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15-826: Multimedia Databases and Data Mining

Lecture #13: Power laws - equivalences
Addendum
C. Faloutsos

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Jumping to the conclusion:

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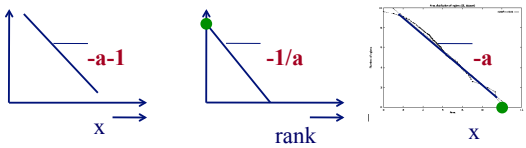
3 versions of P.L.

PDF = frequency-count plot | Zipf plot = Rank-frequency | NCDF = CCDF

IF ONE PLOT IS P.L., SO ARE THE OTHER TWO

Prob(area = x) | area | Prob(area >= x)



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Details, and proof sketches:

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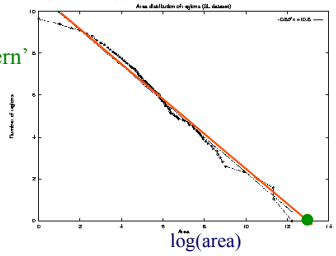
Reminder

More power laws: areas – Korcak's law

log(count(>= area))

'Vaenern'



log(area)

Scandinavian lakes area vs complementary cumulative count (log-log axes)

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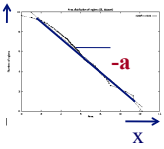
NCDF = CCDF

Prob(area >= x)



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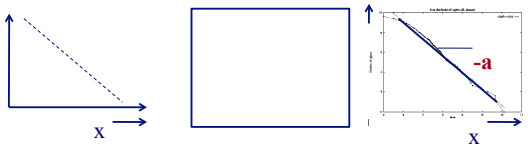

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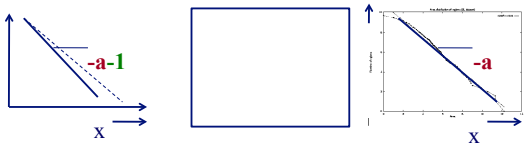

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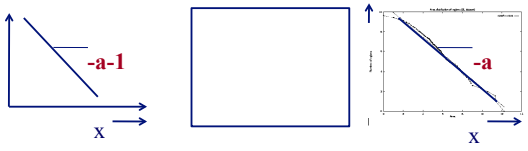

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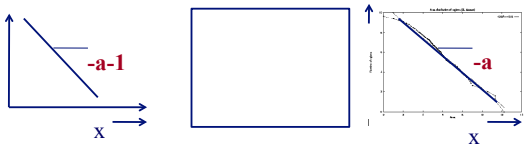

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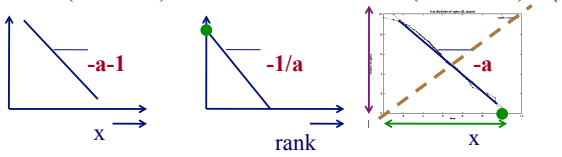

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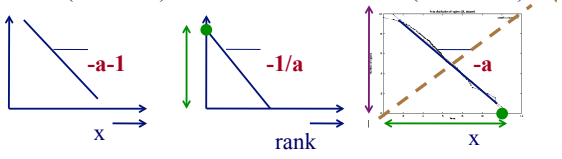

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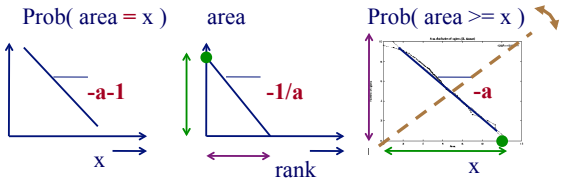

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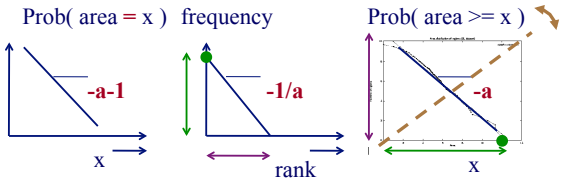

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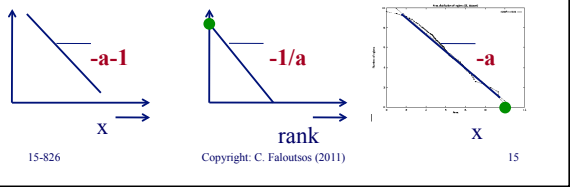

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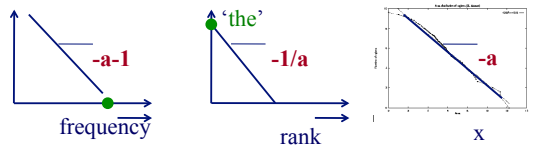
count | frequency | Prob(area >= x)



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Sanity check:

- Zipf showed that if
  - Slope of rank-frequency is -1
  - Then slope of freq-count is -2
- Check it!

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3 versions of P.L.

PDF = frequency-count plot | Zipf plot = Rank-frequency | NCDF = CCDF

slope = -2 $\Leftrightarrow$ slope = -1

count | frequency | Prob(area >= x)

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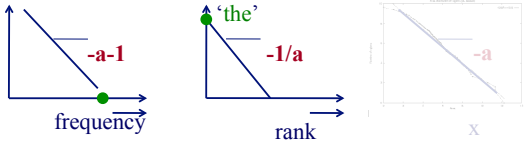
3 versions of P.L.

PDF = frequency-count plot | Zipf plot = Rank-frequency | NCDF = CCDF

✓ slope = -2 ⟺ slope = -1

count | frequency | Prob(area >= x)



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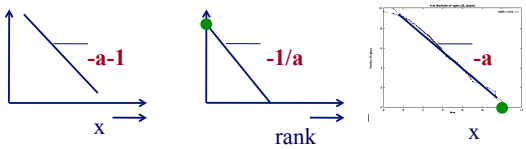

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References

- *Zipf, Power-laws, and Pareto - a ranking tutorial*, Lada A. Adamic www.hpl.hp.com/research/idl/papers/ranking/ranking.html
- L.A. Adamic and B.A. Huberman, *'Zipf's law and the Internet'*, *Glottometrics* 3, 2002,143-150
- *Human Behavior and Principle of Least Effort*, G.K. Zipf, Addison Wesley (1949)