; brain-scans

Log(#octants)





octree levels

More apps: Medical images

[Burdett et al, SPIE ‘93]:

- benign tumors: fd ~ 2.37
- malignant: fd ~ 2.56

More fractals:

- cardiovascular system: 3 (!)
- lungs: 2.9

Fractals & power laws:

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- **geographical / geological**
- social
- computer-system related

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More fractals:

- Coastlines: 1.2-1.58

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More fractals:

- the fractal dimension for the Amazon river is 1.85 (Nile: 1.4)

[ems.gphys.unc.edu/nonlinear/fractals/examples.html]

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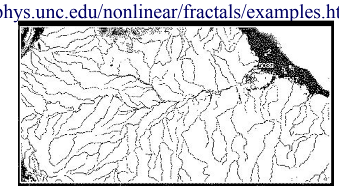

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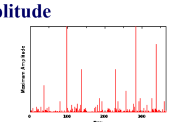
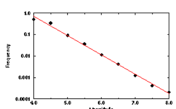
More power laws

- Energy of earthquakes (Gutenberg-Richter law) [simscience.org]





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Fractals & power laws:

appear in numerous settings:

- medical
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- **social**
- computer-system related

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More fractals:

stock prices (LYCOS) - random walks: 1.5

1 year

2 years

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Even more power laws:

- Income distribution (Pareto's law)
- size of firms
- publication counts (Lotka's law)

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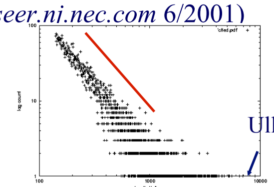
Even more power laws:

library science (Lotka's law of publication count); and citation counts: (*citeseer.ni.nec.com* 6/2001)

log(count)



Ullman

log(#citations)

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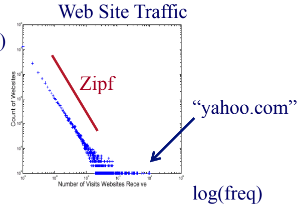
Even more power laws:

- web hit counts [w/ A. Montgomery]



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- social
- **computer-system related**

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Power laws, cont'd

- In- and out-degree distribution of web sites [Barabasi], [IBM-CLEVER]

from [Ravi Kumar, Prabhakar Raghavan, Sridhar Rajagopalan, Andrew Tomkins]

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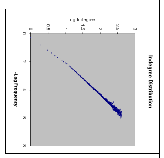
Power laws, cont'd

- In- and out-degree distribution of web sites [Barabasi], [IBM-CLEVER]

log(freq)



log indegree

from [Ravi Kumar, Prabhakar Raghavan, Sridhar Rajagopalan, Andrew Tomkins]

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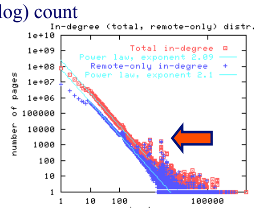
"Foiled by power law"

- [Broder+, WWW'00]

(log) count



(log) in-degree

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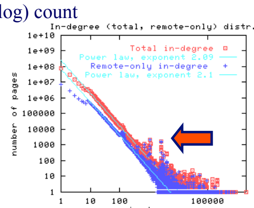
"Foiled by power law"

- [Broder+, WWW'00]

(log) count



"The anomalous bump at 120 on the *x*-axis is due a large clique formed by a single spammer"

(log) in-degree

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Power laws, cont'd

- In- and out-degree distribution of web sites [Barabasi], [IBM-CLEVER]
- length of file transfers [Crovella+Bestavros '96]
- duration of UNIX jobs [Harchol-Balter]

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Even more power laws:

- Distribution of UNIX file sizes
- web hit counts [Huberman]

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What else can they solve?

- ✓ separability [KDD'02]
- forecasting [CIKM'02]
- dimensionality reduction [SBBD'00]
- non-linear axis scaling [KDD'02]
- ✓ disk trace modeling [Wang+'02]
- selectivity of spatial/multimedia queries [PODS'94, VLDB'95, ICDE'00]
- ...

Settings for fractals:

Points; areas (-> fat fractals), eg:

Settings for fractals:

Points; areas, eg:

- cities/stores/hospitals, over earth's surface
- time-stamps of events (customer arrivals, packet losses, criminal actions) over time
- regions (sales areas, islands, patches of habitats) over space

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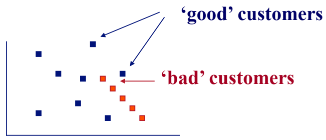
Settings for fractals:

- customer feature vectors (age, income, frequency of visits, amount of sales per visit)



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Some uses of fractals:

- Detect non-existence of rules (if points are uniform)
- Detect non-homogeneous regions (eg., legal login time-stamps may have different fd than intruders')
- Estimate number of neighbors / customers / competitors within a radius

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Multi-Fractals

Setting: points or objects, w/ some value, eg:

- cities w/ populations
- positions on earth and amount of gold/water/oil underneath
- product ids and sales per product
- people and their salaries
- months and count of accidents

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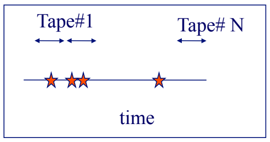
Use of multifractals:

- Estimate tape/disk accesses
  - *how many of the 100 tapes contain my 50 phonecall records?*
  - *how many days without an accident?*



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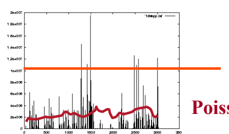
Use of multifractals

- how often do we exceed the threshold?

#bytes



time

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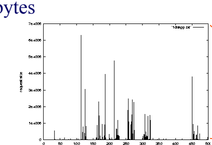
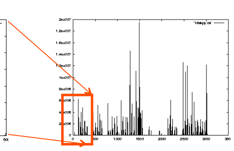
Use of multifractals cont'd

- Extrapolations for/from samples

#bytes





time

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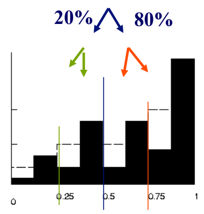
Use of multifractals cont'd

- *How many distinct products account for 90% of the sales?*



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Conclusions

- Real data often **disobey** textbook assumptions (Gaussian, Poisson, uniformity, independence)

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Conclusions - cont'd

Self-similarity & power laws: appear in **many** cases

Bad news:
lead to skewed distributions
(no Gaussian, Poisson,
uniformity, independence,
mean, variance)

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Conclusions - cont'd

Self-similarity & power laws: appear in **many** cases

Bad news:
lead to skewed distributions
(no Gaussian, Poisson,
uniformity, X independence,
mean, variance)

Good news: 

- 'correlation integral' for separability
- rank/frequency plots
- 80-20 (multifractals)
- (Hurst exponent,
- strange attractors,
- renormalization theory,
- ++)

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Conclusions

- **tool#1: (for points) 'correlation integral'**: (#pairs within <= r) vs (distance r)
- **tool#2: (for categorical values) rank-frequency** plot (a'la Zipf)
- **tool#3: (for numerical values) CCDF:** Complementary cumulative distr. function (#of elements with value >= a)

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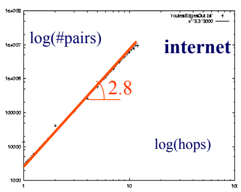
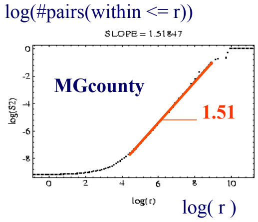
Practitioner’s guide:

- **tool#1**: #pairs vs distance, for a **set of objects**, with a distance function (slope = intrinsic dimensionality)





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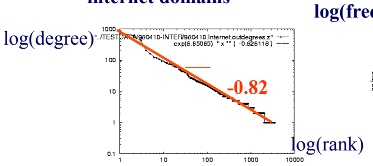
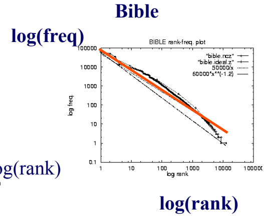
Practitioner’s guide:

- **tool#2**: rank-frequency plot (for **categorical attributes**)

internet domains



Bible



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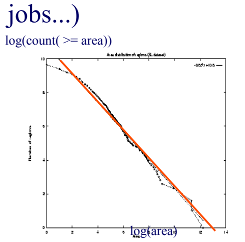
Practitioner’s guide:

- **tool#3**: CCDF, for (skewed) **numerical attributes**, eg. areas of islands/lakes, UNIX jobs...)



scandinavian lakes

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Resources:

- Software for fractal dimension
  - www.cs.cmu.edu/~christos/software.html
  - And specifically 'fdnq_h':
  - www.cs.cmu.edu/~christos/SRC/fdnq_h.zip
- Also, in 'R': 'fdim' package

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Books

- Strongly recommended intro book:
  - Manfred Schroeder *Fractals, Chaos, Power Laws: Minutes from an Infinite Paradise* W.H. Freeman and Company, 1991
- Classic book on fractals:
  - B. Mandelbrot *Fractal Geometry of Nature,* W.H. Freeman, 1977

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– [vldb96] Christos Faloutsos, Yossi Matias and Avi Silberschatz, *Modeling Skewed Distributions Using Multifractals and the `80-20 Law'* Conf. on Very Large Data Bases (VLDB), Bombay, India, Sept. 1996.

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- [Wang+'02] Mengzhi Wang, Anastassia Ailamaki and Christos Faloutsos, Capturing the spatio-temporal behavior of real traffic data Performance 2002 (IFIP Int. Symp. on Computer Performance Modeling, Measurement and Evaluation), Rome, Italy, Sept. 2002

Appendix - Gory details

- Bad news: There are more than one fractal dimensions
  - Minkowski fd; Hausdorff fd; Correlation fd; Information fd
- Great news:
  - they can all be computed fast!
  - they usually have nearby values

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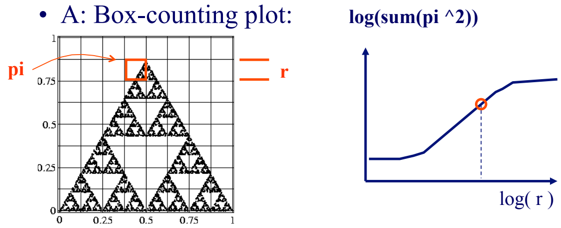
Fast estimation of fd(s):

- How, for the (correlation) fractal dimension?
- A: Box-counting plot:



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Definitions

- *pi* : the percentage (or count) of points in the *i*-th cell
- *r*: the side of the grid

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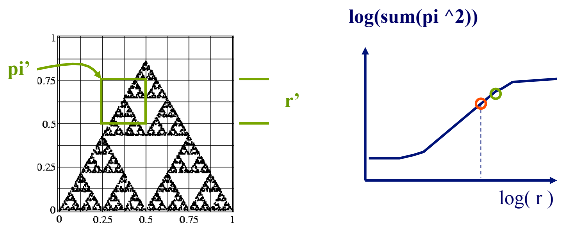
Fast estimation of fd(s):

- compute sum(pi^2) for another grid side, r'



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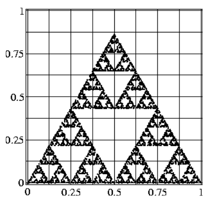
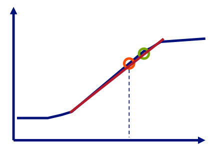
Fast estimation of fd(s):

- etc; if the resulting plot has a linear part, its slope is the correlation fractal dimension D2





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Definitions (cont'd)

- Many more fractal dimensions Dq (related to Renyi entropies):

$$D_q = \frac{1}{q-1} \frac{\partial \log(\sum p_i^q)}{\partial \log(r)} \qquad q \neq 1$$

$$D_1 = \frac{\partial \sum p_i \log(p_i)}{\partial \log(r)}$$

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Hausdorff or box-counting fd:

- Box counting plot: Log(N (r)) vs Log (r)
- r: grid side
- N (r): count of non-empty cells
- (Hausdorff) fractal dimension D0:

$$D_0 = -\frac{\partial \log(N(r))}{\partial \log(r)}$$

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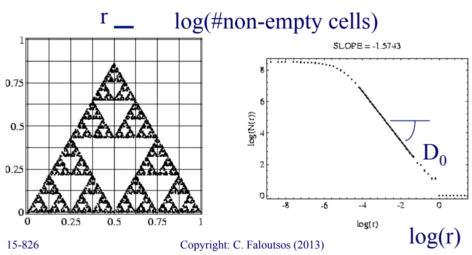
Definitions (cont'd)

- Hausdorff fd:



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Observations

- q=0: Hausdorff fractal dimension
- q=2: Correlation fractal dimension (**identical** to the exponent of the number of neighbors vs radius)
- q=1: Information fractal dimension

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Observations, cont'd

- in general, the Dq's take similar, but not identical, values.
- except for perfectly self-similar point-sets, where Dq=Dq' for any *q, q'*

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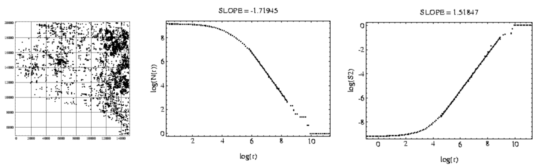
Examples:MG county

- Montgomery County of MD (road end-points)



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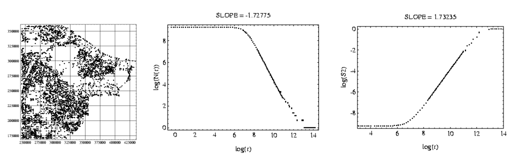
Examples:LB county

- Long Beach county of CA (road end-points)



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Conclusions

- many fractal dimensions, with nearby values
- can be computed quickly
  (O(N) or O(N log(N))
- (code: on the web:
  – www.cs.cmu.edu/~christos/SRC/fdnq_h.zip
  – Or `R' ('fdim' package)