



CMU SCS

15-826: Multimedia Databases and Data Mining

SVD - part I (definitions)
C. Faloutsos



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Outline

Goal: 'Find similar / interesting things'

- Intro to DB
- ➔ • Indexing - similarity search
- Data Mining

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Indexing - Detailed outline

- primary key indexing
- secondary key / multi-key indexing
- spatial access methods
- fractals
- text
- ➔ • Singular Value Decomposition (SVD)
- multimedia
- ...

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SVD - Detailed outline

- ➔ • Motivation
- Definition - properties
- Interpretation
- Complexity
- Case studies
- Additional properties

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SVD - Motivation

- problem #1: text - LSI: find 'concepts'
- problem #2: compression / dim. reduction

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SVD - Motivation

- problem #1: text - LSI: find 'concepts'

	term	data	information	retrieval	brain	lung
document						
CS-TR1	1	1	1	0	0	
CS-TR2	2	2	2	0	0	
CS-TR3	1	1	1	0	0	
CS-TR4	5	5	5	0	0	
MED-TR1	0	0	0	2	2	
MED-TR2	0	0	0	3	3	
MED-TR3	0	0	0	1	1	

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SVD - Motivation

- problem #2: compress / reduce dimensionality

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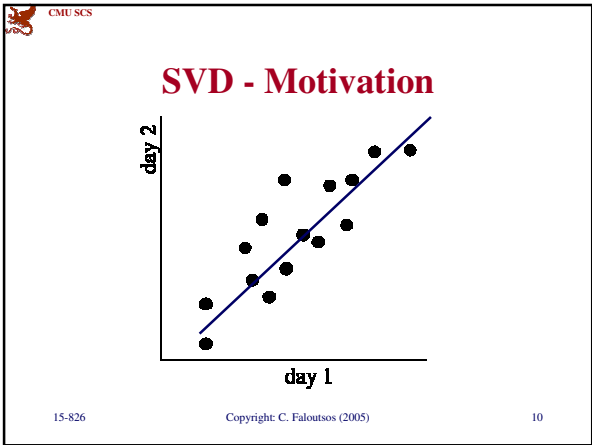
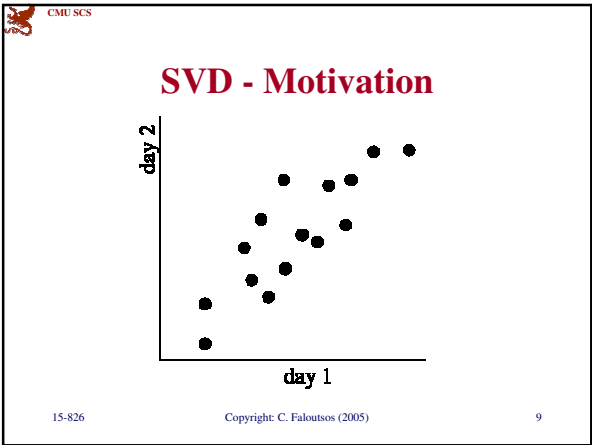
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Problem - specs

- ~10*6 rows; ~10*3 columns; no updates;
- random access to any cell(s) ; small error: OK

customer	day	We	Th	Fr	Sa	Su
		7/10/06	7/11/06	7/12/06	7/13/06	7/14/06
ABC Inc.		1	1	1	0	0
DEF Ltd.		2	2	2	0	0
GHI Inc.		1	1	1	0	0
KLM Co.		5	5	5	0	0
Smith		0	0	0	2	2
Johansen		0	0	0	3	3
Thompson		0	0	0	1	1

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SVD - Detailed outline

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SVD - Definition

(reminder: matrix multiplication)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

3 x 2 2 x 1

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SVD - Definition

(reminder: matrix multiplication)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

$3 \times 2 \quad 2 \times 1 \quad 3 \times 1$

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SVD - Definition

(reminder: matrix multiplication)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ \\ \end{bmatrix}$$

$3 \times 2 \quad 2 \times 1 \quad 3 \times 1$

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SVD - Definition

(reminder: matrix multiplication)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ \end{bmatrix}$$

$3 \times 2 \quad 2 \times 1 \quad 3 \times 1$

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SVD - Definition

(reminder: matrix multiplication)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

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SVD - Definition

$A_{[n \times m]} = U_{[n \times r]} \Lambda_{[r \times r]} (V_{[m \times r]})^T$

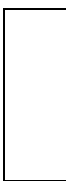
- A : $n \times m$ matrix (eg., n documents, m terms)
- U : $n \times r$ matrix (n documents, r concepts)
- Λ : $r \times r$ diagonal matrix (strength of each ‘concept’) (r : rank of the matrix)
- V : $m \times r$ matrix (m terms, r concepts)

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SVD - Definition

- $A = U \Lambda V^T$ - example:

A	U	Λ	V^T
Men	Men	rank	men
		$\times \begin{bmatrix} \square & & \\ & \square & \\ & & \square \end{bmatrix} \times$	

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SVD - Properties

THEOREM [Press+92]: always possible to decompose matrix **A** into **A** = **U** **Λ** **V**^T, where

- **U**, **Λ**, **V**: unique (*)
- **U**, **V**: column orthonormal (ie., columns are unit vectors, orthogonal to each other)
 - **U**^T **U** = **I**; **V**^T **V** = **I** (**I**: identity matrix)
- **Λ**: singular are positive, and sorted in decreasing order

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SVD - Example

• **A** = **U** **Λ** **V**^T - example:

↑ CS

↓ MD

data

retrieval
inf.↓ brain lung

1 1 1 0 0

2 2 2 0 0

1 1 1 0 0

5 5 5 0 0

0 0 0 2 2

0 0 0 3 3

0 0 0 1 1

0.18 0

0.36 0

0.18 0

0.90 0

0 0.53

0 0.80

0 0.27

x

9.64 0

0 5.29

x

0.58 0.58 0.58 0 0

0 0 0 0.71 0.71

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SVD - Example

• **A** = **U** **Λ** **V**^T - example:

↑ CS

↓ MD

data

retrieval
inf.↓ brain lung

1 1 1 0 0

2 2 2 0 0

1 1 1 0 0

5 5 5 0 0

0 0 0 2 2

0 0 0 3 3

0 0 0 1 1

0.18 0

0.36 0

0.18 0

0.90 0

0 0.53

0 0.80

0 0.27

x

9.64 0

0 5.29

x

0.58 0.58 0.58 0 0

0 0 0 0.71 0.71

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SVD - Example

• **A** = **U** **Λ** **V**^T - example: doc-to-concept similarity matrix

↑ CS

↓ MD

data

retrieval
inf.↓ brain lung

1 1 1 0 0

2 2 2 0 0

1 1 1 0 0

5 5 5 0 0

0 0 0 2 2

0 0 0 3 3

0 0 0 1 1

0.18 0

0.36 0

0.18 0

0.90 0

0 0.53

0 0.80

0 0.27

x

9.64 0

0 5.29

x

0.58 0.58 0.58 0 0

0 0 0 0.71 0.71

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SVD - Example

• **A** = **U** **Λ** **V**^T - example: 'strength' of CS-concept

↑ CS

↓ MD

data

retrieval
inf.↓ brain lung

1 1 1 0 0

2 2 2 0 0

1 1 1 0 0

5 5 5 0 0

0 0 0 2 2

0 0 0 3 3

0 0 0 1 1

0.18 0

0.36 0

0.18 0

0.90 0

0 0.53

0 0.80

0 0.27

x

9.64 0

0 5.29

x

0.58 0.58 0.58 0 0

0 0 0 0.71 0.71

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SVD - Example

• **A** = **U** **Λ** **V**^T - example: term-to-concept similarity matrix

↑ CS

↓ MD

data

retrieval
inf.↓ brain lung

1 1 1 0 0

2 2 2 0 0

1 1 1 0 0

5 5 5 0 0

0 0 0 2 2

0 0 0 3 3

0 0 0 1 1

0.18 0

0.36 0

0.18 0

0.90 0

0 0.53

0 0.80

0 0.27

x

9.64 0

0 5.29

x

0.58 0.58 0.58 0 0

0 0 0 0.71 0.71

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SVD - Example

- $A = U \Lambda V^T$ - example:

retrieval
inf. ↓ brain lung

data

↑ CS
↓ MD

1	1	1	0	0
2	2	2	0	0
1	1	1	0	0
5	5	5	0	0
0	0	0	2	2
0	0	0	3	3
0	0	0	1	1

term-to-concept
similarity matrix

0.18	0
0.36	0
0.18	0
0.90	0
0	0.53
0	0.80
0	0.27

CS-concept

$\times \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \times$

$\rightarrow \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$

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SVD - Interpretation #1

‘documents’, ‘terms’ and ‘concepts’:

- U : document-to-concept similarity matrix
- V : term-to-concept sim. matrix
- Λ : its diagonal elements: ‘strength’ of each concept

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SVD - Interpretation #2

- best axis to project on: (‘best’ = min sum of squares of projection errors)

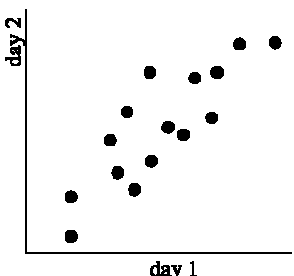
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SVD - Motivation



A scatter plot with 'day 1' on the horizontal axis and 'day 2' on the vertical axis. It shows approximately 15 black dots representing data points, which are scattered but show a general upward trend.

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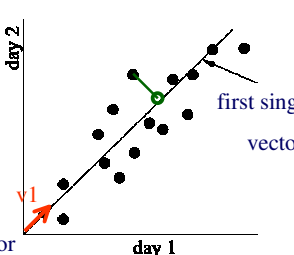
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SVD - interpretation #2

SVD: gives best axis to project



A scatter plot similar to the previous one, but with a black line of best fit passing through the data points. A green vector labeled 'v1' originates from the origin and points along the line of best fit. A label 'first singular vector' points to this line.

- minimum RMS error

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SVD - Interpretation #2

customer	day	We 7/10/96	Th 7/11/96	Fr 7/12/96	Sa 7/13/96	Su 7/14/96
ABC Inc.		1	1	1	0	0
DEF Ltd.		2	2	2	0	0
GHI Inc.		1	1	1	0	0
KLM Co.		5	5	5	0	0
Smith		0	0	0	2	2
Johnson		0	0	0	3	3
Thompson		0	0	0	1	1

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SVD - Interpretation #2

• $A = U \Lambda V^T$ - example:

1

1

1

0

0

2

2

2

0

0

1

1

1

0

0

5

5

5

0

0

0

0

0

2

2

0

0

0

3

3

0

0

0

1

1

0.18

0

0.36

0

0.18

0

0.90

0

0

0.53

0

0.80

0

0.27

9.64

0

0

5.29

0.58

0.58

0.58

0

0

0

0

0

0.71

0.71

v1

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SVD - Interpretation #2

• $A = U \Lambda V^T$ - example:

variance ('spread') on the v1 axis

1

1

1

0

0

2

2

2

0

0

1

1

1

0

0

5

5

5

0

0

0

0

0

2

2

0

0

0

3

3

0

0

0

1

1

0.18

0

0.36

0

0.18

0

0.90

0

0

0.53

0

0.80

0

0.27

9.64

0

0

5.29

0.58

0.58

0.58

0

0

0

0

0

0.71

0.71

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SVD - Interpretation #2

• $A = U \Lambda V^T$ - example:

– $U \Lambda$ gives the coordinates of the points in the projection axis

1

1

1

0

0

2

2

2

0

0

1

1

1

0

0

5

5

5

0

0

0

0

0

2

2

0

0

0

3

3

0

0

0

1

1

0.18

0

0.36

0

0.18

0

0.90

0

0

0.53

0

0.80

0

0.27

9.64

0

0

5.29

0.58

0.58

0.58

0

0

0

0

0

0.71

0.71

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SVD - Interpretation #2

- More details
- Q: how exactly is dim. reduction done?

1

1

1

0

0

2

2

2

0

0

1

1

1

0

0

5

5

5

0

0

0

0

0

2

2

0

0

0

3

3

0

0

0

1

1

0.18

0

0.36

0

0.18

0

0.90

0

0

0.53

0

0.80

0

0.27

9.64

0

0

5.29

0.58

0.58

0.58

0

0

0

0

0

0.71

0.71

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SVD - Interpretation #2

- More details
- Q: how exactly is dim. reduction done?
- A: set the smallest singular values to zero:

1

1

1

0

0

2

2

2

0

0

1

1

1

0

0

5

5

5

0

0

0

0

0

2

2

0

0

0

3

3

0

0

0

1

1

0.18

0

0.36

0

0.18

0

0.90

0

0

0.53

0

0.80

0

0.27

9.64

0

0

~~5.29~~

0.58

0.58

0.58

0

0

0

0

0

0.71

0.71

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SVD - Interpretation #2

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

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SVD - Interpretation #2

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

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SVD - Interpretation #2

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 0.18 \\ 0.36 \\ 0.18 \\ 0.90 \\ 0 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 9.64 \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \end{bmatrix}$$

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SVD - Interpretation #2

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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SVD - Interpretation #2

Exactly equivalent:
'spectral decomposition' of the matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

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SVD - Interpretation #2

Exactly equivalent:
'spectral decomposition' of the matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} | & | \\ u_1 & u_2 \\ | & | \end{bmatrix} \times \begin{bmatrix} \lambda_1 & \emptyset \\ \emptyset & \lambda_2 \end{bmatrix} \times \begin{bmatrix} \text{---} v_1 \text{---} \\ \text{---} v_2 \text{---} \end{bmatrix}$$

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SVD - Interpretation #2

Exactly equivalent:
'spectral decomposition' of the matrix:

$$\begin{matrix} \leftarrow m \rightarrow \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{matrix} = \lambda_1 u_1 v_1^T + \lambda_2 u_2 v_2^T + \dots$$

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SVD - Interpretation #2

Exactly equivalent:
'spectral decomposition' of the matrix:

$$\begin{matrix} \leftarrow m \rightarrow \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{matrix} = \lambda_1 u_1 v_1^T + \lambda_2 u_2 v_2^T + \dots$$

$\xrightarrow{\text{r terms}}$

$n \times 1$ $1 \times m$

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SVD - Interpretation #2

approximation / dim. reduction:
by keeping the first few terms (Q: how many?)

$$\begin{matrix} \leftarrow m \rightarrow \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{matrix} = \lambda_1 u_1 v_1^T + \lambda_2 u_2 v_2^T + \dots$$

assume: $\lambda_1 \geq \lambda_2 \geq \dots$

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SVD - Interpretation #2

A (heuristic - [Fukunaga]): keep 80-90% of
'energy' (= sum of squares of λ_i 's)

$$\begin{matrix} \leftarrow m \rightarrow \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{matrix} = \lambda_1 u_1 v_1^T + \lambda_2 u_2 v_2^T + \dots$$

assume: $\lambda_1 \geq \lambda_2 \geq \dots$

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SVD - Detailed outline

- Motivation
- Definition - properties
- Interpretation
 - #1: documents/terms/concepts
 - #2: dim. reduction
 - #3: picking non-zero, rectangular 'blobs'
- ➔ Complexity
- Case studies
- Additional properties

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SVD - Interpretation #3

- finds non-zero 'blobs' in a data matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

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SVD - Interpretation #3

- finds non-zero 'blobs' in a data matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ \hline 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

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SVD - Interpretation #3

- Drill: find the SVD, 'by inspection'!
- Q: rank = ??

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} ?? \\ ?? \\ ?? \\ ?? \\ ?? \end{bmatrix} \times \begin{bmatrix} ?? & 0 \\ 0 & ?? \end{bmatrix} \times \begin{bmatrix} ?? & 0 & 0 & 0 & 0 \\ 0 & ?? & 0 & 0 & 0 \\ 0 & 0 & ?? & 0 & 0 \\ 0 & 0 & 0 & ?? & 0 \\ 0 & 0 & 0 & 0 & ?? \end{bmatrix}$$

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SVD - Interpretation #3

- A: rank = 2 (2 linearly independent rows/cols)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} | & | & | & | & | \\ | & | & | & | & | \\ | & | & | & | & | \\ | & | & | & | & | \\ | & | & | & | & | \end{bmatrix} \times \begin{bmatrix} ?? & 0 \\ 0 & ?? \end{bmatrix} \times \begin{bmatrix} \text{---} & ?? & \text{---} \\ \text{---} & ?? & \text{---} \end{bmatrix}$$

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SVD - Interpretation #3

- A: rank = 2 (2 linearly independent rows/cols)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} ?? & 0 \\ 0 & ?? \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

orthogonal??

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SVD - Interpretation #3

- column vectors: are orthogonal - but not unit vectors:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} & 0 \\ 1/\sqrt{3} & 0 \\ 1/\sqrt{3} & 0 \\ 0 & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} \end{bmatrix} \times \begin{bmatrix} ?? & 0 \\ 0 & ?? \end{bmatrix} \times \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} & 0 & 0 \\ 0 & 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

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SVD - Interpretation #3

- and the singular values are:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} & 0 \\ 1/\sqrt{3} & 0 \\ 1/\sqrt{3} & 0 \\ 0 & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} \end{bmatrix} \times \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \times \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} & 0 & 0 \\ 0 & 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

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SVD - Interpretation #3

• Q: How to check we are correct?

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} & 0 \\ 1/\sqrt{3} & 0 \\ 1/\sqrt{3} & 0 \\ 0 & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} \end{bmatrix} \times \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \times \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} & 0 & 0 \\ 0 & 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

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SVD - Interpretation #3

• A: SVD properties:

- matrix product should give back matrix **A**
- matrix **U** should be column-orthonormal, i.e., columns should be unit vectors, orthogonal to each other
- ditto for matrix **V**
- matrix **Λ** should be diagonal, with positive values

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SVD - Detailed outline

- Motivation
- Definition - properties
- Interpretation
- ➔ • Complexity
- Case studies
- Additional properties

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SVD - Complexity

- $O(n * m * m)$ or $O(n * n * m)$ (whichever is less)
- less work, if we just want singular values
- or if we want first k singular vectors
- or if the matrix is sparse [Berry]
- Implemented: in any linear algebra package (LINPACK, matlab, Splus, mathematica ...)

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SVD - conclusions so far

- SVD: $\mathbf{A} = \mathbf{U} \mathbf{\Lambda} \mathbf{V}^T$: unique (*)
- **U**: document-to-concept similarities
- **V**: term-to-concept similarities
- **Λ**: strength of each concept
- dim. reduction: keep the first few strongest singular values (80-90% of 'energy')
 - SVD: picks up linear correlations
- SVD: picks up non-zero 'blobs'

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