

Anomaly detection in large graphs

Christos Faloutsos

CMU

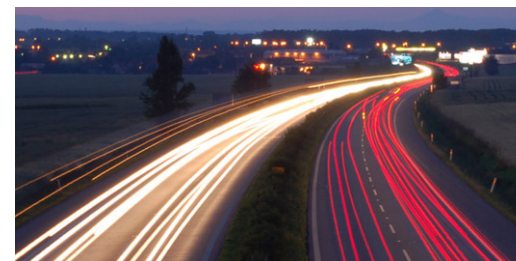
Thank you!

- Annette Jiang (IEEE)
- Evan Butterfield (IEEE)
- Tina Huang (Tencent)



Roadmap

- ➔ • Introduction – Motivation
 - Why study (big) graphs?
- Part#1: Patterns in graphs
- Part#2: time-evolving graphs; tensors
- Conclusions



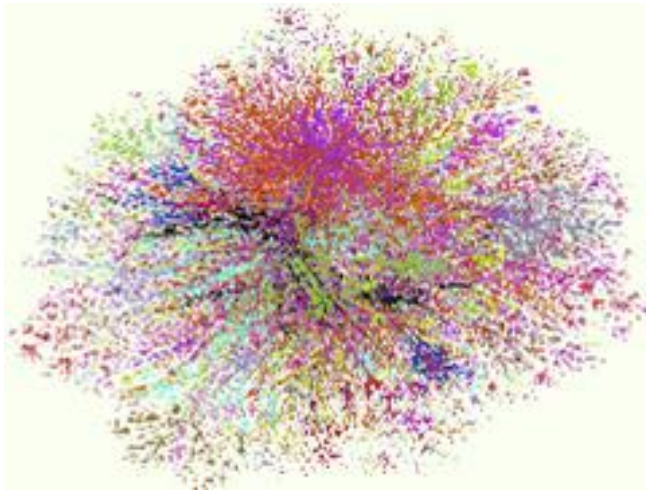
Graphs - why should we care?



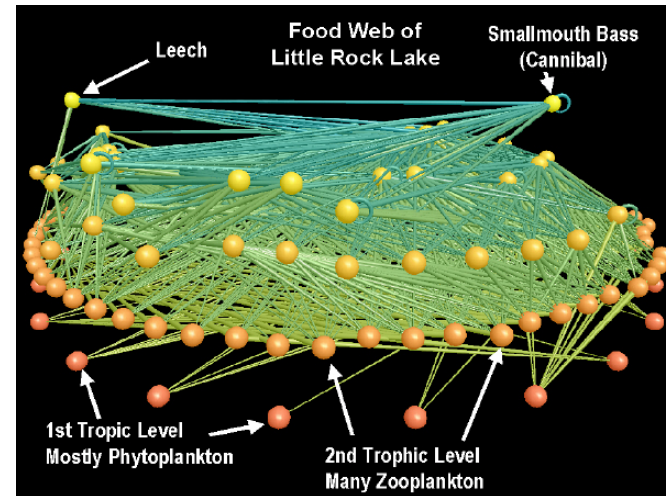
>\$10B; ~1B users



Graphs - why should we care?



Internet Map
[lumeta.com]



Food Web
[Martinez '91]

Graphs - why should we care?

- web-log ('blog') news propagation
- computer network security: email/IP traffic and anomaly detection
- Recommendation systems
- Who-bought-from-whom (ebay, Alibaba)
-



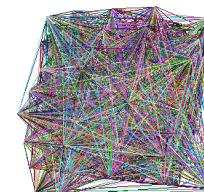
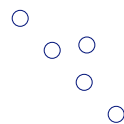
NETFLIX



Many-to-many db relationship -> graph

Motivating problems

- P1: patterns? Fraud detection?



- P2: patterns in time-evolving graphs / tensors

destination

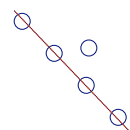


source

time

Motivating problems

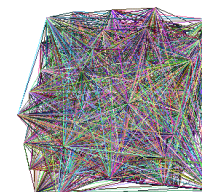
- P1: patterns? Fraud detection?



Patterns



anomalies



- P2: patterns in time-evolving graphs / tensors

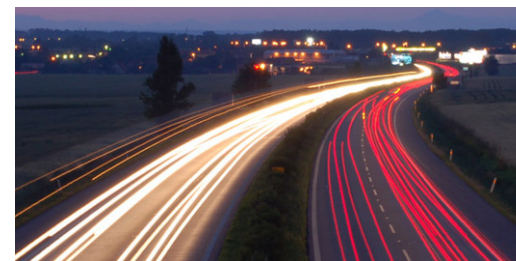
destination



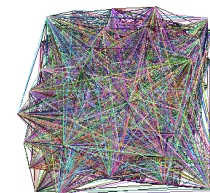
source

time

Roadmap



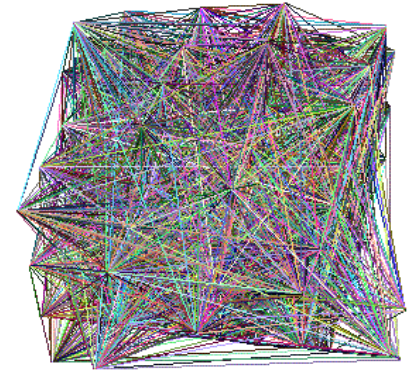
- Introduction – Motivation
 - Why study (big) graphs?
- ➔ • Part#1: Patterns & fraud detection
- Part#2: time-evolving graphs; tensors
- Conclusions



Part 1: Patterns, & fraud detection

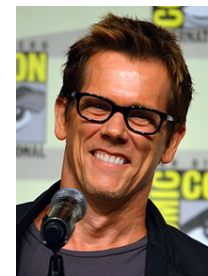
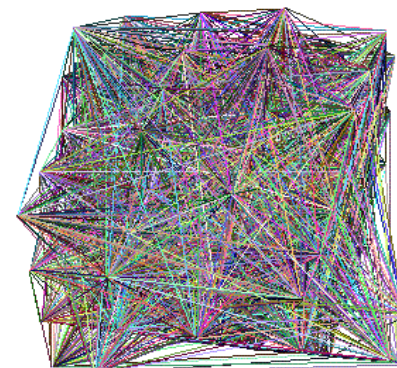
Laws and patterns

- Q1: Are real graphs random?



Laws and patterns

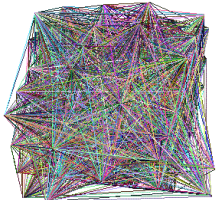
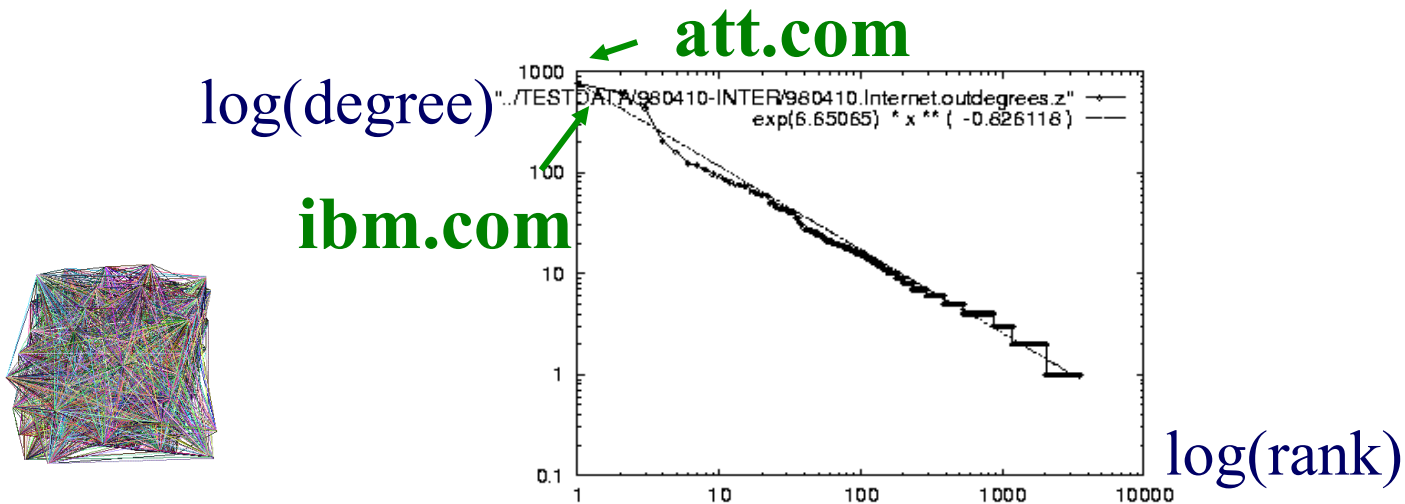
- Q1: Are real graphs random?
- A1: NO!!
 - Diameter ('6 degrees'; 'Kevin Bacon')
 - in- and out- degree distributions
 - other (surprising) patterns
- So, let's look at the data



Solution# S.1

- Power law in the degree distribution [Faloutsos x 3 SIGCOMM99]

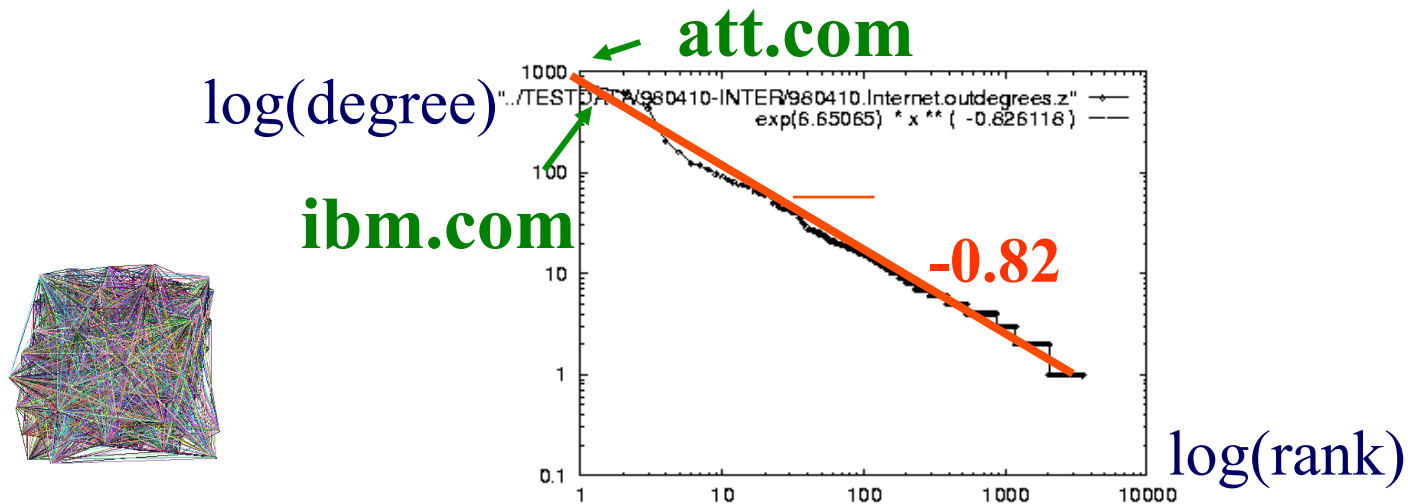
internet domains



Solution# S.1

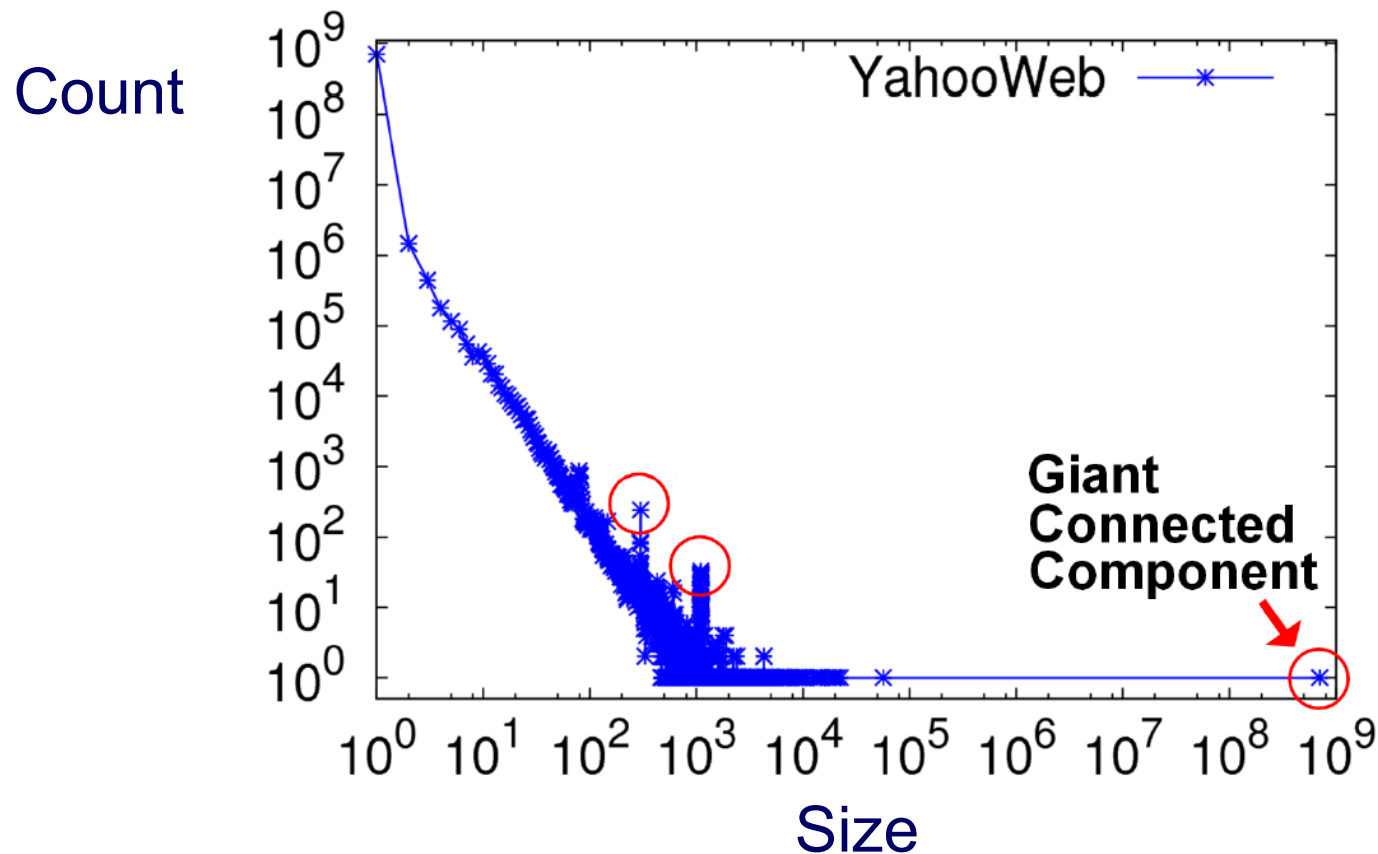
- Power law in the degree distribution [Faloutsos x 3 SIGCOMM99]

internet domains



S2: connected component sizes

- Connected Components – 4 observations:

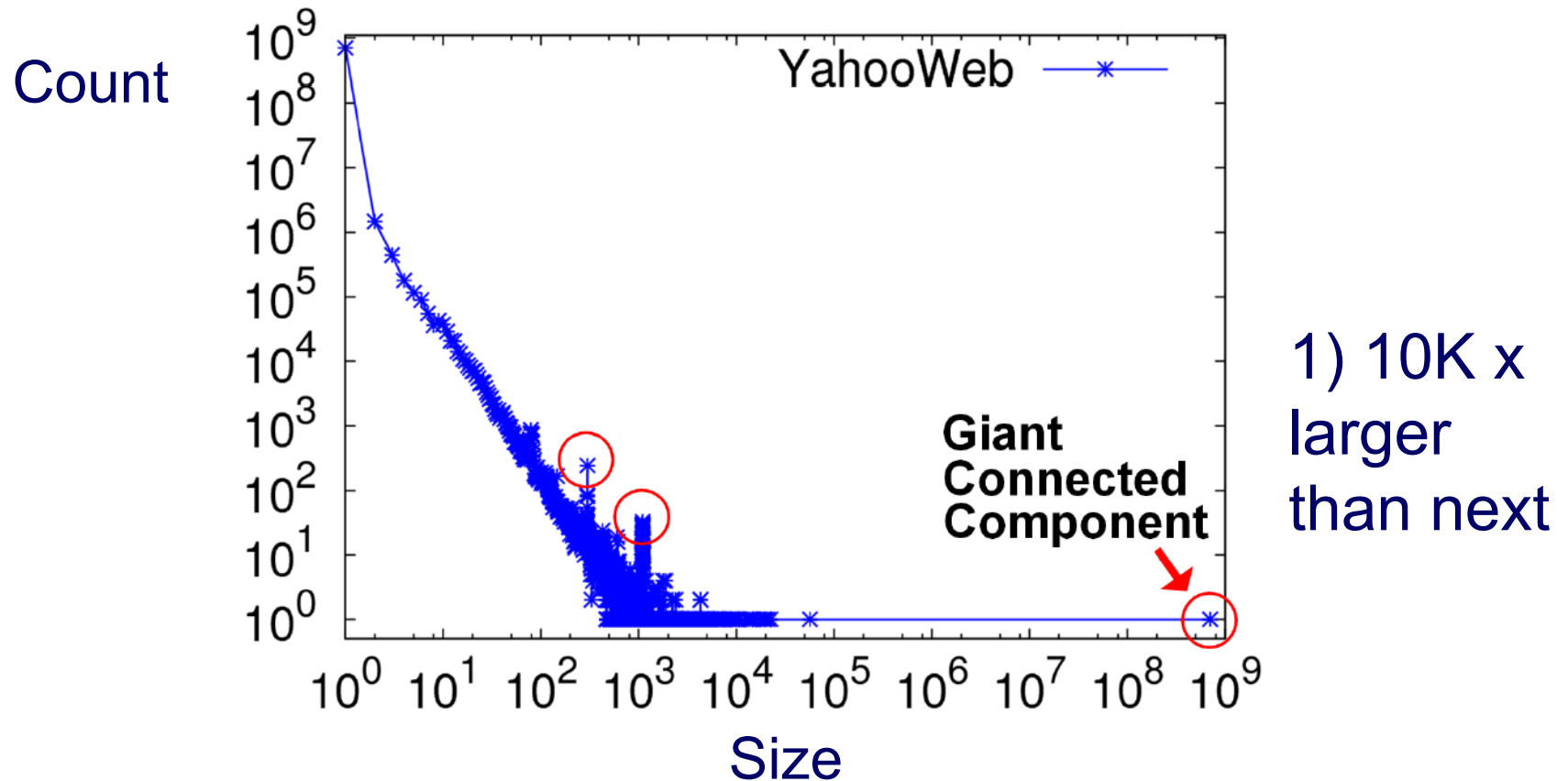


1.4B nodes
6B edges

S2: connected component sizes



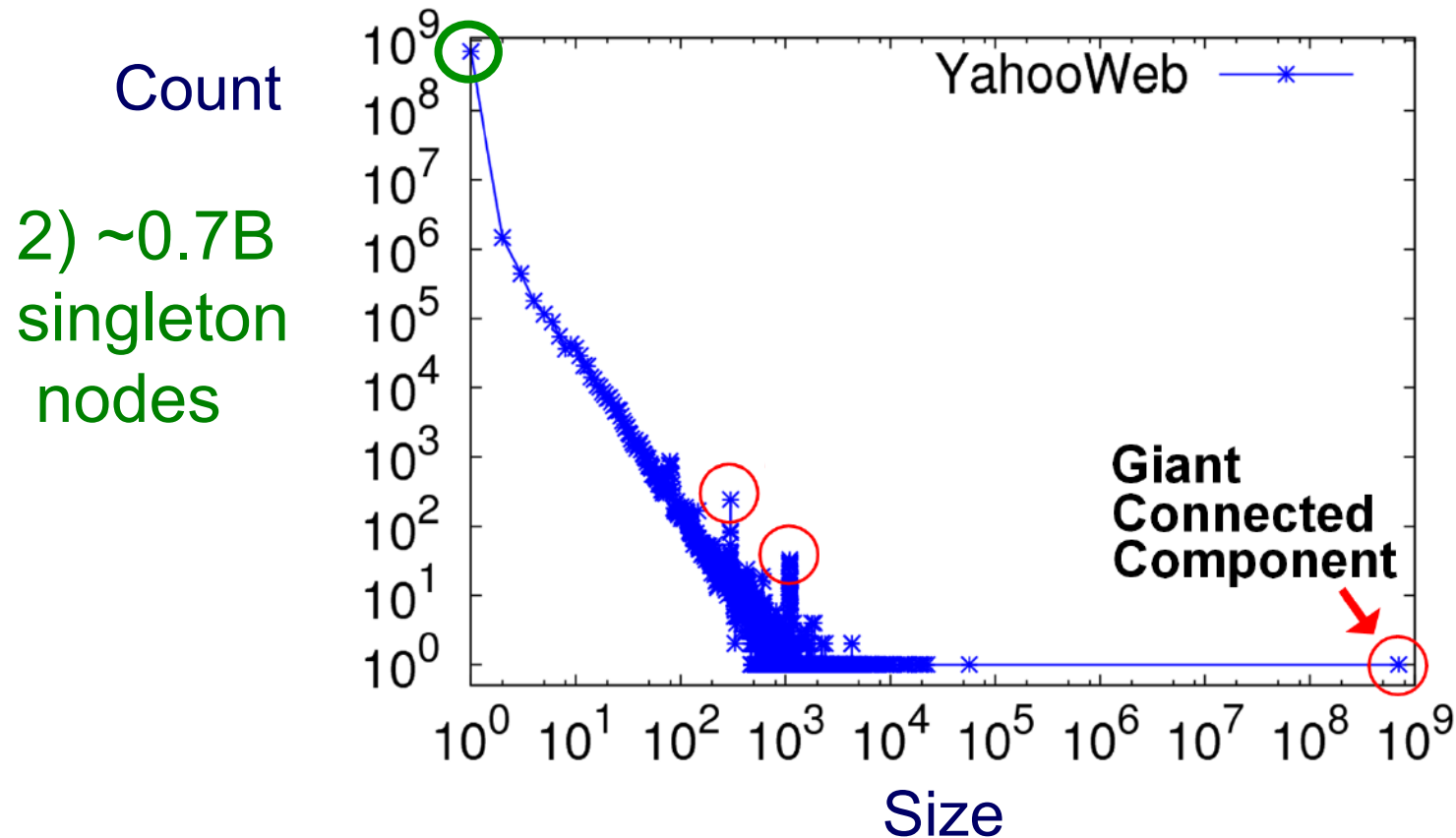
- Connected Components



S2: connected component sizes



- Connected Components

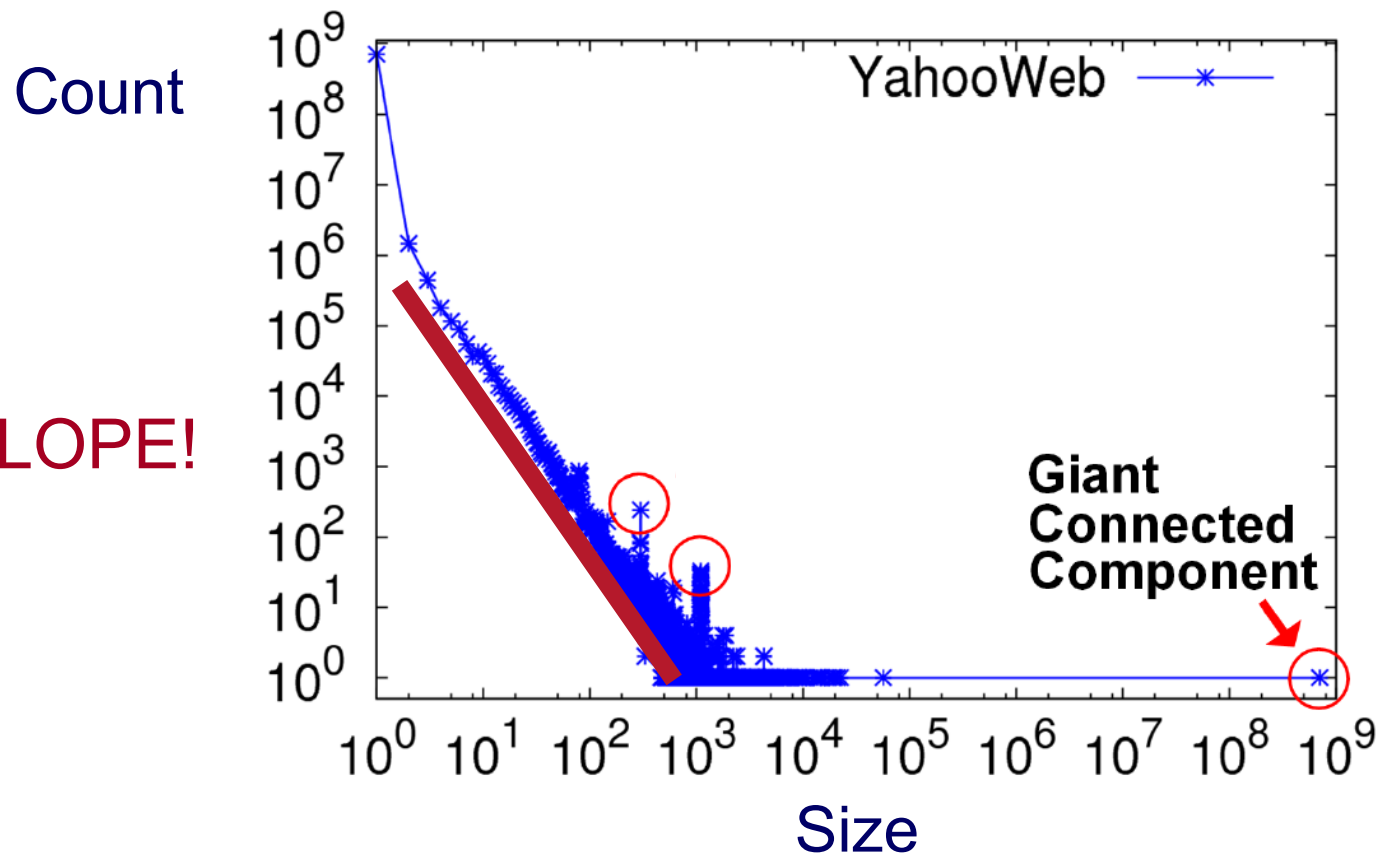


S2: connected component sizes



- Connected Components

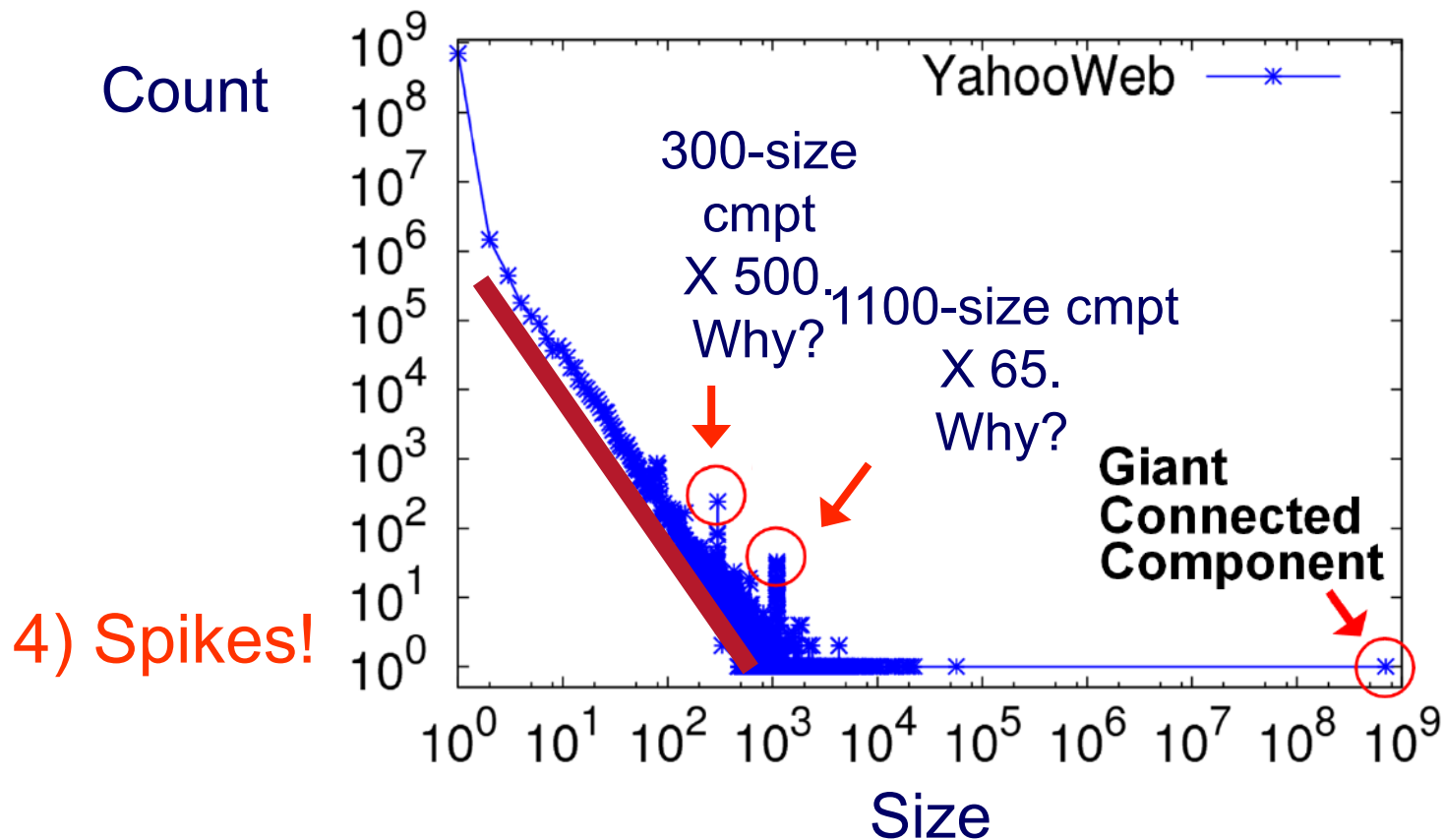
3) SLOPE!



S2: connected component sizes



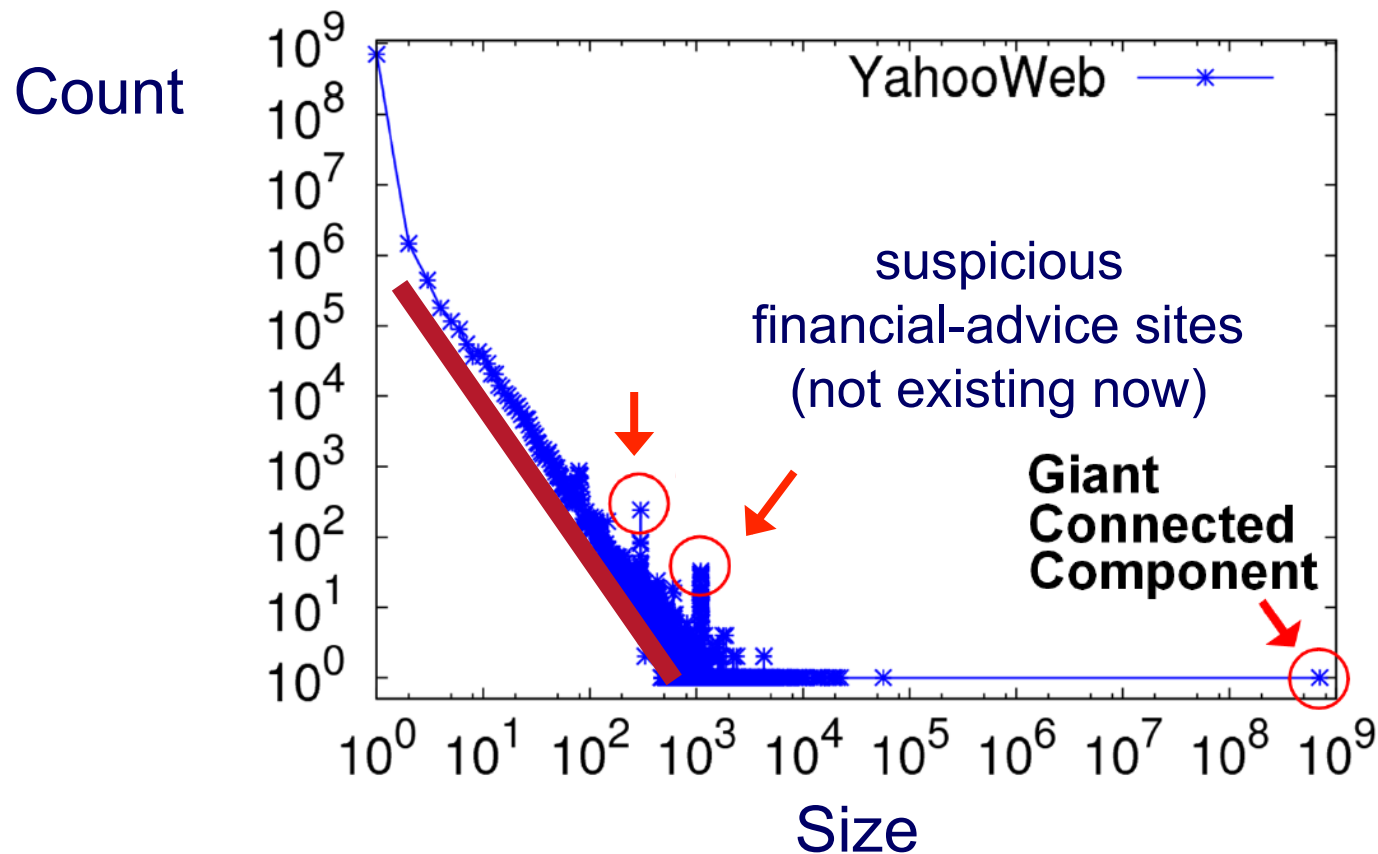
- Connected Components



S2: connected component sizes



- Connected Components



MORE Graph Patterns

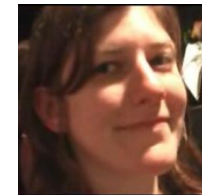
	Unweighted	Weighted
Static	<p> L01. Power-law degree distribution [Faloutsos et al. '99, Kleinberg et al. '99, Chakrabarti et al. '04, Newman '04]</p> <p>L02. Triangle Power Law (TPL) [Tsourakakis '08]</p> <p> L03. Eigenvalue Power Law (EPL) [Siganos et al. '03]</p> <p>L04. Community structure [Flake et al. '02, Girvan and Newman '02]</p>	<p>L10. Snapshot Power Law (SPL) [McGlohon et al. '08]</p>
Dynamic	<p>L05. Densification Power Law (DPL) [Leskovec et al. '05]</p> <p>L06. Small and shrinking diameter [Albert and Barabási '99, Leskovec et al. '05]</p> <p>L07. Constant size 2nd and 3rd connected components [McGlohon et al. '08]</p> <p>L08. Principal Eigenvalue Power Law (λ_1PL) [Akoglu et al. '08]</p> <p>L09. Bursty/self-similar edge/weight additions [Gomez and Santonja '98, Gribble et al. '98, Crovella and</p>	<p>L11. Weight Power Law (WPL) [McGlohon et al. '08]</p>

RTG: A Recursive Realistic Graph Generator using Random Typing Leman Akoglu and Christos Faloutsos. *PKDD'09*.

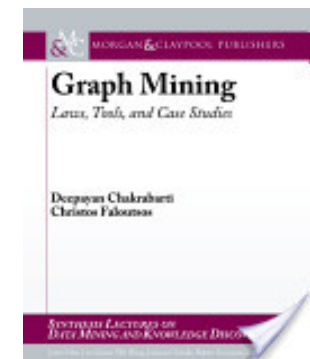
MORE Graph Patterns

	Unweighted	Weighted
Static	<p>L01. Power-law degree distribution [Faloutsos et al. '99, Kleinberg et al. '99, Chakrabarti et al. '04, Newman '04]</p> <p>L02. Triangle Power Law (TPL) [Tsourakakis '08]</p> <p>L03. Eigenvalue Power Law (EPL) [Siganos et al. '03]</p> <p>L04. Community structure [Flake et al. '02, Girvan and Newman '02]</p>	<p>L10. Snapshot Power Law (SPL) [McGlohon et al. '08]</p>
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- Mary McGlohon, Leman Akoglu, Christos Faloutsos. *Statistical Properties of Social Networks*. in "Social Network Data Analytics" (Ed.: Charu Aggarwal)

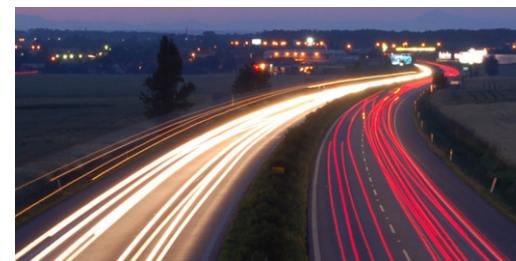


- Deepayan Chakrabarti and Christos Faloutsos, [*Graph Mining: Laws, Tools, and Case Studies*](#) Oct. 2012, Morgan Claypool.



Roadmap

- Introduction – Motivation
- Part#1: Patterns in graphs
 - P1.1: Patterns
 - ➔ – P1.2: Anomaly / fraud detection
 - No labels – spectral
 - With labels: Belief Propagation
- Part#2: time-evolving graphs; tensors
- Conclusions



Patterns

anomalies

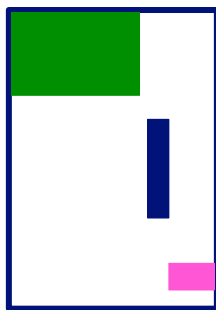
How to find ‘suspicious’ groups?

- ‘blocks’ are normal, right?



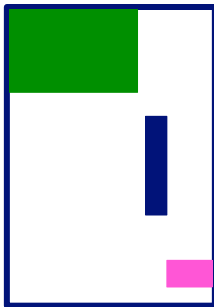
idols

fans



Except that:

- ‘blocks’ are normal, ~~right?~~
- ‘hyperbolic’ communities are more realistic [Araujo+, PKDD’14]



Tencent, 6/22



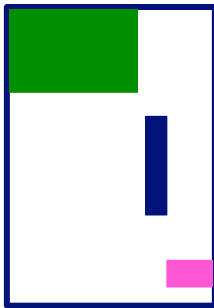
(c) C. Faloutsos, 2017

Except that:

- ‘blocks’ are usually **suspicious**
- ‘hyperbolic’ communities are more realistic [Araujo+, PKDD’14]



Q: Can we spot blocks, easily?



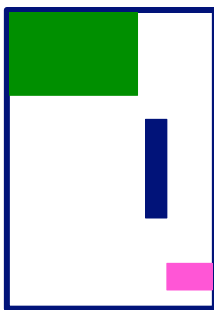
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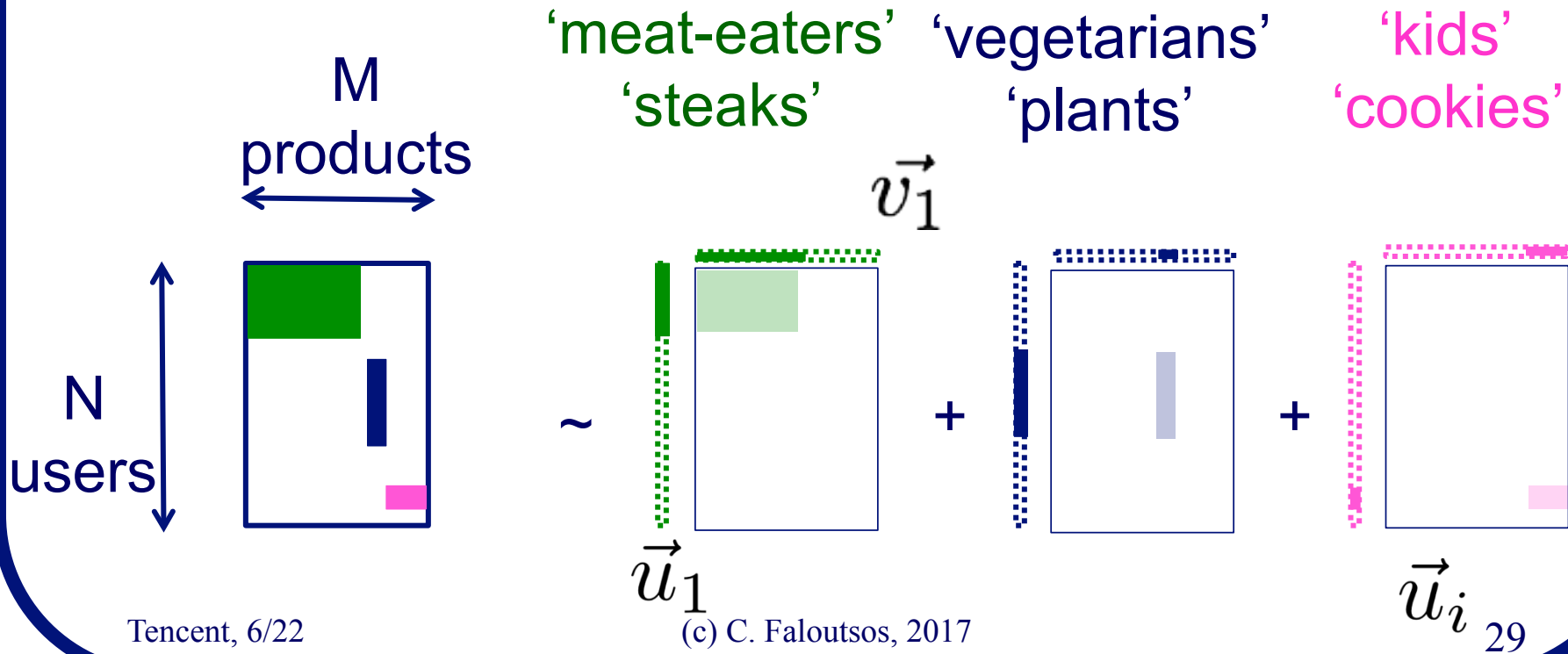
Q: Can we spot blocks, easily?

A: Silver bullet: SVD!



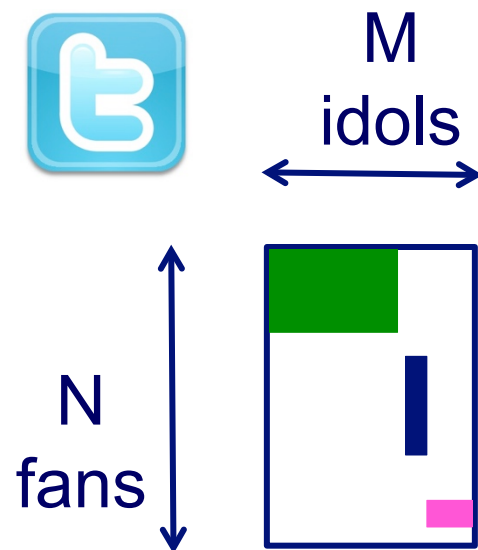
Crush intro to SVD

- Recall: (SVD) matrix factorization: finds blocks



Crush intro to SVD

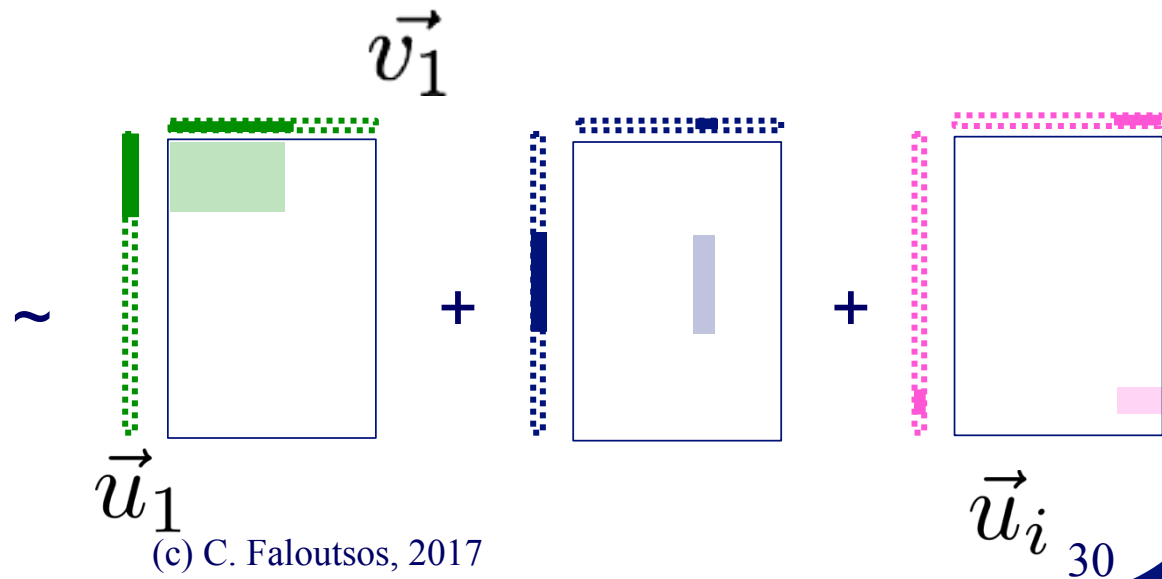
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'music lovers' 'singers' \vec{v}_1

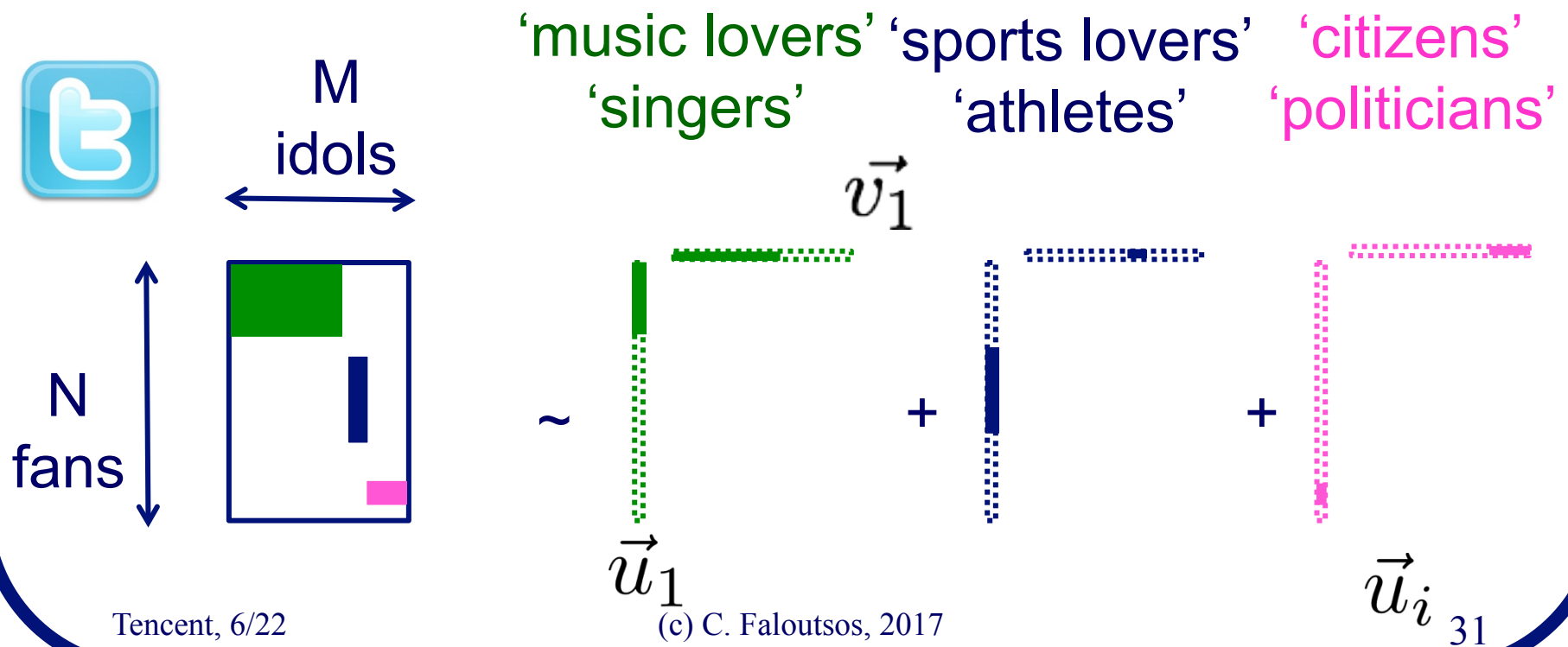
'sports lovers' 'athletes'

'citizens' 'politicians' \vec{u}_i



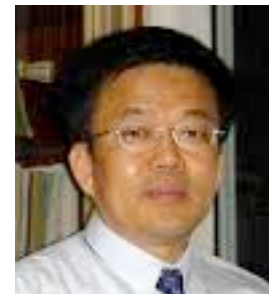
Crush intro to SVD

- Recall: (SVD) matrix factorization: finds blocks



Inferring Strange Behavior from Connectivity Pattern in Social Networks

PAKDD'14



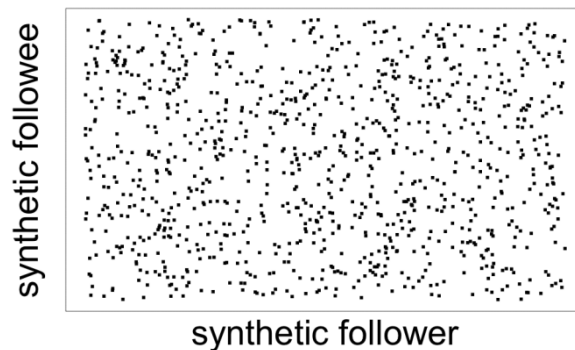
Meng Jiang, Peng Cui, Shiqiang Yang (Tsinghua)
Alex Beutel, Christos Faloutsos (CMU)



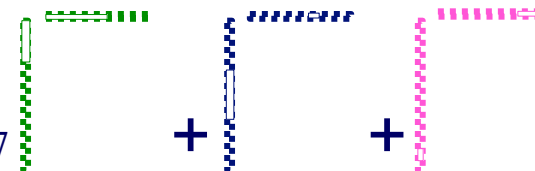
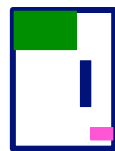
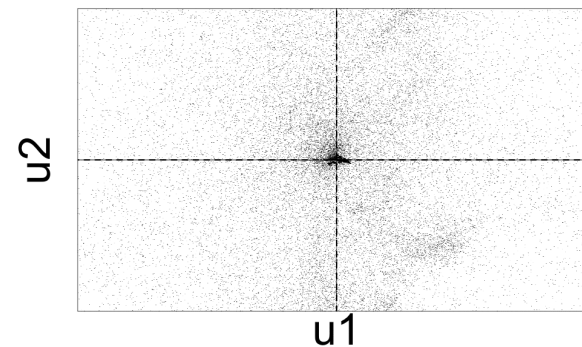
Lockstep and Spectral Subspace Plot

- Case #0: No lockstep behavior in random power law graph of 1M nodes, 3M edges
- Random \longleftrightarrow “Scatter”

Adjacency Matrix



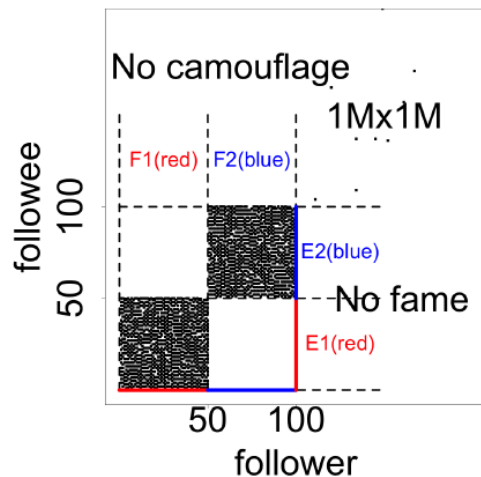
Spectral Subspace Plot



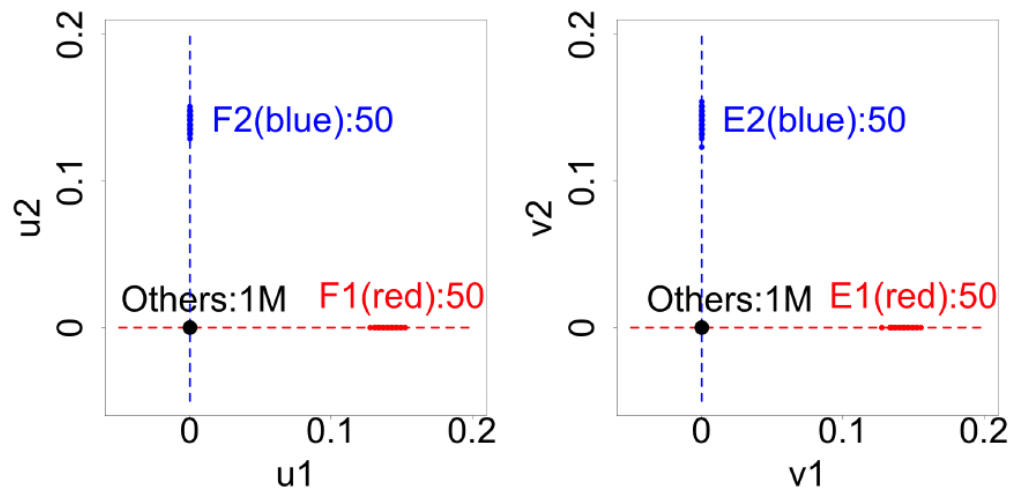
Lockstep and Spectral Subspace Plot

- Case #1: non-overlapping lockstep
- “Blocks” \longleftrightarrow “Rays”

Adjacency Matrix



Spectral Subspace Plot

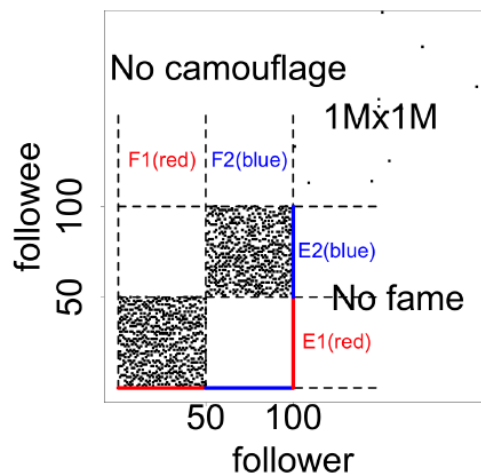


Rule 1 (short “rays”): two blocks, high density (90%), no “camouflage”, no “fame”

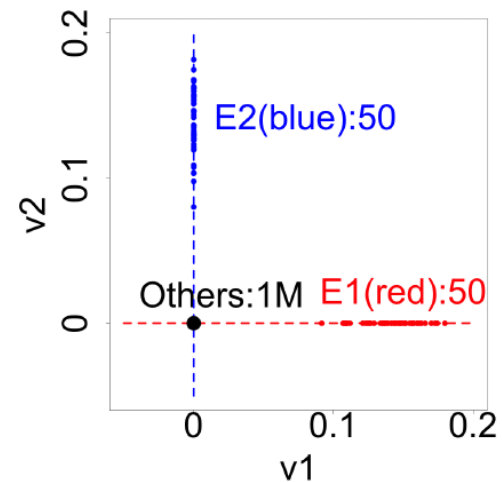
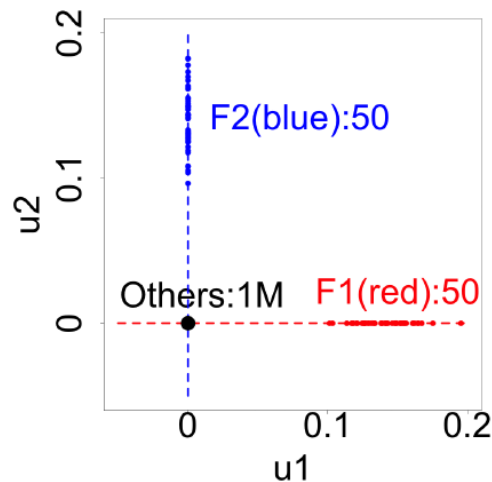
Lockstep and Spectral Subspace Plot

- Case #2: non-overlapping lockstep
- “Blocks; low density” \longleftrightarrow Elongation

Adjacency Matrix



Spectral Subspace Plot



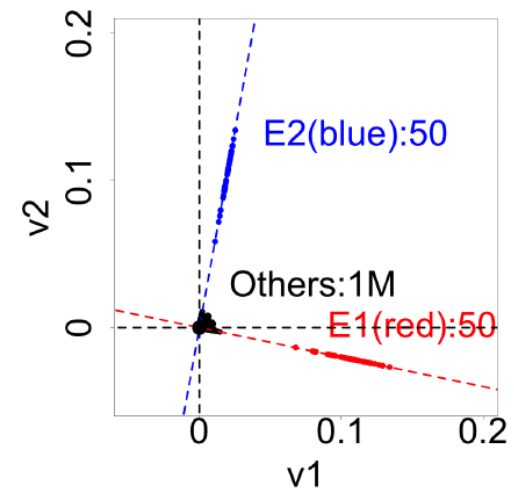
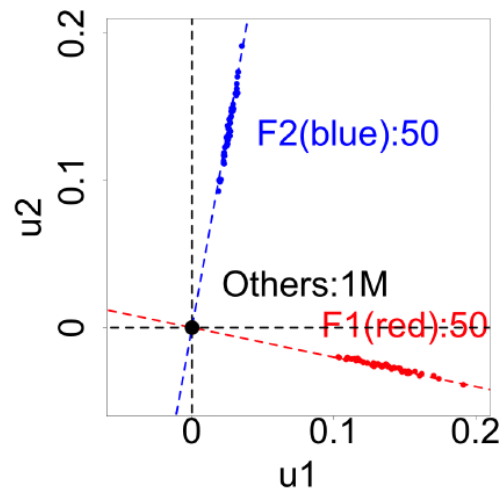
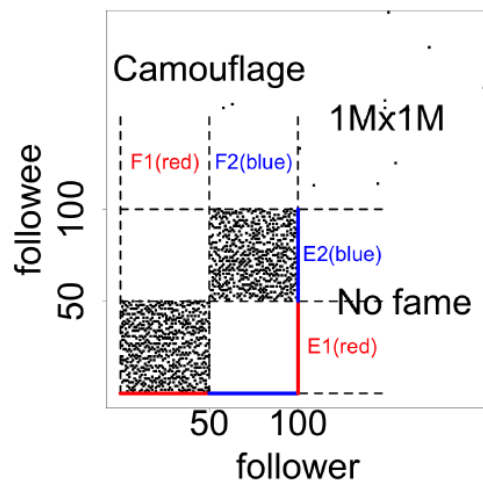
Rule 2 (long “rays”): two blocks, low density (50%), no “camouflage”, no “fame”

Lockstep and Spectral Subspace Plot

- Case #3: non-overlapping lockstep
- “Camouflage” (or “Fame”) \longleftrightarrow Tilting
“Rays”

Adjacency Matrix

Spectral Subspace Plot

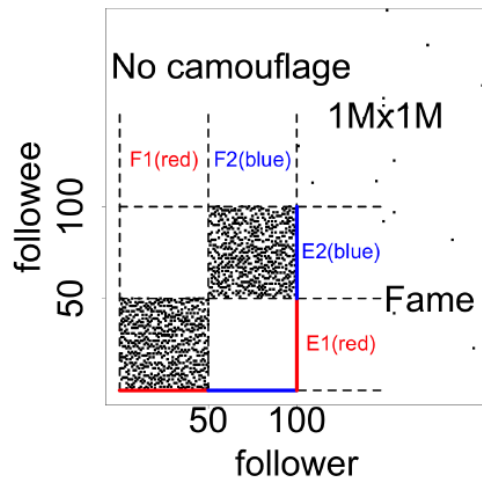


Rule 3 (tilting “rays”): two blocks, with “camouflage”, no “fame”

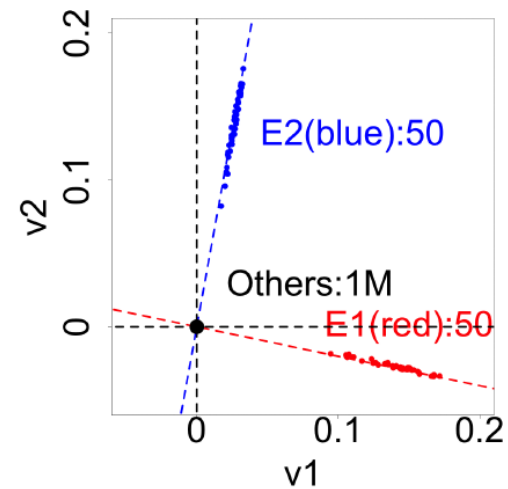
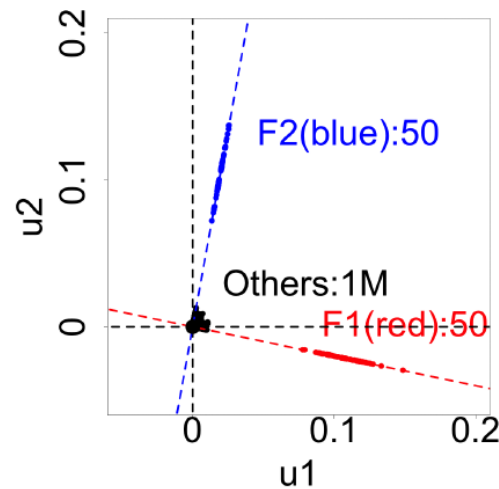
Lockstep and Spectral Subspace Plot

- Case #3: non-overlapping lockstep
- “Camouflage” (or “**Fame**”) \longleftrightarrow Tilting
“Rays”

Adjacency Matrix




Spectral Subspace Plot



Rule 3 (tilting “rays”): two blocks, no “camouflage”, with “fame”

Dataset

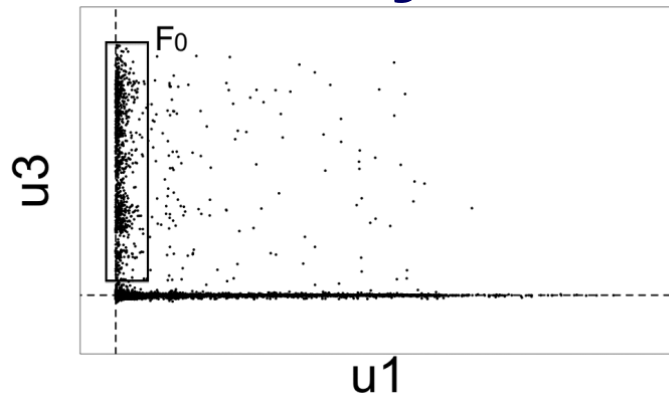
- Tencent Weibo 
- 117 million nodes (with profile and UGC data)
- 3.33 billion directed edges



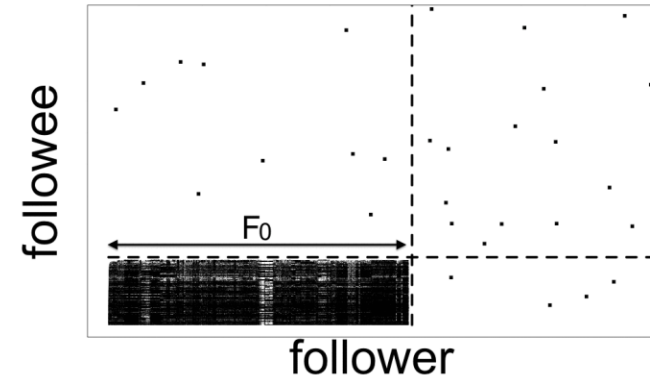
Real Data



“Rays”

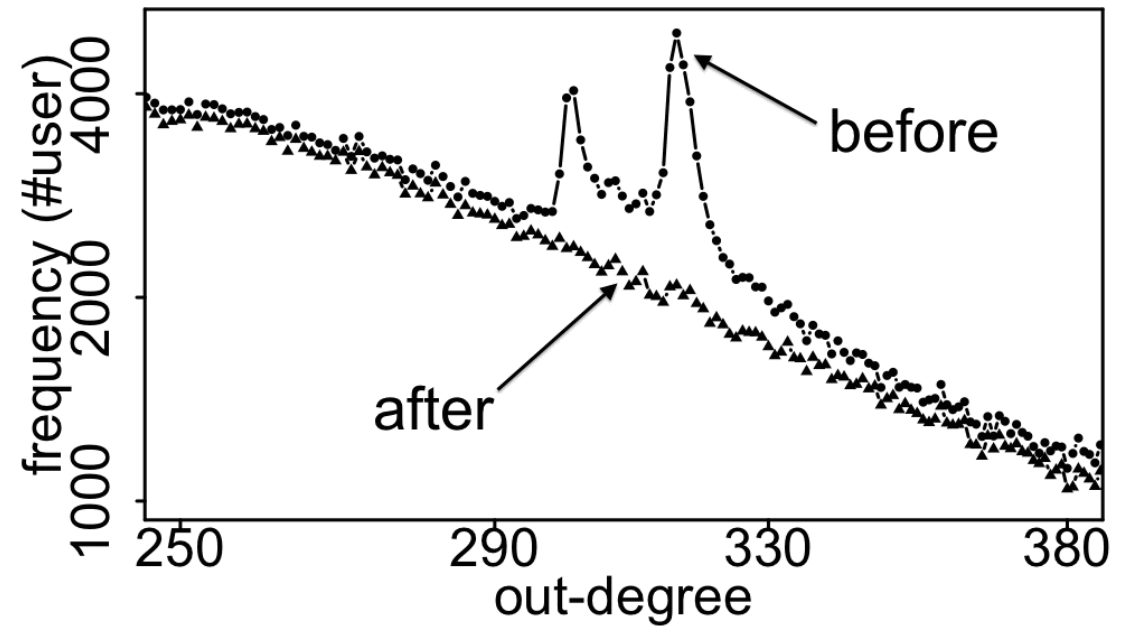
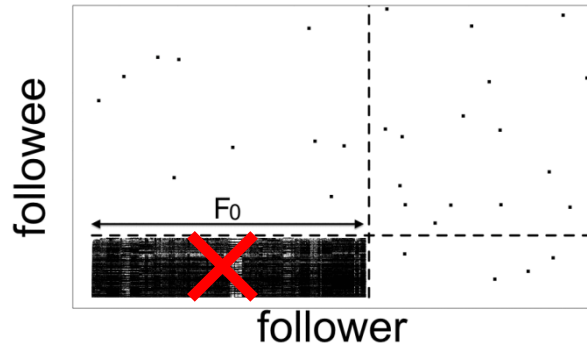


“Block”



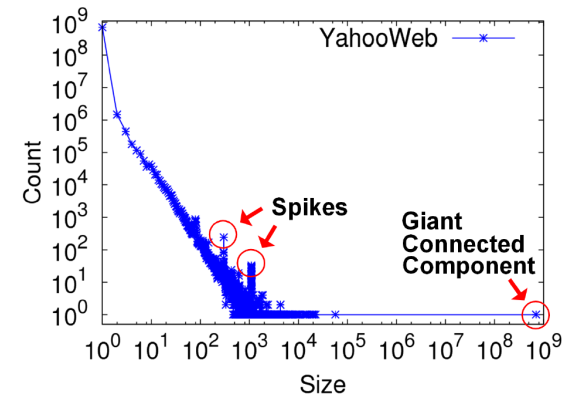
Real Data

- Spikes on the out-degree distribution



Summary of Part#1

- *many* patterns in real graphs
 - Power-laws everywhere
 - Long (and growing) list of tools for anomaly/ fraud detection



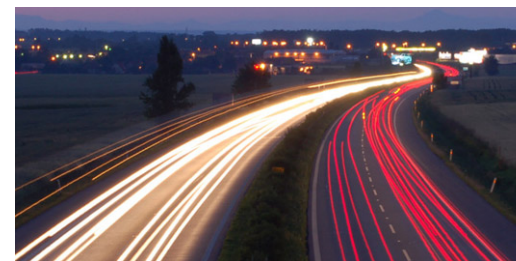
Patterns



anomalies

Roadmap

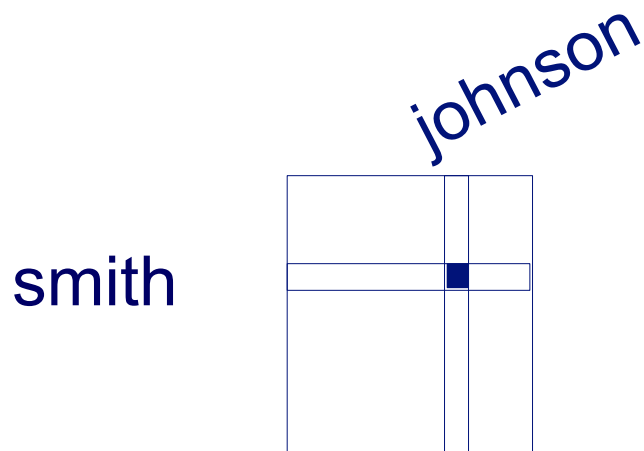
- Introduction – Motivation
- Part#1: Patterns in graphs
- Part#2: time-evolving graphs
 - ➔ – P2.1: tools/tensors
 - P2.2: other patterns
- Conclusions



Part 2: Time evolving graphs; tensors

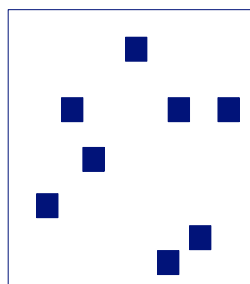
Graphs over time -> tensors!

- Problem #2.1:
 - Given who calls whom, and when
 - Find patterns / anomalies



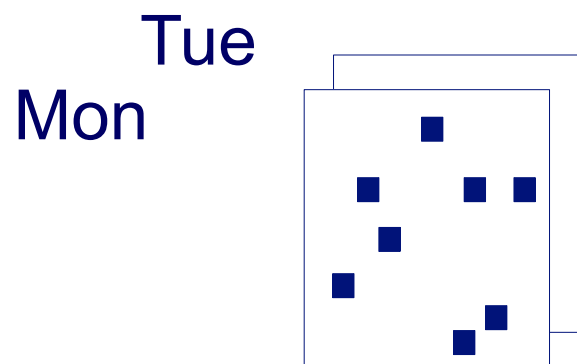
Graphs over time -> tensors!

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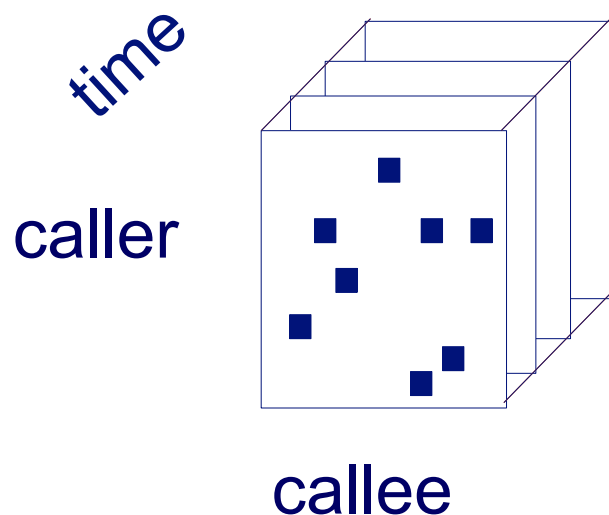
Graphs over time -> tensors!

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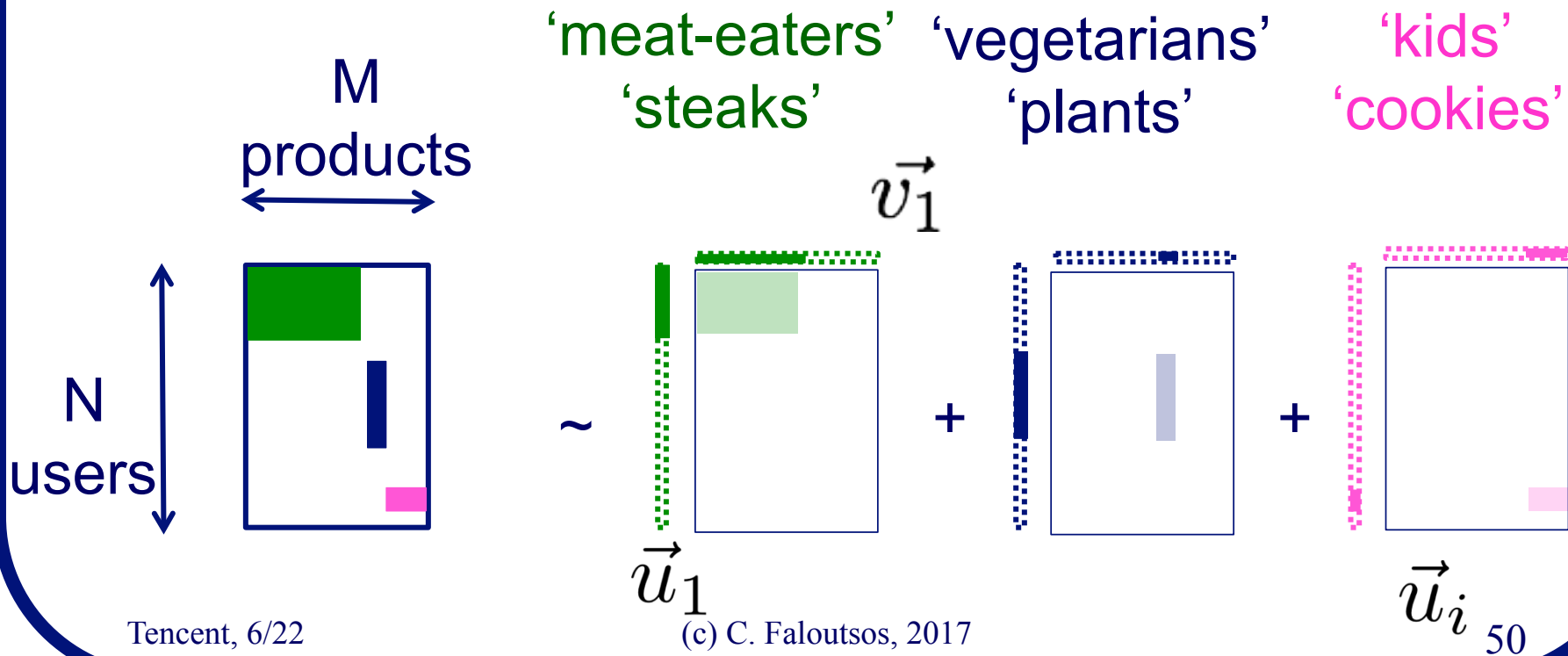
Graphs over time -> tensors!

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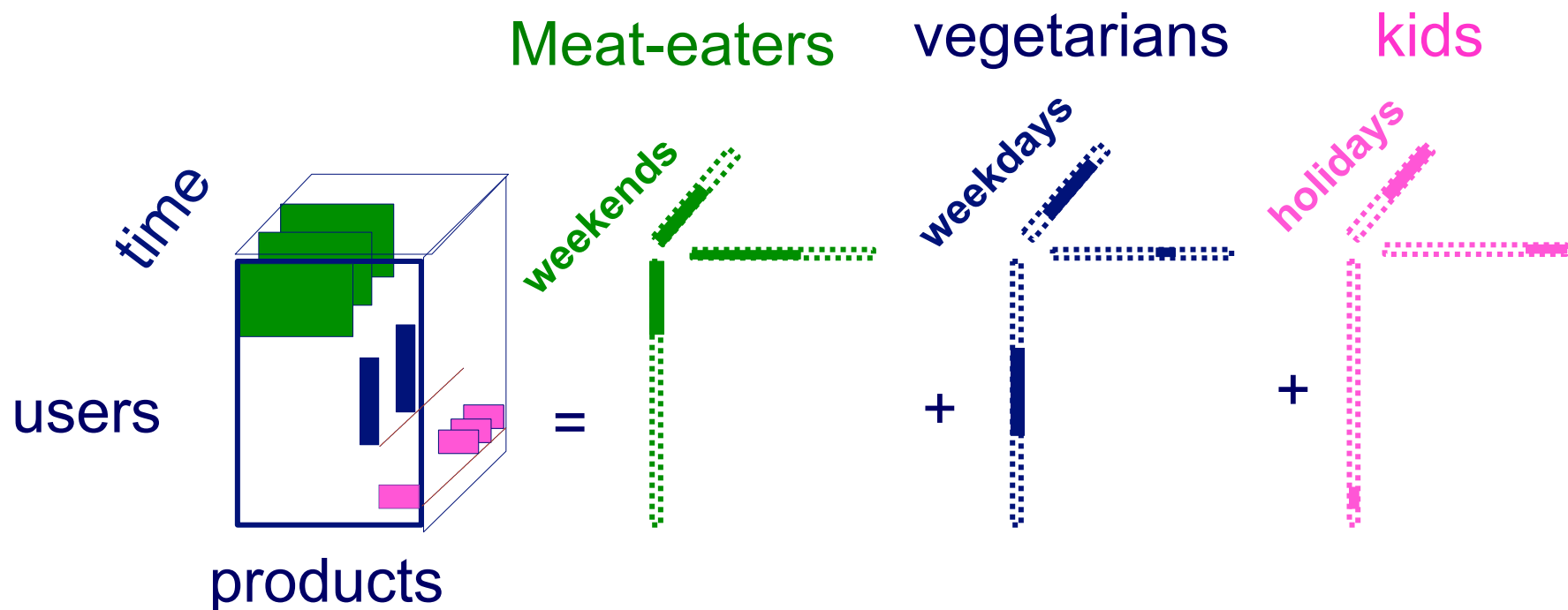
Answer : tensor factorization

- Recall: (SVD) matrix factorization: finds blocks



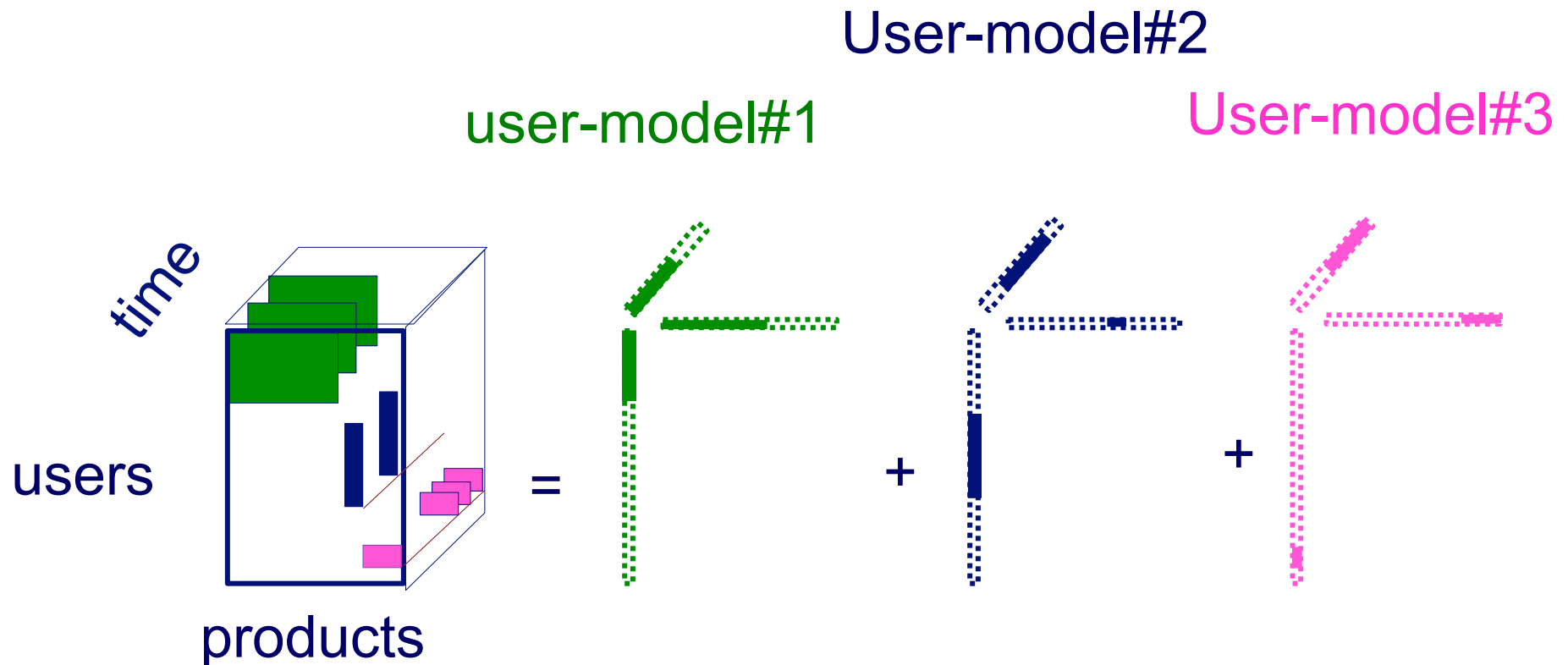
Answer: tensor factorization

- PARAFAC decomposition



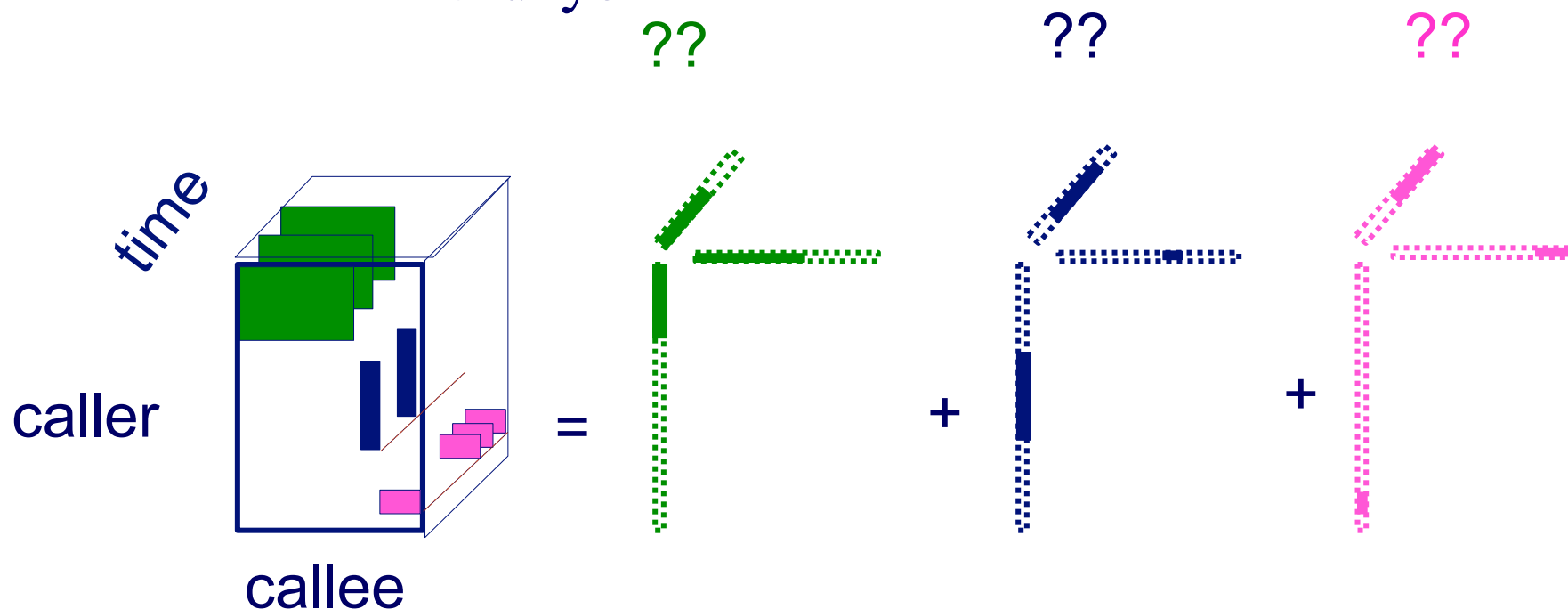
Answer: tensor factorization

- PARAFAC decomposition

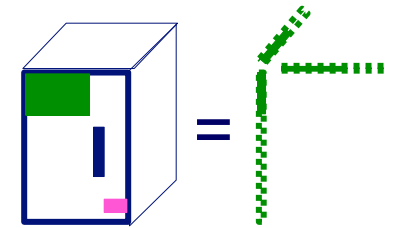


Answer: tensor factorization

- PARAFAC decomposition
- Results for who-calls-whom-when
 - 4M x 15 days

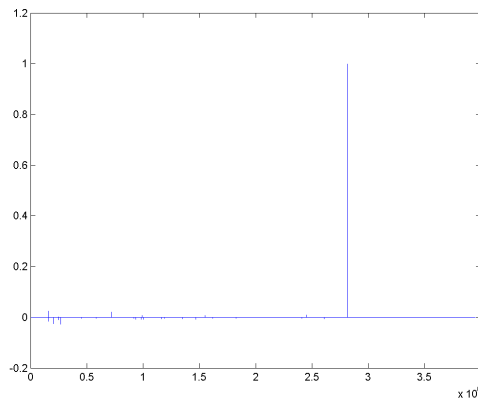


Anomaly detection in time-evolving graphs

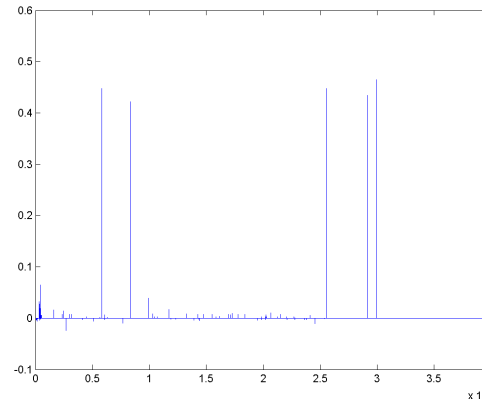


- Anomalous communities in phone call data:
 - European country, 4M clients, data over 2 weeks

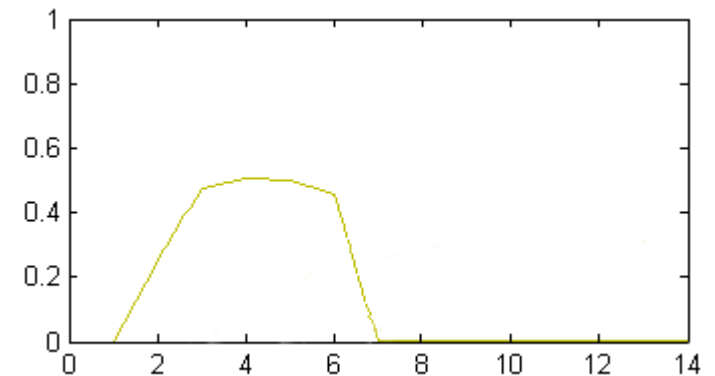
1 caller



5 receivers

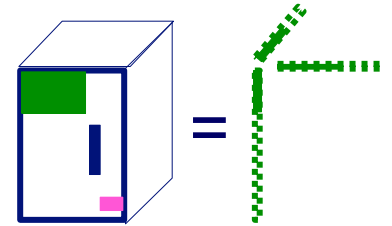


4 days of activity



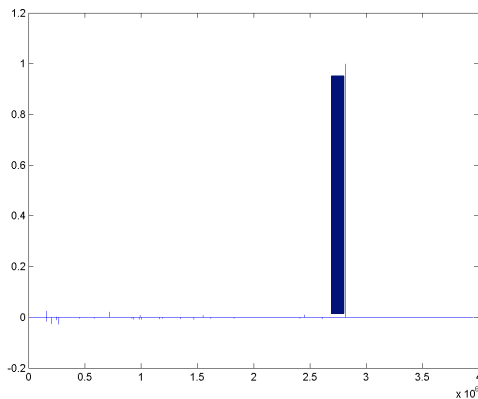
~200 calls to EACH receiver on EACH day!

Anomaly detection in time-evolving graphs

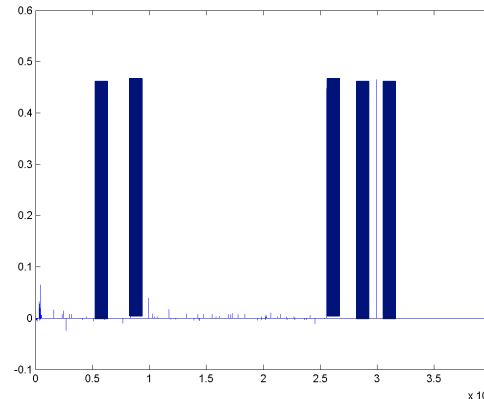


- Anomalous communities in phone call data:
 - European country, 4M clients, data over 2 weeks

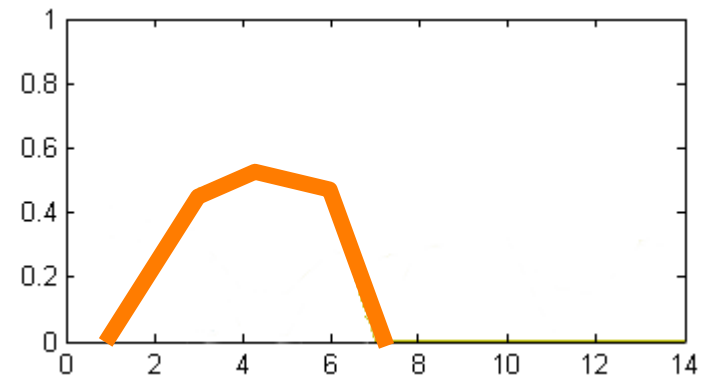
1 caller



5 receivers

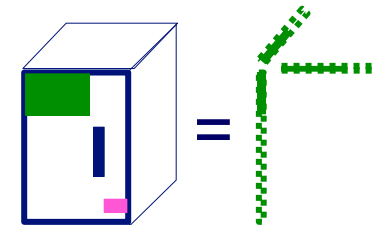


4 days of activity



~200 calls to EACH receiver on EACH day!

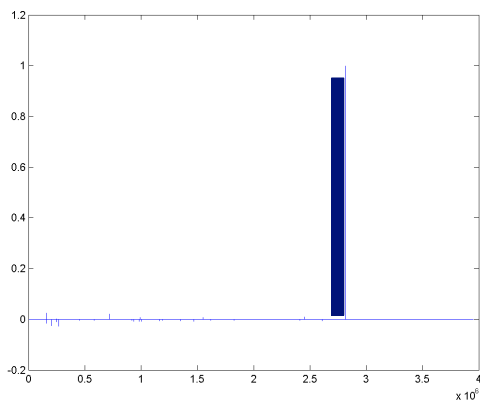
Anomaly detection in time-evolving graphs



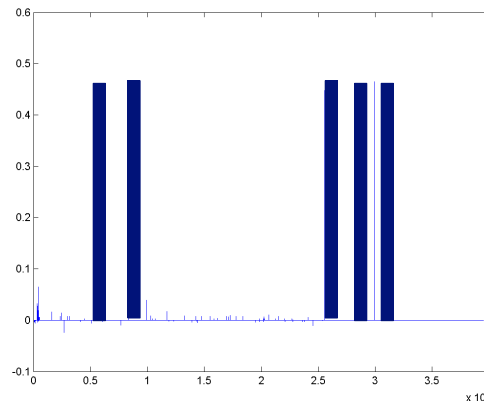
- Anomalous communities in phone call data:
 - European country, 4M clients, data over 2 weeks



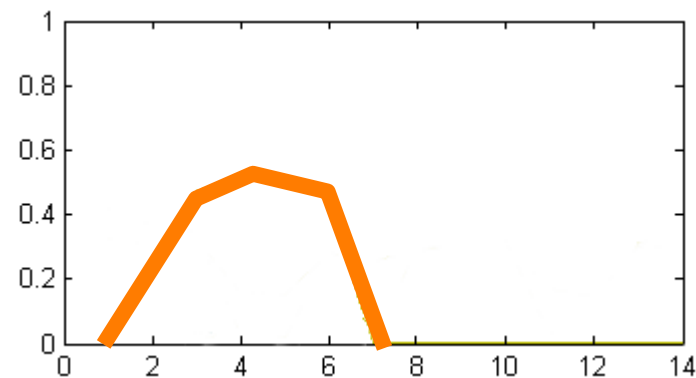
1 caller



5 receivers

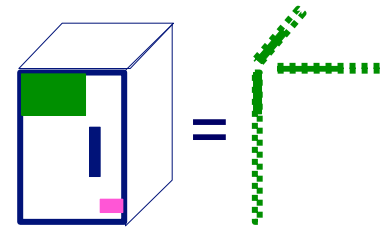


4 days of activity



~200 calls to EACH receiver on EACH day!

Anomaly detection in time-evolving graphs



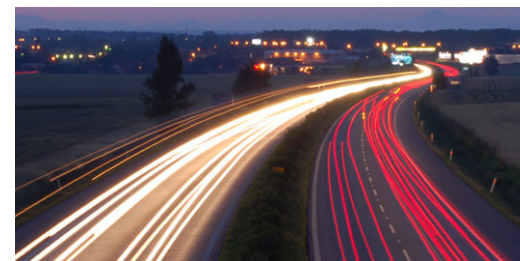
- Anomalous communities in phone call data:
 - European country, 4M clients, data over 2 weeks



Miguel Araujo, Spiros Papadimitriou, Stephan Günnemann, Christos Faloutsos, Prithwish Basu, Ananthram Swami, Evangelos Papalexakis, Danai Koutra. *Com2: Fast Automatic Discovery of Temporal (Comet) Communities*. PAKDD 2014, Tainan, Taiwan.

Roadmap

- Introduction – Motivation
- Part#1: Patterns in graphs
- Part#2: time-evolving graphs
 - P2.1: tools/tensors
 - P2.2: other patterns
 - inter-arrival time
 - Network growth
 - Group evolution
- Conclusions





Beyond Sigmoids: the NetTide Model for Social Network Growth and its Applications

KDD'16

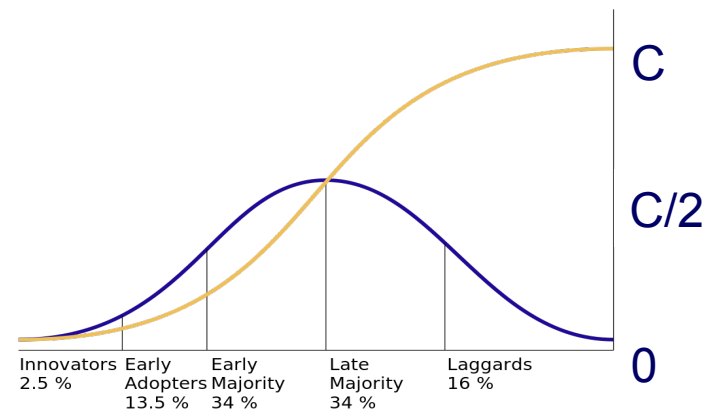
Chengxi Zang 臧承熙, Peng Cui, CF



PROBLEM: $n(t)$ and $e(t)$, over time?

- $n(t)$: the number of nodes.
- $e(t)$: the number of edges.
- E.g.:
 - How many members will  have next month?
 - How many friendship links will  have next year?

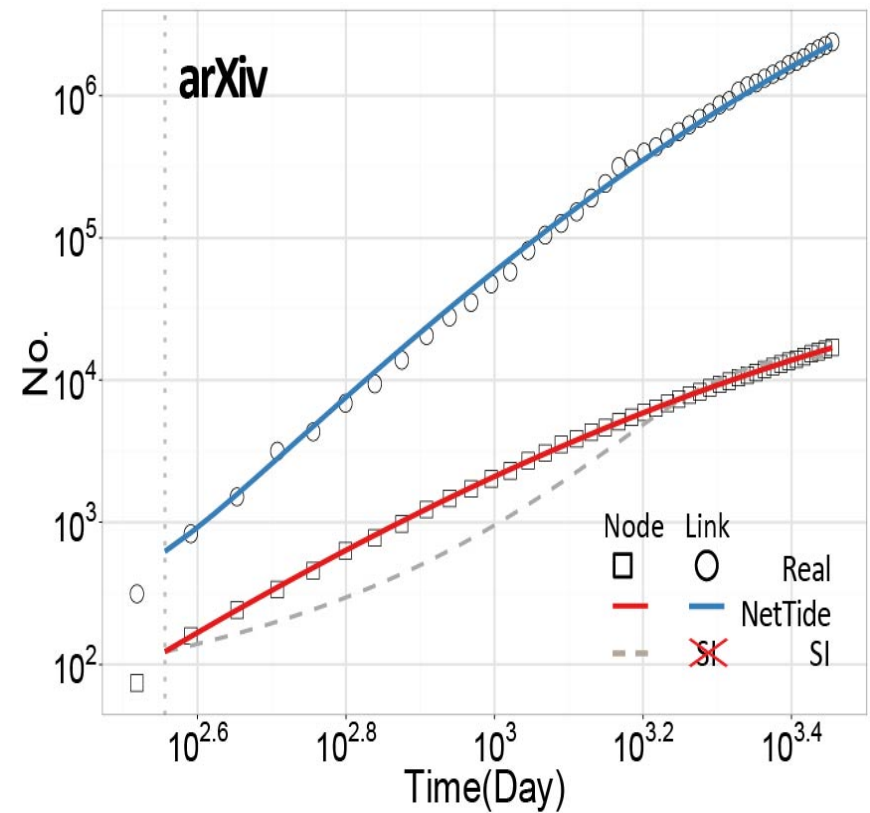
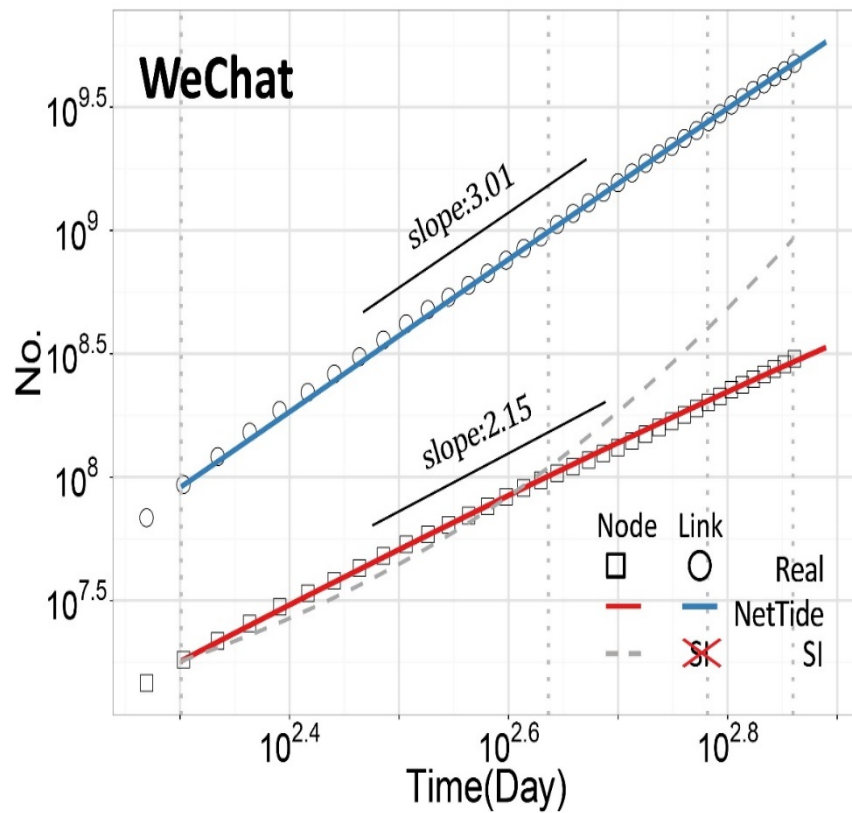
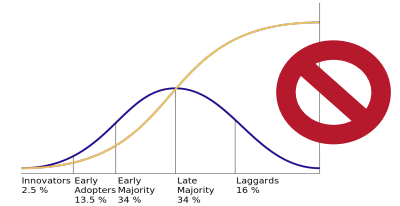
- Linear?
- Exponential?
- Sigmoid?



Datasets

- **WeChat 2011/1-2013/1 300M nodes, 4.75B links**
- ArXiv 1992/3-2002/3 17k nodes, 2.4M links
- Enron 1998/1-2002/7 86K nodes, 600K links
- Weibo 2006 165K nodes, 331K links

A: Power Law Growth



Cumulative growth (Log-Log scale)

Proposed: NetTide Model

- Nodes $n(t)$

$$\frac{dn(t)}{dt} = \frac{\beta}{t^\theta} n(t) (N - n(t))$$

- Links $e(t)$

$$\frac{de(t)}{dt} = \frac{\beta'}{t^\theta} n(t) \left(\alpha(n(t) - 1)^\gamma - \frac{e(t)}{n(t)} \right) + 2 \frac{dn(t)}{dt}$$

NetTide-Node Model



$$\frac{dn(t)}{dt} = \frac{\beta}{t^\theta} n(t) (N - n(t))$$

#nodes(t)
Total population

- Intuition:
 - Rich-get-richer
 - Limitation
 - Fizzling nature
- } = SI; ~Bass

NetTide-Node Model



$$\frac{dn(t)}{dt} = \frac{\beta}{t^\theta} n(t) (N - n(t))$$

#nodes(t)
Total population

- Intuition:
 - Rich-get-richer
 - **Limitation**
 - Fizzling nature
- } = SI; ~Bass

NetTide-Node Model



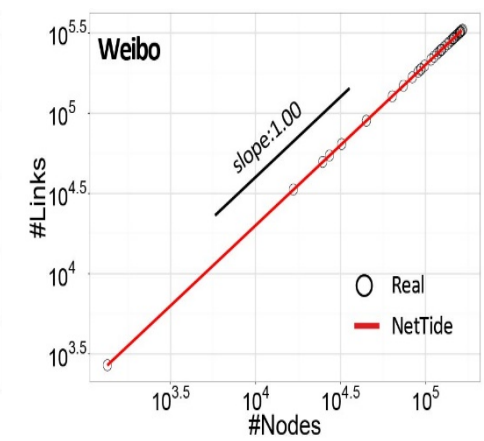
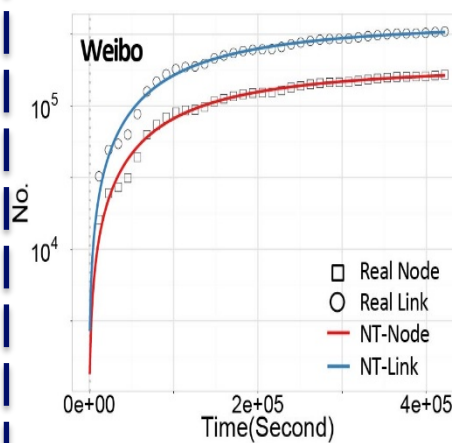
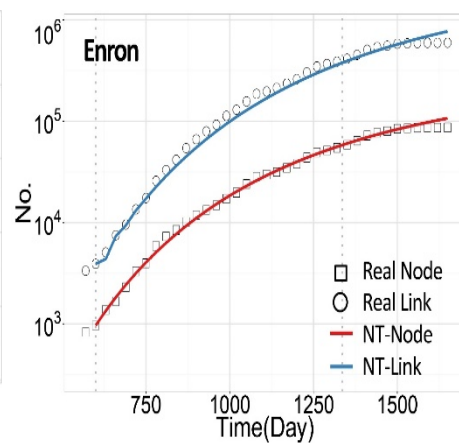
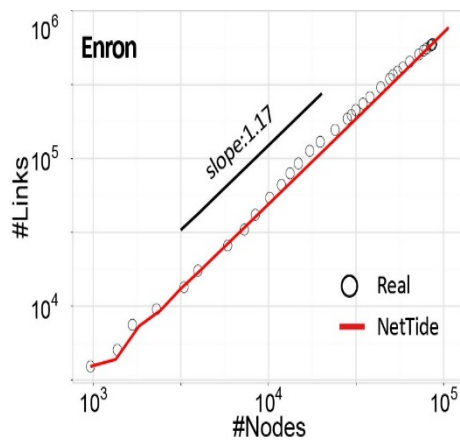
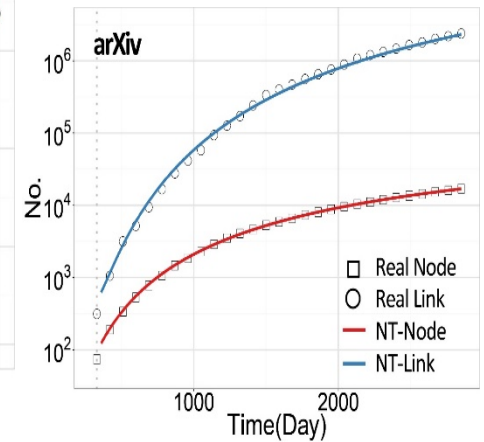
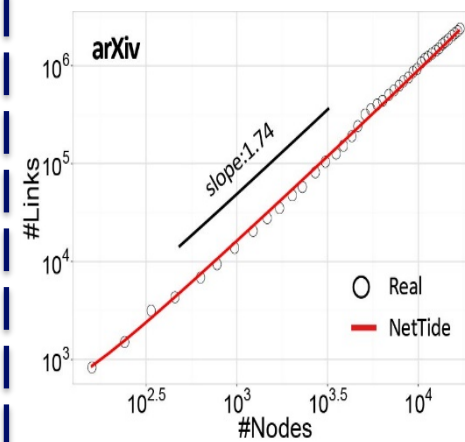
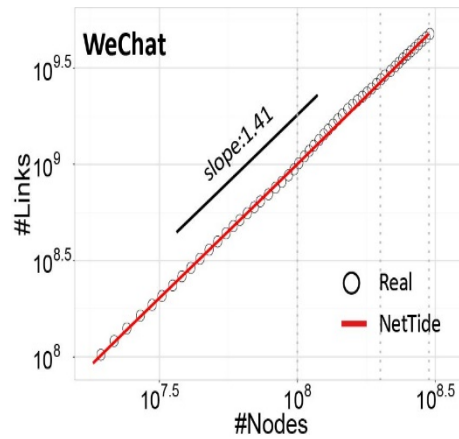
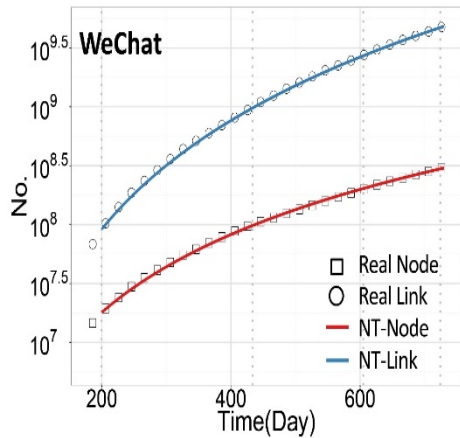
$$\frac{dn(t)}{dt} = \frac{\beta}{t^\theta} n(t) (N - n(t))$$

#nodes(t)

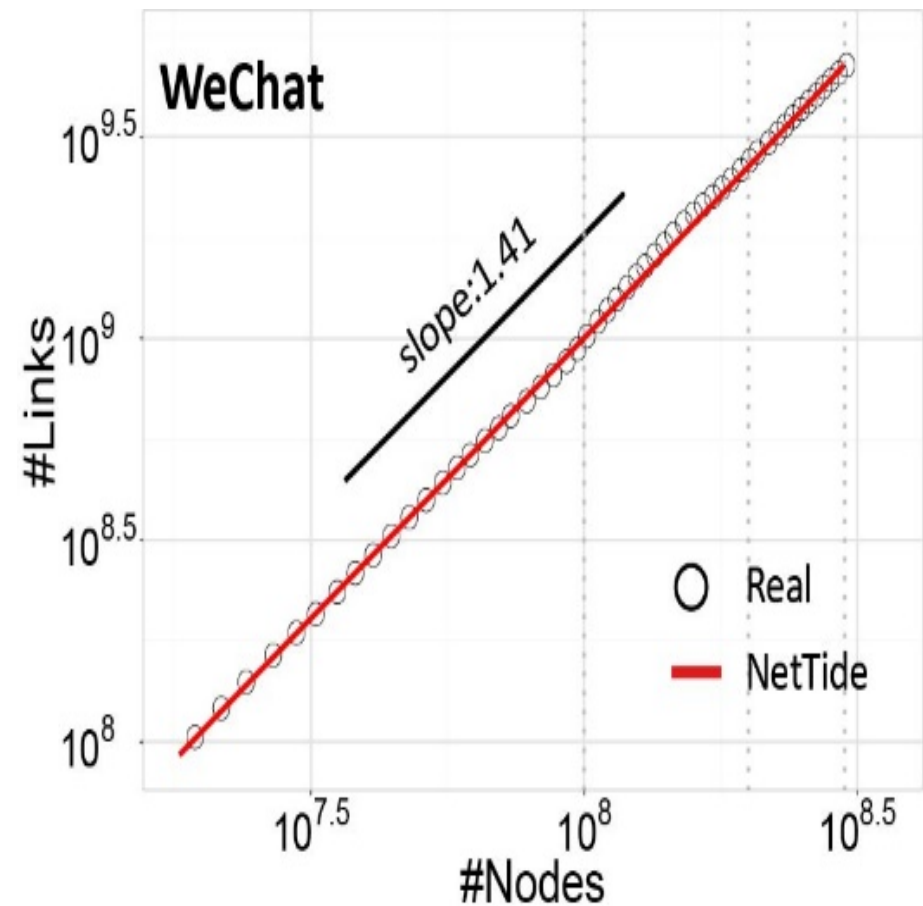
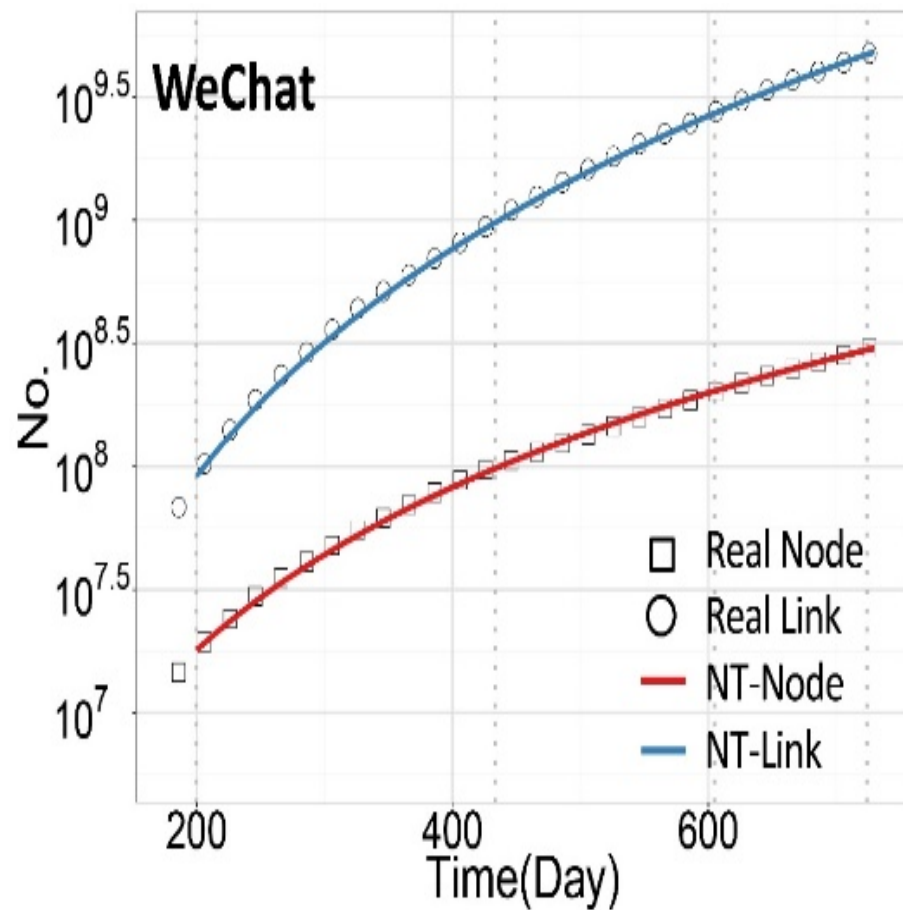
Total population

- Intuition:
 - Rich-get-richer
 - Limitation
 - **Fizzling nature**
- } = SI; ~Bass

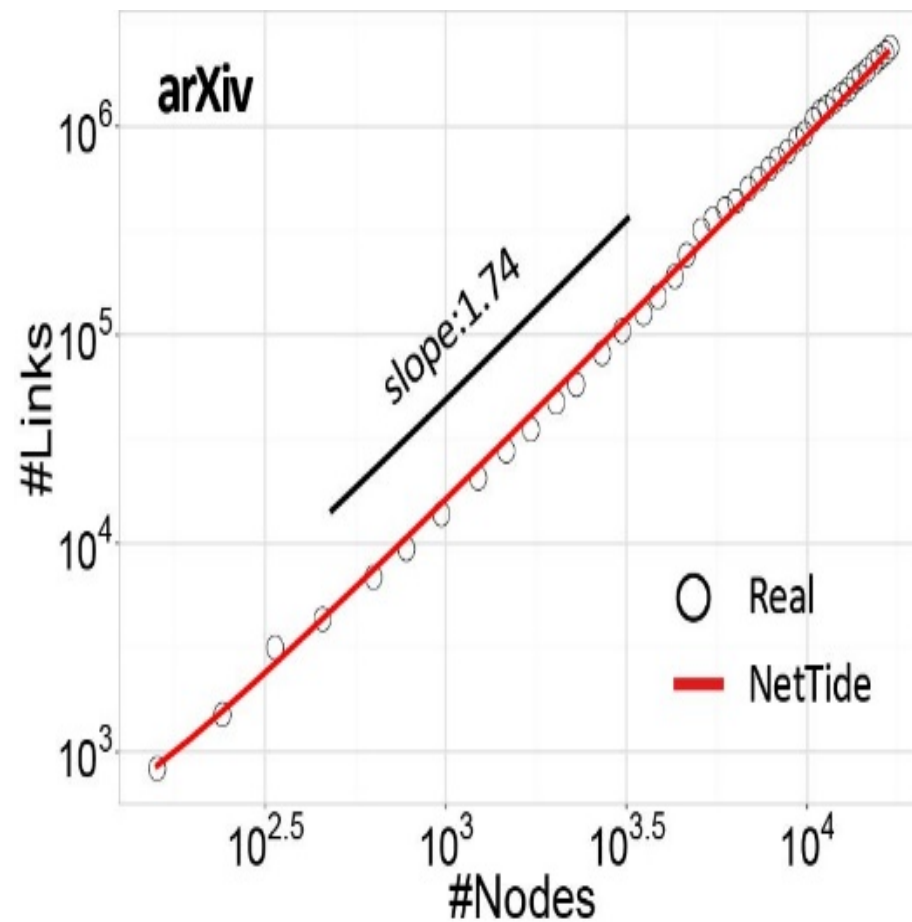
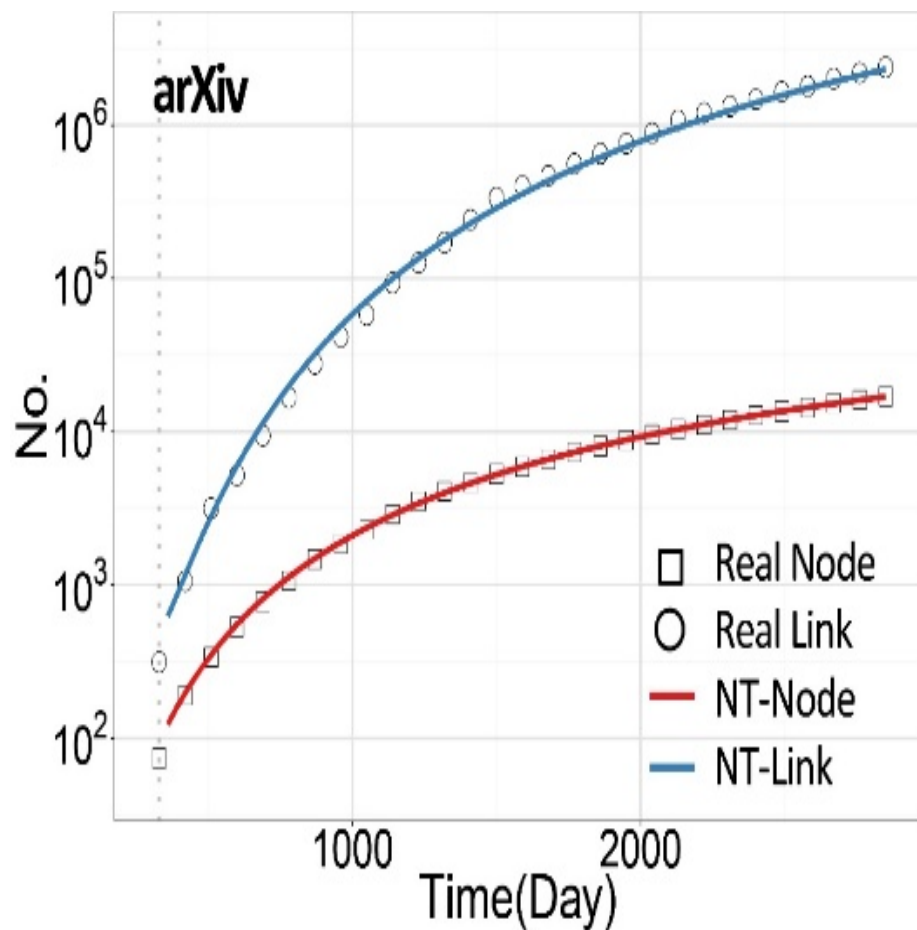
Results: Accuracy



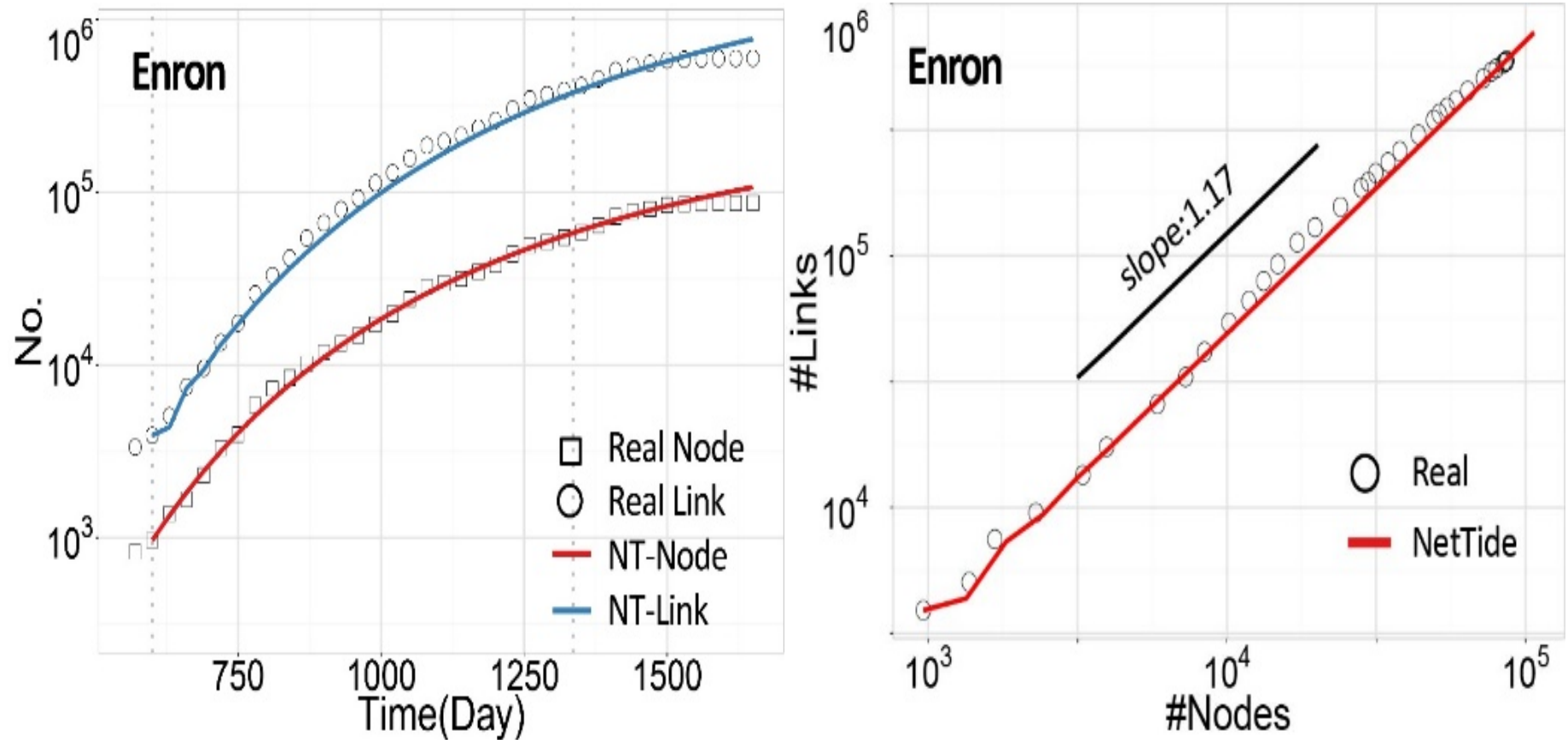
Results: Accuracy



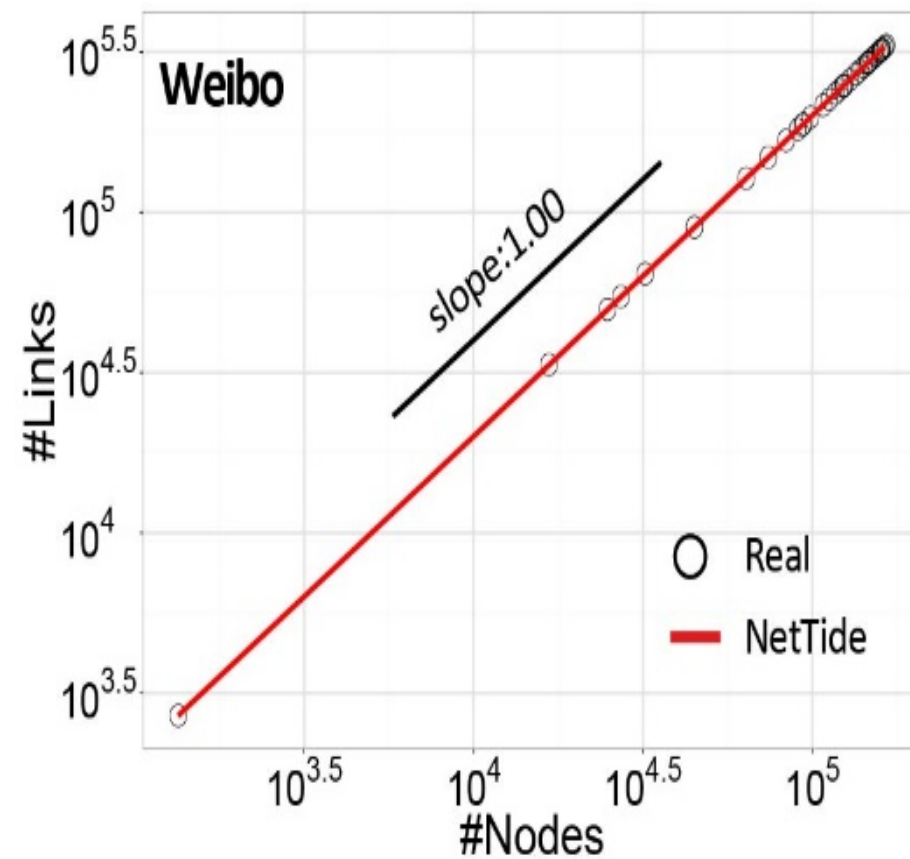
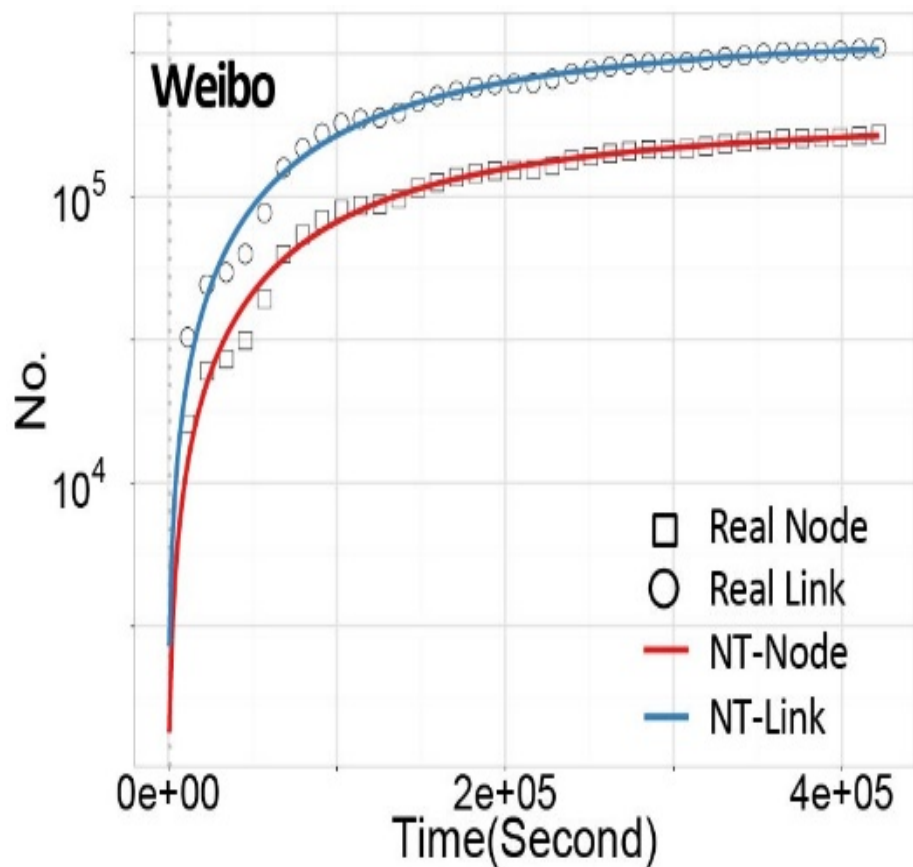
Results: Accuracy



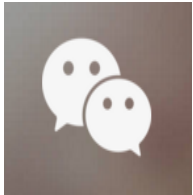
Results: Accuracy



Results: Accuracy

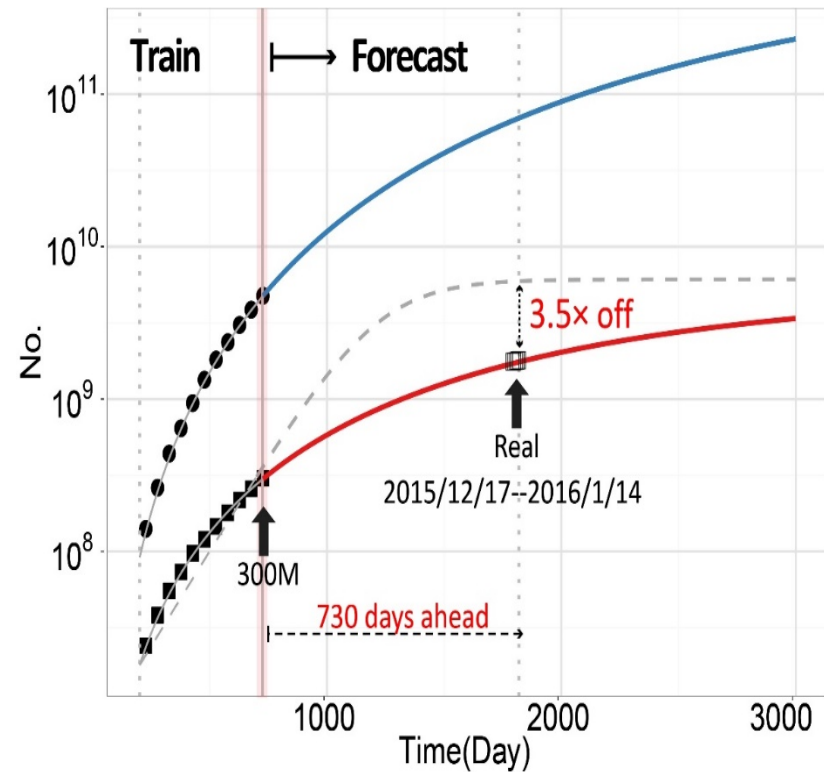
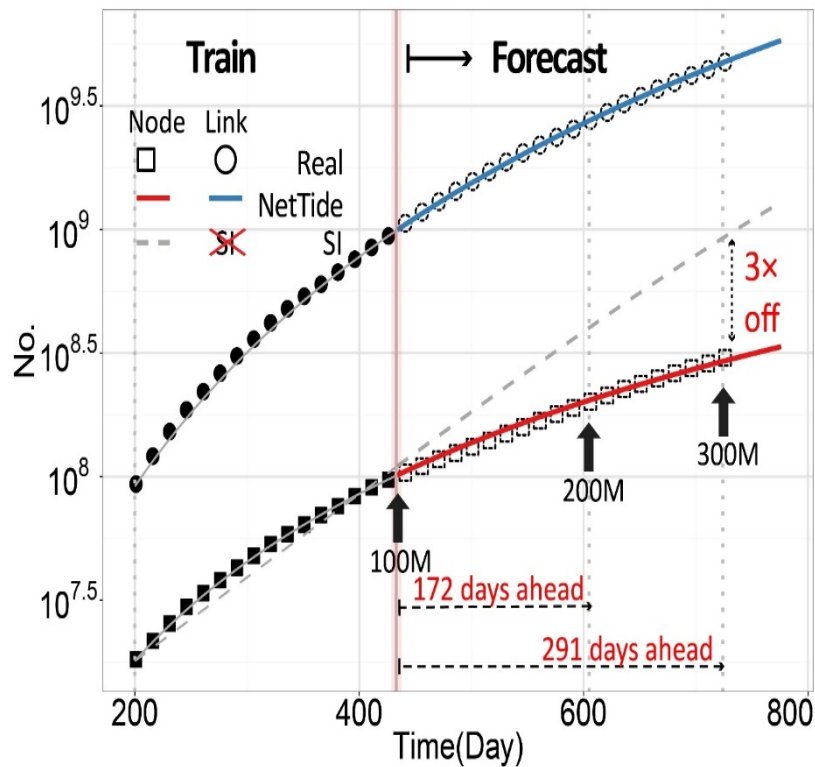


Results: Forecast



WeChat from 100 million to 300 million

730 days ahead



Roadmap

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 - P2.2: other patterns
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 - Network growth
 - Group evolution
- Conclusions





Carnegie
Mellon
University



manLab
MEDIA AND NETWORK

Come-and-Go Patterns of Group Evolution: A Dynamic Model



Tianyang Zhang, Peng Cui, Christos Faloutsos
Yunfei Lu, Hao Ye, Wenwu Zhu, Shiqiang Yang

KDD'16, San Francisco, CA

Social Group Dynamics – An open problem



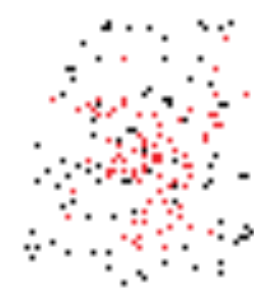
1 hour
N = 5



3 days
N = 77



3 weeks
N = 98



2 months
N = 83

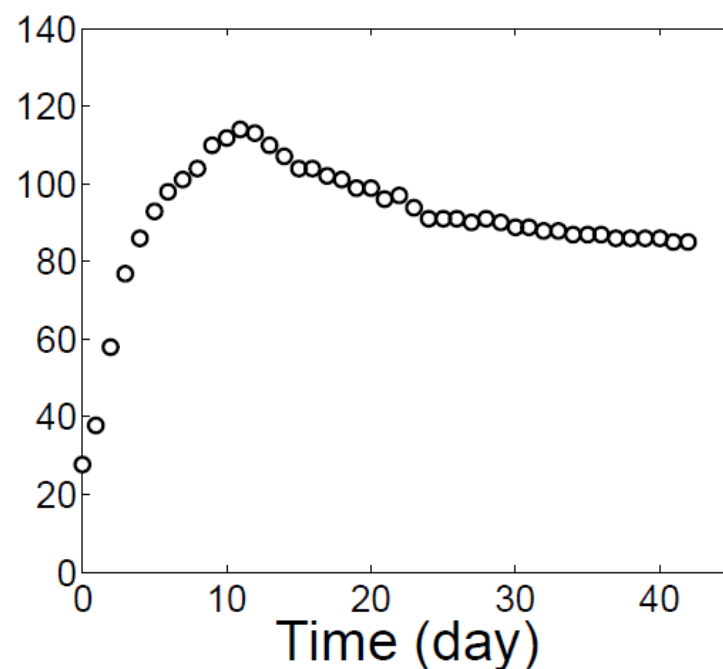
- Will it grow larger or decline?
- Forecast group size after one month?

Our Problem: Group Evolution Process

□ Goals:

- **G1: Discover Patterns**
- **G2: Reveal Mechanisms**
- **G3: Model Evolution Process**

members

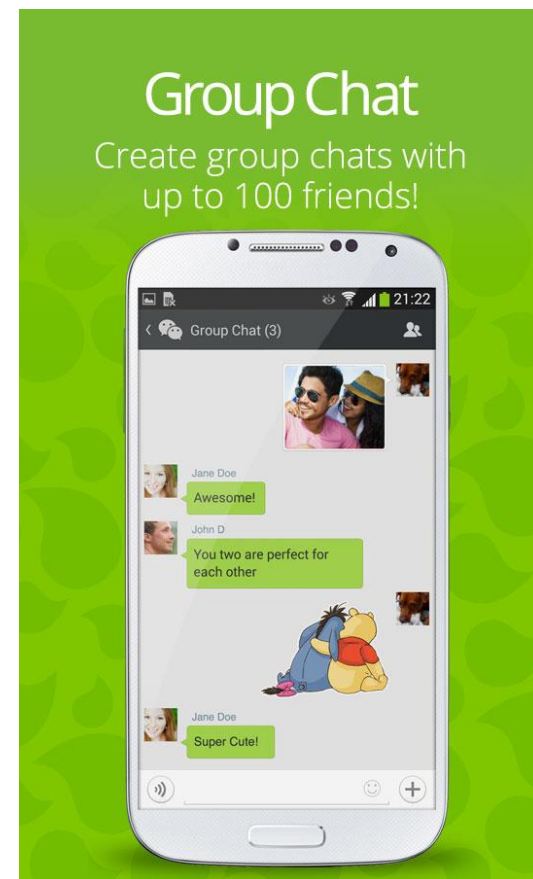


Group Evolution Process

G1: Discover Patterns

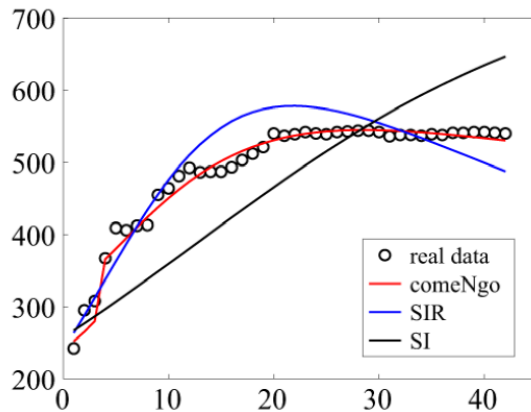
□ Wechat Group dataset

- Largest social network in China
- Sample 100K social groups
- 42 days since established
- 15M records
 - Join / Quit log
 - Temporal information

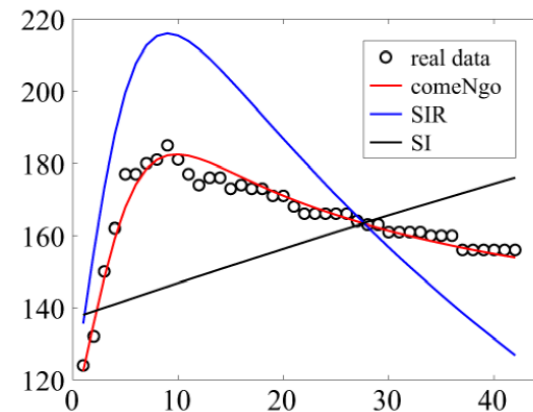


G1: Discover Patterns

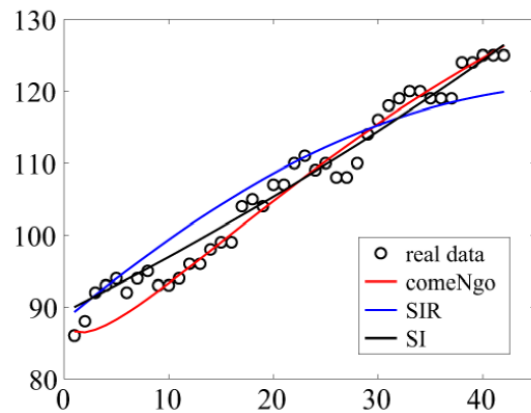
□ Recurring Patterns



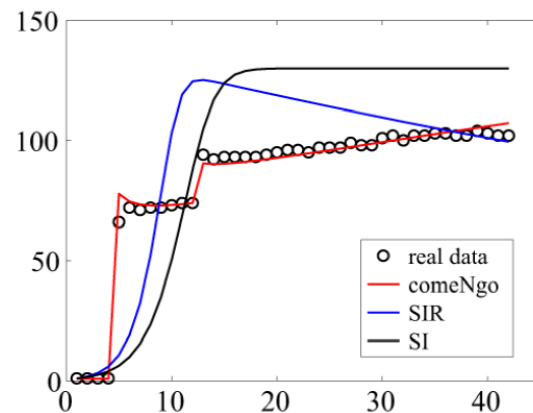
rise and stay



come and go



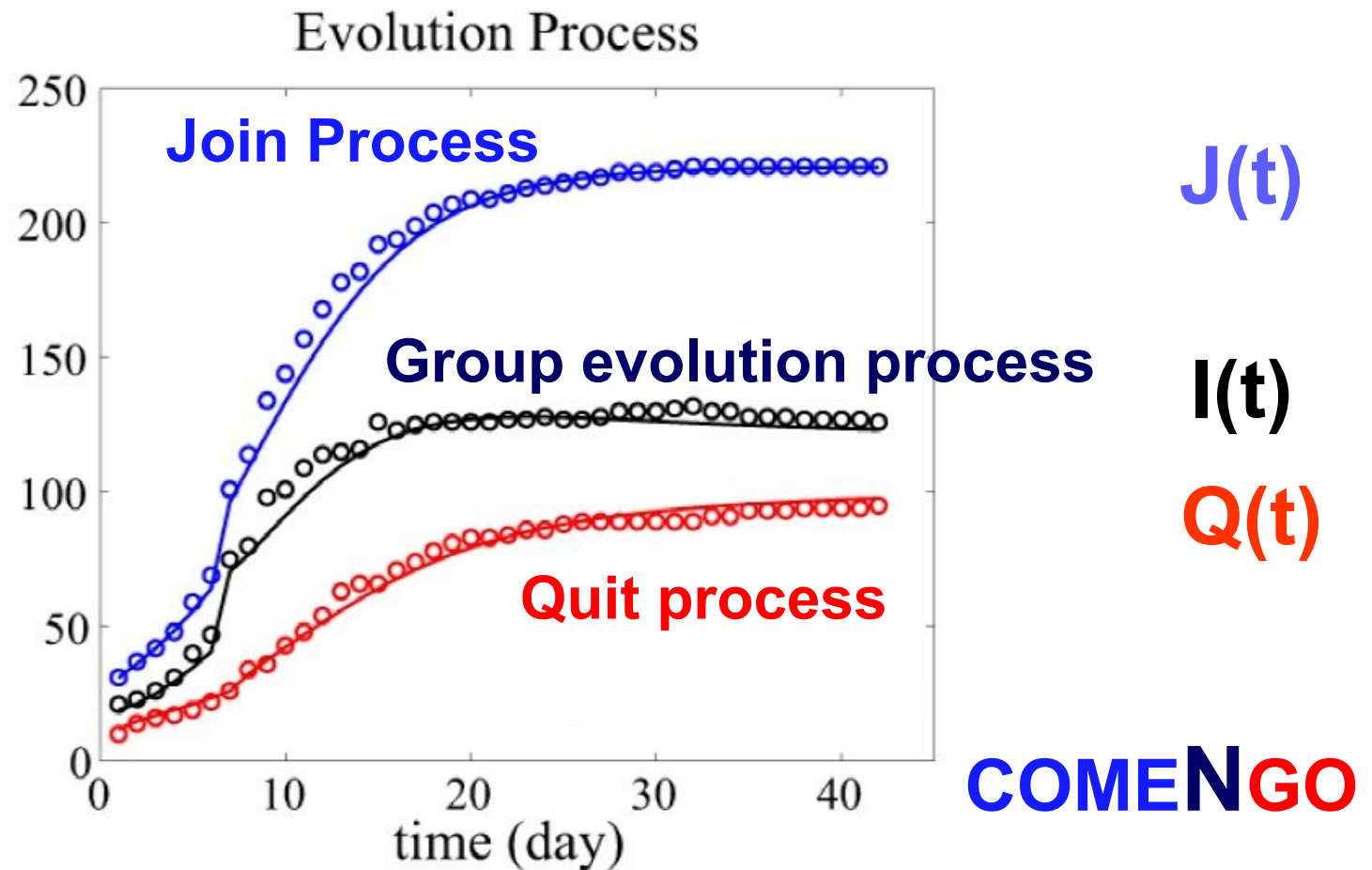
continuous increase



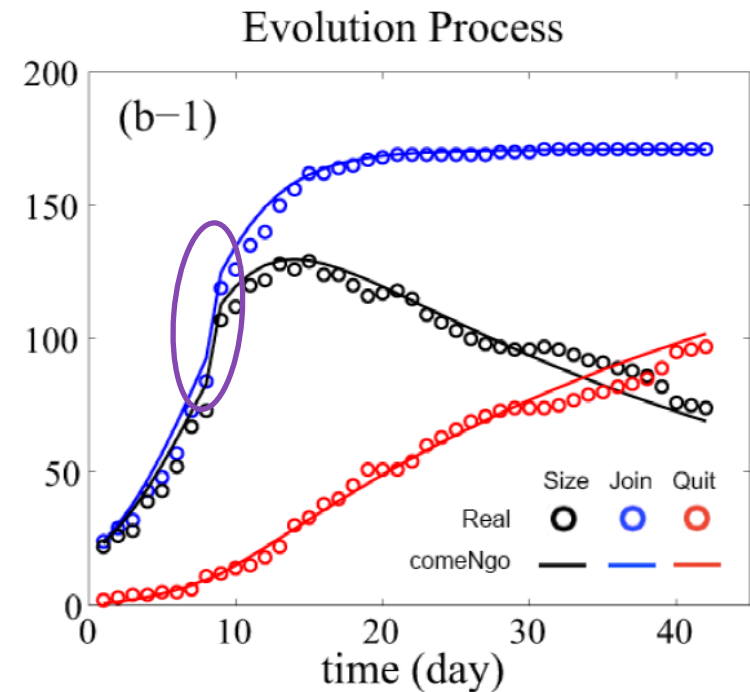
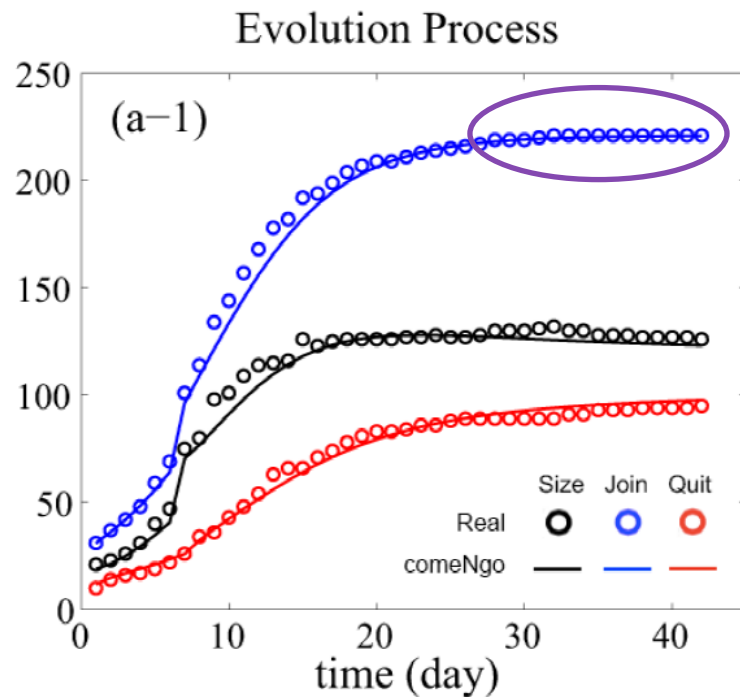
rocket increase

G2: Reveal Mechanisms

□ Join/quit logs

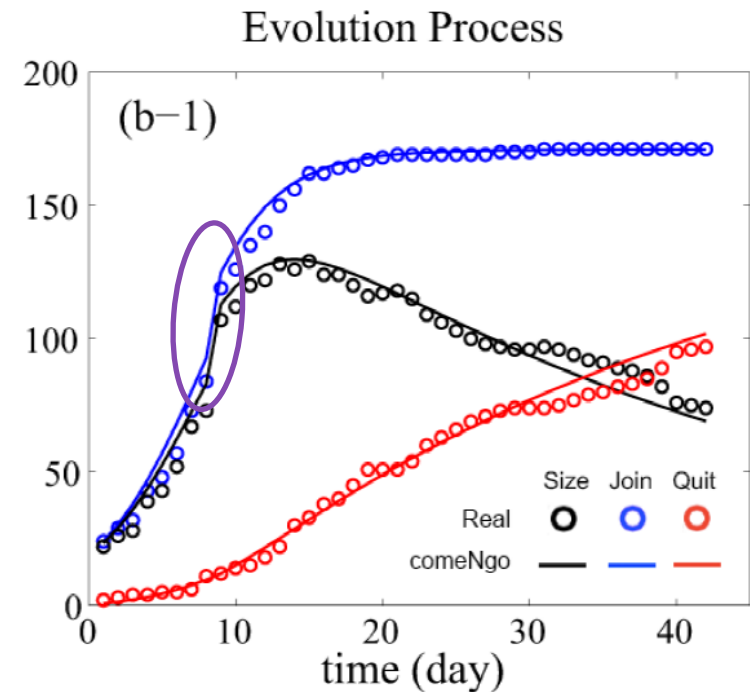
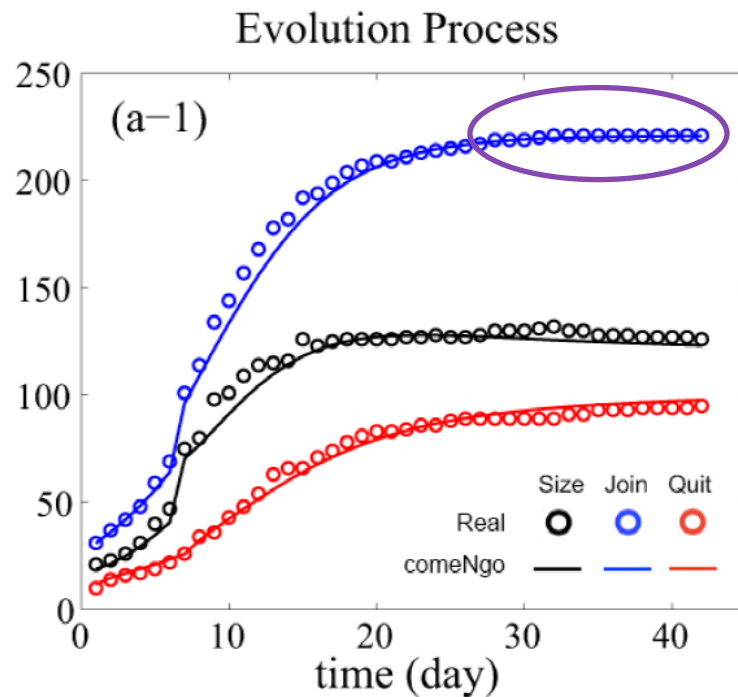


G2: Reveal Mechanisms



- **Q: Can we find (simple) equations, that can fit all these patterns ($J(t)$, $Q(t)$)?**

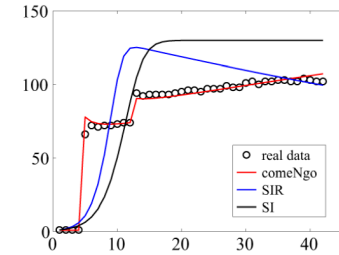
G2: Reveal Mechanisms



- **Q: Can we find (simple) equations, that can fit all these patterns ($J(t)$, $Q(t)$)?**
- **A: Yes!**

G3-1: Dynamic Model – Join

- Join process



diffusion growth

bursty growth

$$J'(t) = \frac{dJ}{dt} = \beta(N - J(t))I(t) + \sum \lambda_i \delta(t - t_i)$$

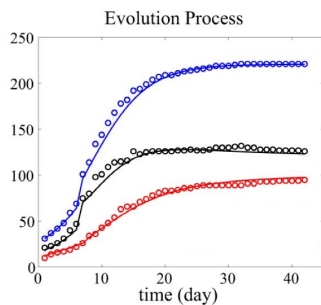
attractiveness

effect strength

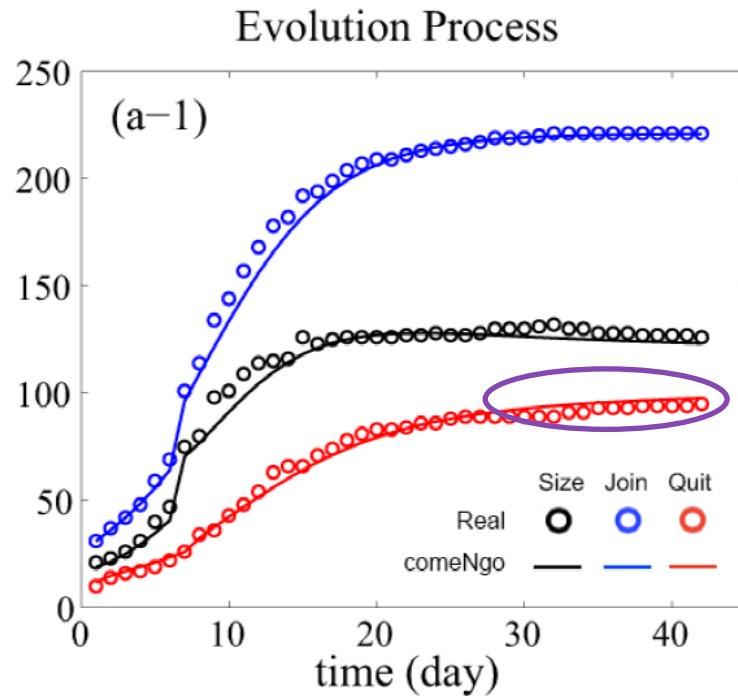
group size

population

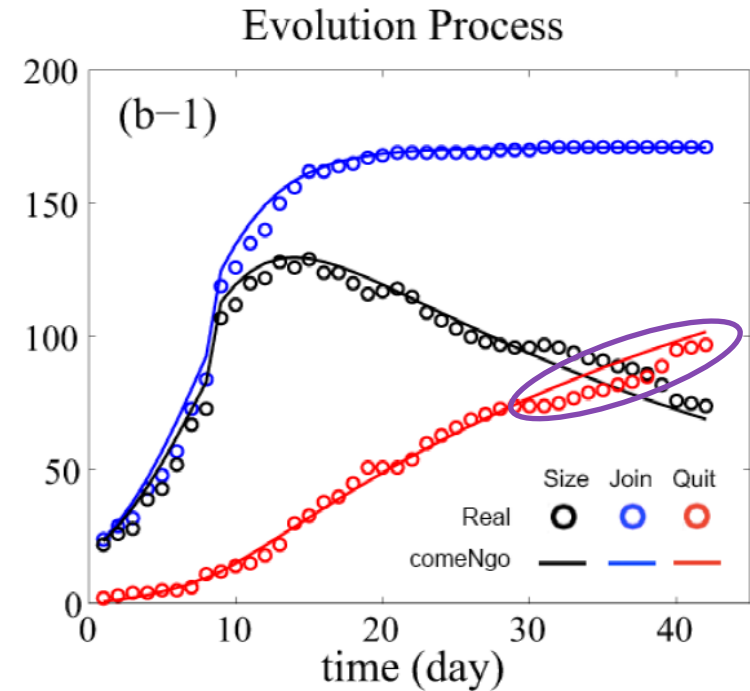
time of external shock



G2-2: Reveal Mechanisms – Quit

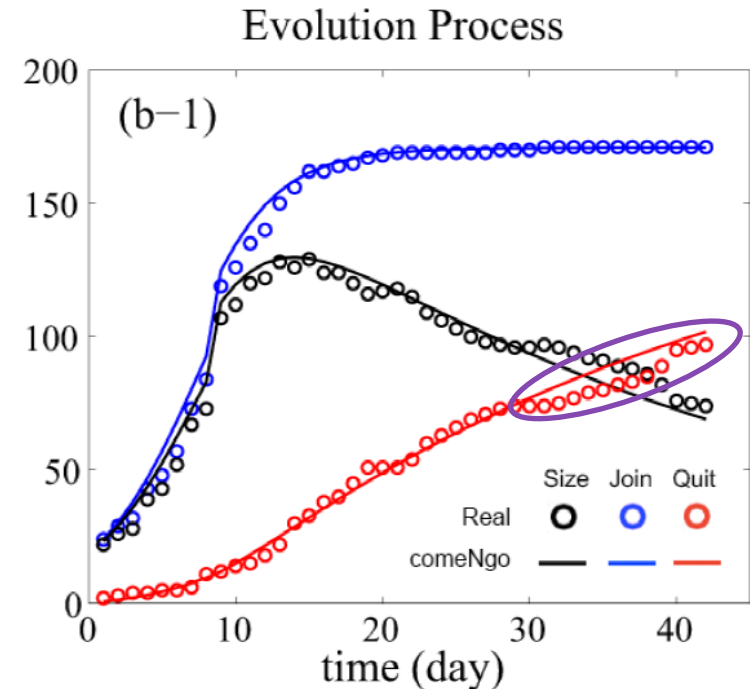
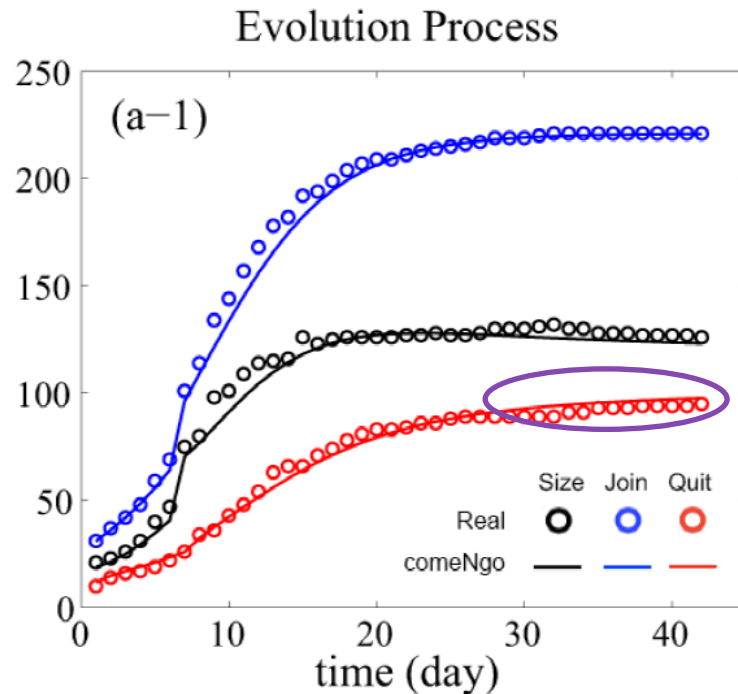


Stabilizing



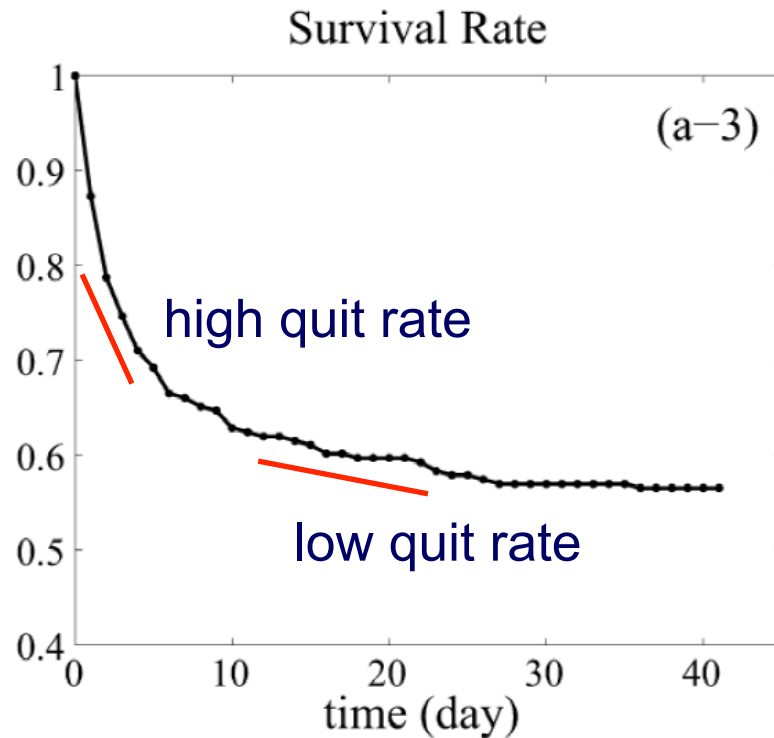
declining

G2-2: Reveal Mechanisms – Quit

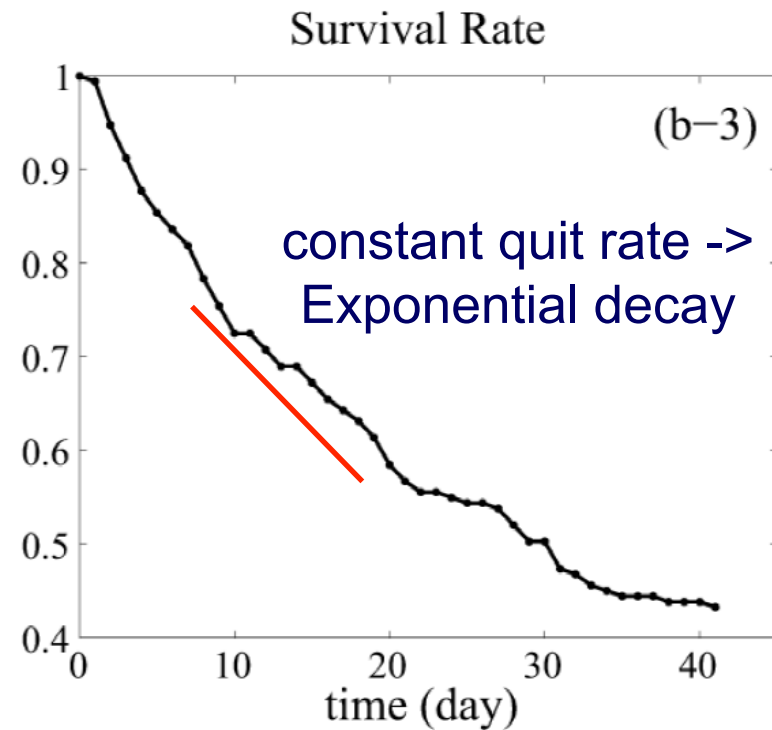


Q: Quitting: exponential ('half life' == SIR) ?

G2-2: Reveal Mechanisms – Quit Process



Gradually stable
85% of all groups:
Decay over holding time



Continuous decline
15% of all groups:
Constant quit rate

G3-2: Dynamic Model – Quit Process

□ Quit process – Quit rate:

$$\underline{\gamma(\tau)} = \gamma_0 \tau^{-\alpha} \quad \left\{ \begin{array}{l} \alpha=0, \text{ exact exponential distribution} \\ 0 < \alpha < 1, \text{ exponential like distribution} \\ \alpha > 1, \text{ power-law distribution} \end{array} \right.$$

G3-2: Dynamic Model – Quit Process

□ Quit process

- Quit rate may decrease over holding time τ
- Power-law or Exponential distributed holding time

$$\underline{\gamma(\tau)} = \gamma_0 \tau^{-\alpha} \quad \left\{ \begin{array}{l} \alpha=0, \text{ exact exponential distribution} \\ 0 < \alpha < 1, \text{ exponential like distribution} \\ \alpha > 1, \text{ power-law distribution} \end{array} \right.$$

Quit Rate

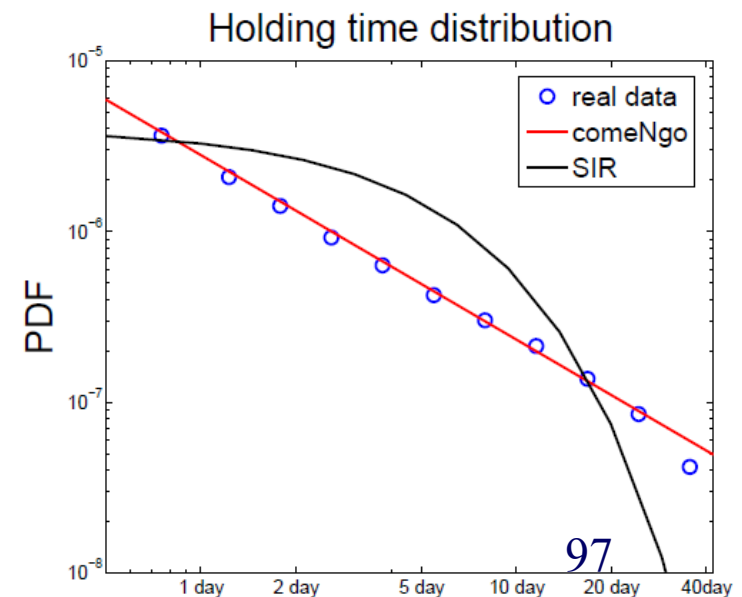
$$\underline{f(\tau)} = c \tau^{-\alpha} \exp\left(\frac{\gamma_0 \tau^{1-\alpha}}{\alpha - 1}\right)$$

p.d.f of holding time τ

- γ_0 : short time satisfaction degree
- α : long time dependence

Tencent, 6/22

(c) C. Faloutsos, 2017



G3: Dynamic Model - COMENGO

$$J(t) = ? \quad Q(t) = ?$$

group size: $I(t) = J(t) - Q(t)$

join process: $J'(t) = \frac{dJ}{dt} = \beta(N - J(t))I(t) + \sum \lambda_i \delta(t - t_i)$

quit process: $Q'(t) = \frac{dQ}{dt} = \int_0^t J'(x) f(t - x) dx$

holding time: $f(\tau) = c\tau^{-\alpha} \exp\left(\frac{\gamma_0 \tau^{1-\alpha}}{\alpha - 1}\right)$

G3: Dynamic Model - COMENGO

group size: $I(t) = J(t) - Q(t)$

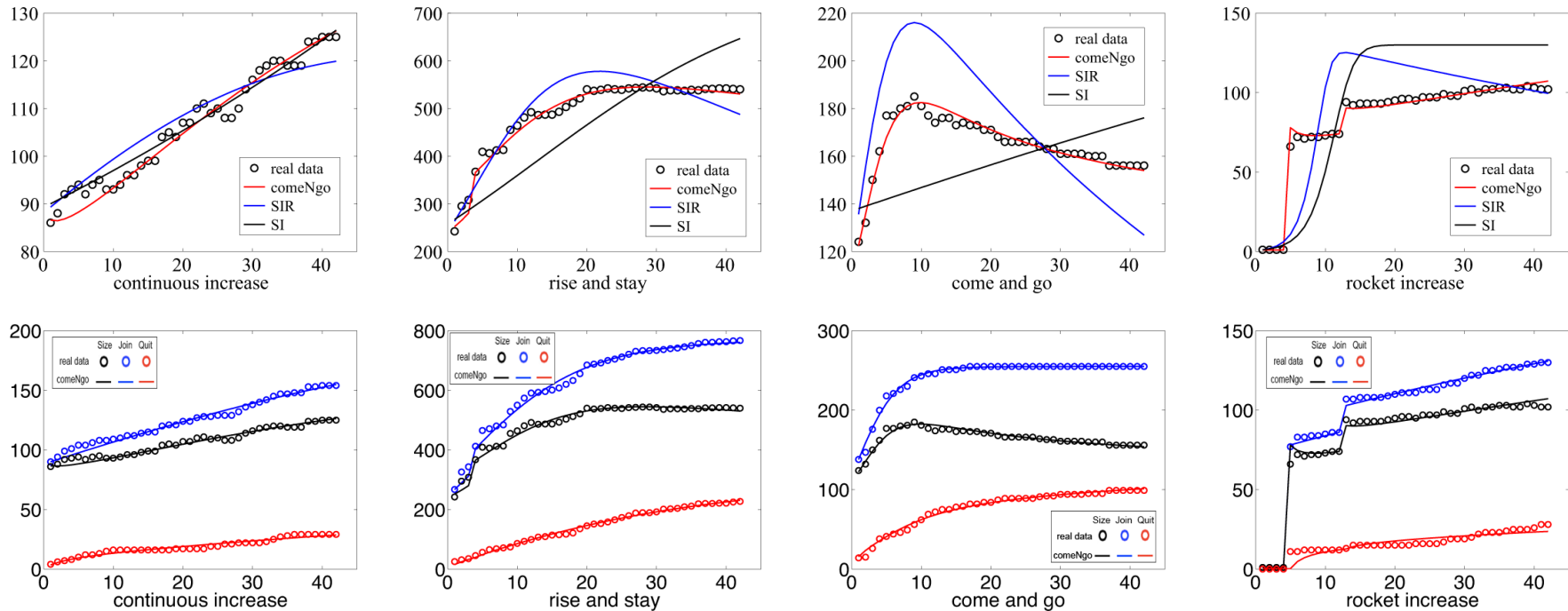
join process: $J'(t) = \frac{dJ}{dt} = \beta(N - J(t))I(t) + \sum \lambda_i \delta(t - t_i)$

quit process: $Q'(t) = \frac{dQ}{dt} = \int_0^t J'(x) f(t - x) dx$

holding time: $f(\tau) = c\tau^{-\alpha} \exp\left(\frac{\gamma_0 \tau^{1-\alpha}}{\alpha - 1}\right)$

Experiment – Fitting Accuracy

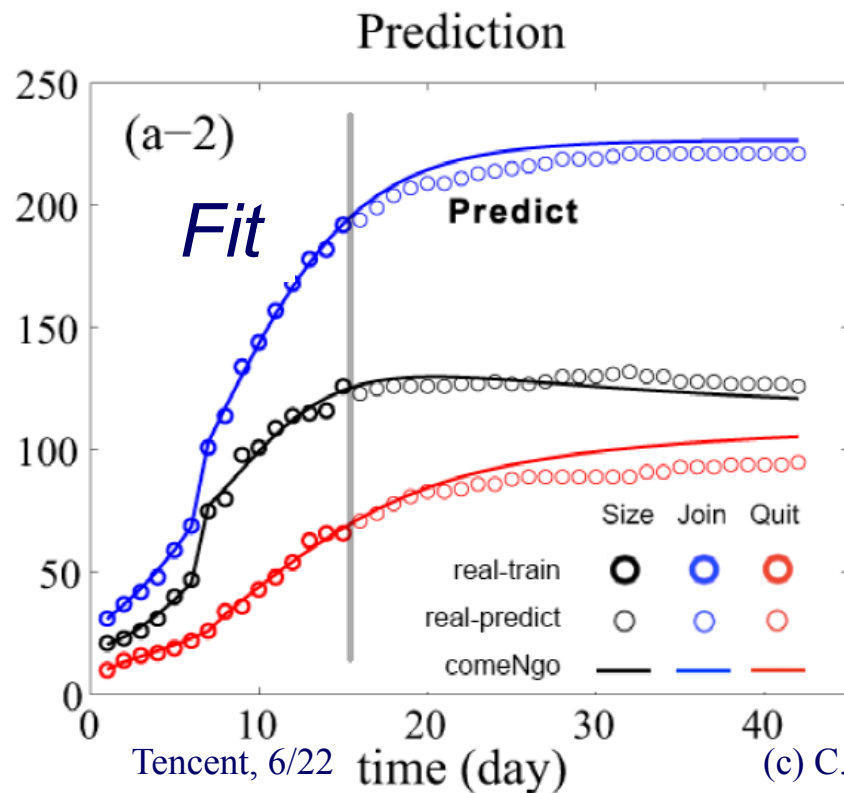
- ▣ Fits all different patterns
- ▣ Fit both join & quit process



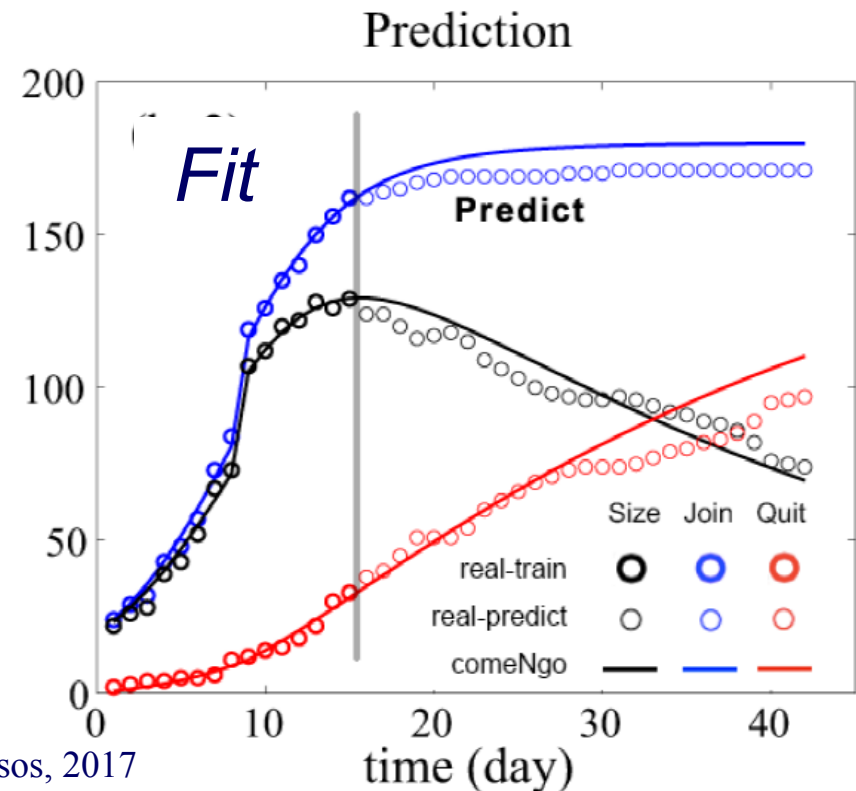
Experiment – Predicting Power

□ Size prediction

- Given early stage data, predict the group size in future



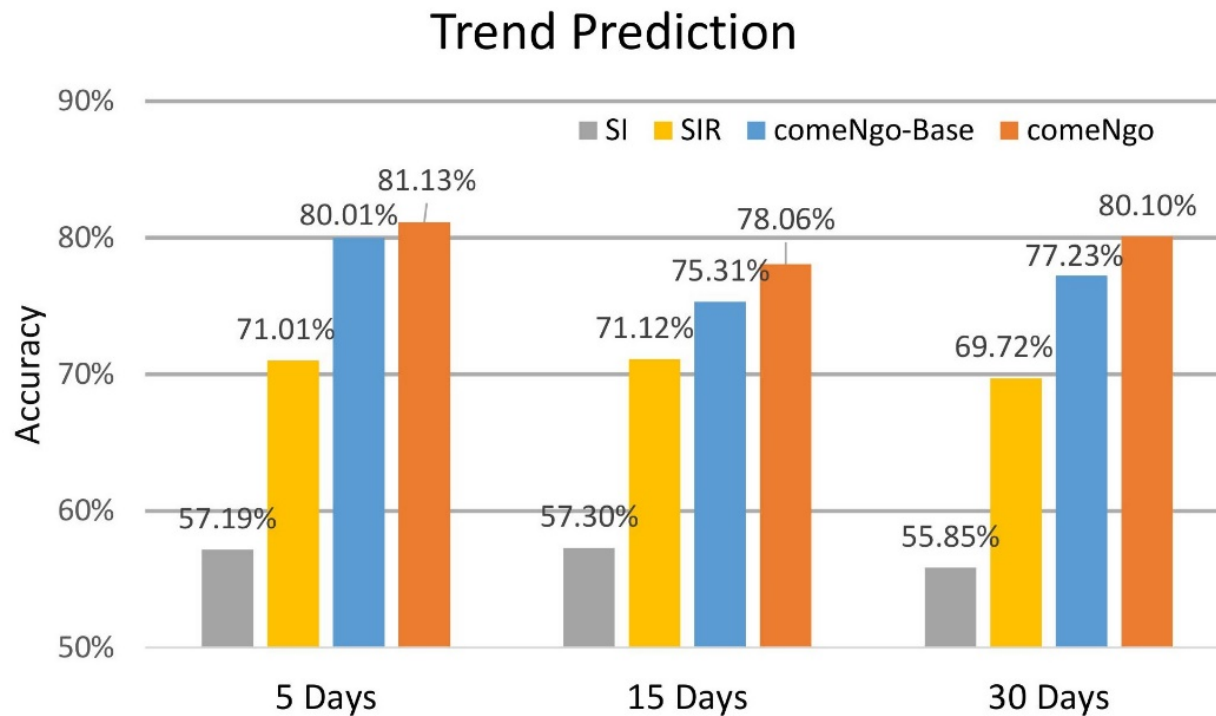
(c) C. Faloutsos, 2017



Experiment – Predicting Power

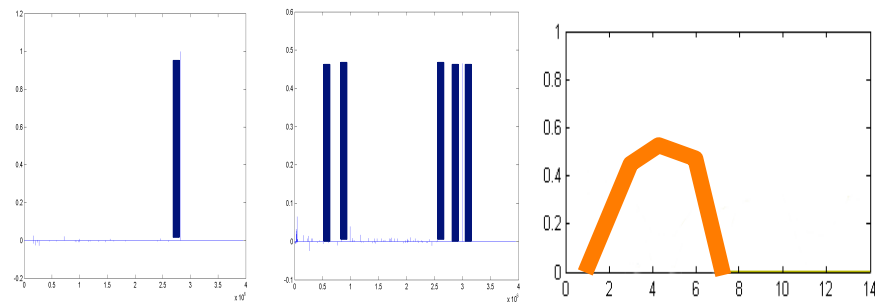
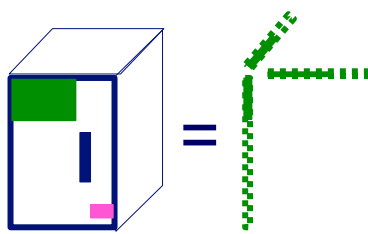
□ Trend prediction

- Given early stage data, predict whether the group will grow
- **14.3%** better accuracy



Part 2: Conclusions

- Time-evolving / heterogeneous graphs \rightarrow tensors
- PARAFAC finds patterns
- Surprising temporal patterns (P.L. growth, comeNgo group evolution)



Roadmap



- Introduction – Motivation
 - Why study (big) graphs?
- Part#1: Patterns in graphs
- Part#2: time-evolving graphs; tensors
- ➔ • Acknowledgements and Conclusions

Thanks



Disclaimer: All opinions are mine; not necessarily reflecting the opinions of the funding agencies

Thanks to: NSF IIS-0705359, IIS-0534205, CTA-INARC; Yahoo (M45), LLNL, IBM, SPRINT, Google, INTEL, HP, iLab

Cast



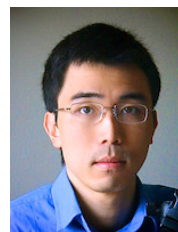
Akoglu,
Leman



Araujo,
Miguel



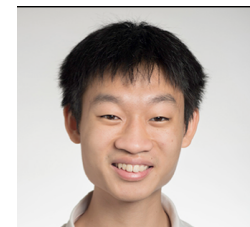
Beutel,
Alex



Chau,
Polo



Eswaran,
Dhivya



Hooi,
Bryan



Kang, U



Koutra,
Danai



Papalexakis,
Vagelis



Shah,
Neil




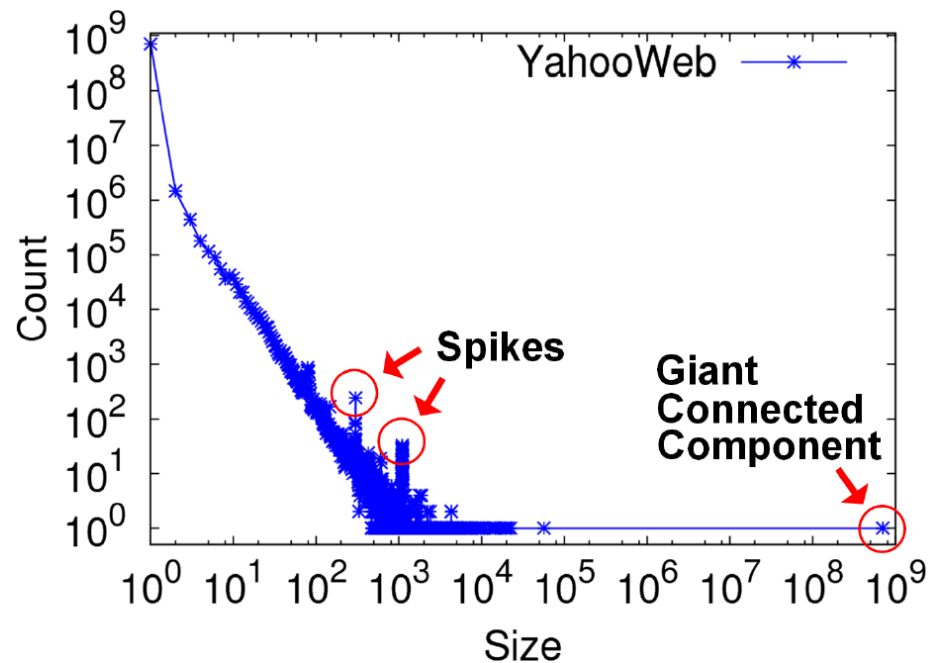
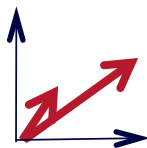
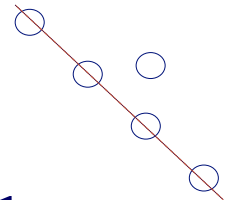
Shin,
Kijung



Song,
Hyun Ah

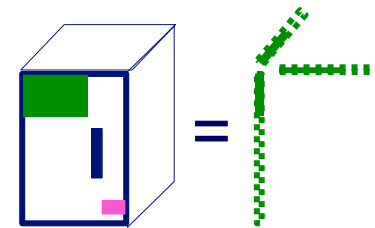
CONCLUSION#1 – Big data

- **Patterns**  **Anomalies**
- **Large datasets reveal patterns/outliers that are invisible otherwise**

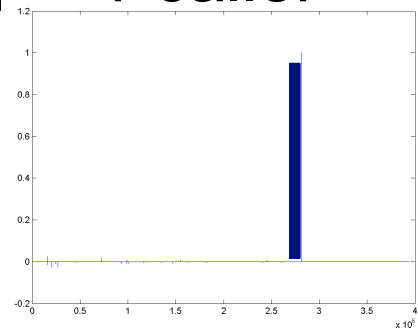


CONCLUSION#2 – tensors

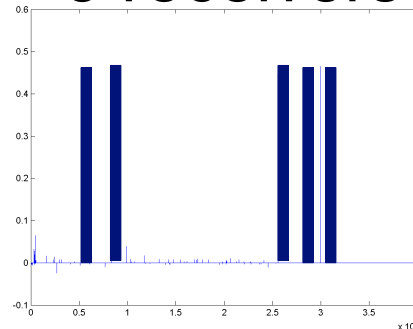
- powerful tool



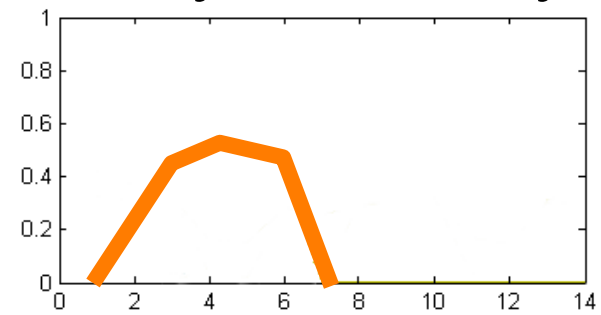
1 caller



5 receivers

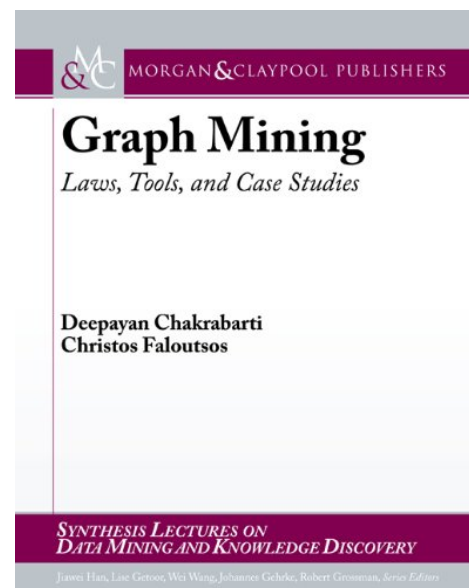


4 days of activity



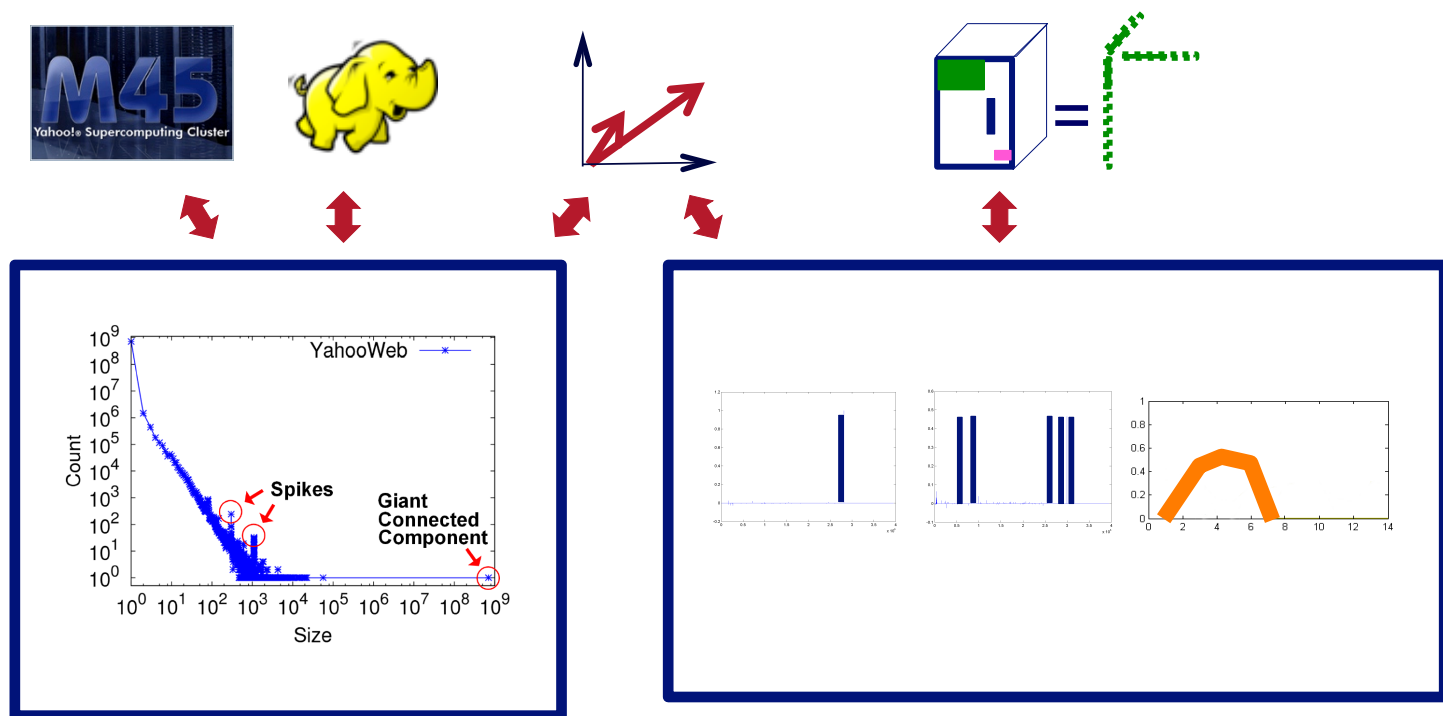
References

- D. Chakrabarti, C. Faloutsos: *Graph Mining – Laws, Tools and Case Studies*, Morgan Claypool 2012
- <http://www.morganclaypool.com/doi/abs/10.2200/S00449ED1V01Y201209DMK006>



TAKE HOME MESSAGE:

Cross-disciplinarity



Thank you!

Cross-disciplinary

